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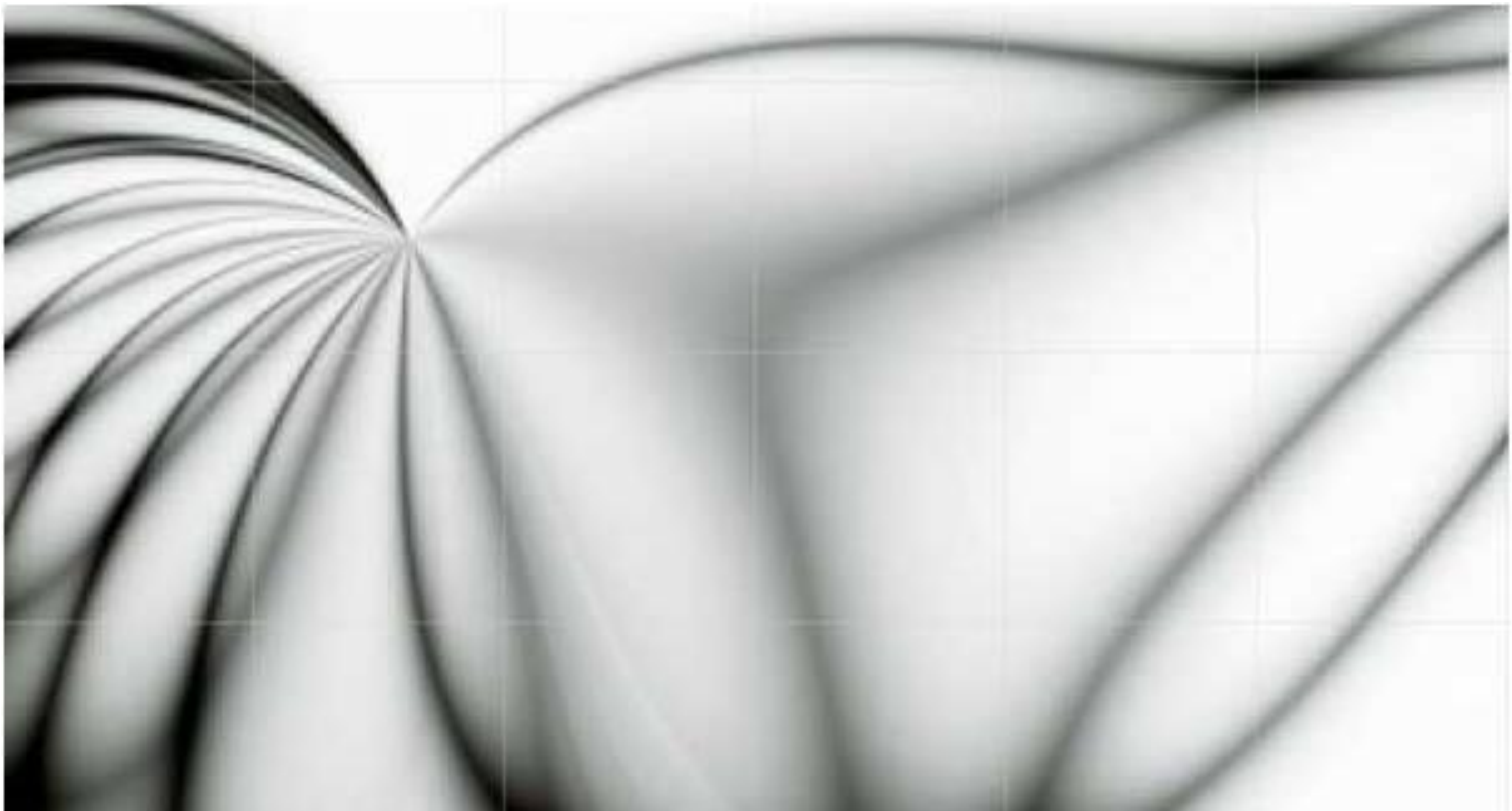

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Analyze the optimal solutions of optimization problems by means of fractional gradient based system using VIM

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Abstract. In this paper, a class of Nonlinear Programming problem is modeled with gradient based system of fractional order differential equations in Caputo's sense. To see the overlap between the equilibrium point of the fractional order dynamic system and the optimal solution of the NLP problem in a longer timespan the Multistage Variational teration Method is applied. The comparisons among the multistage variational iteration method, the variational iteration method and the fourth order Runge-Kutta method in fractional and integer order show that fractional order model and techniques can be seen as an effective and reliable tool for finding optimal solutions of Nonlinear Programming problems.

Keywords: Nonlinear programming problem; penalty function; fractional order dynamic system; variational iteration method; multistage technique.

AMS Classification: 49M37, 90C30, 26A33, 34A08.

1. Introduction

Many problems in modern science and technology are commonly encountered with some class of optimization problems. This is the main reason why optimization is an attractive research area for many scientists in various disciplines. In literature most of efficient methods have been developed for finding the optimal solution of these problems. A detailed and modern discussion for these methods can be found in Luenberger and Sun [1, 2].

Gradient based method is one of these approaches for solving NLP problems. The main idea behind the method is to replace optimization problem to a system of ordinary differential equations (ODEs), which is equipped with optimality conditions, for getting optimal solutions of the NLP problem. The gradient based method was introduced by Arrow and Hurwicz [3], Fiacco and Mccormick [4], Yamashita [5] and Botsaris [6]. In this sense, the method improved by Brown and Bartholomew-Biggs [7], Evtushenko and Zhadan [8] for equality constrained problems. Schropp [9] and Wang et al. [10] improved gradient based method for nonlinear constrained problem using slack variables and Lagrangian formula. Recently, Jin et al. [11, 12], Shikhman and Stein [13] and Özdemir and Evirgen [14] have considered a gradient based method for optimization problems.

The fractional calculus, which is one of the other important research areas of science, has been attracting the attention of many researchers because of its interdisciplinary application and physical meaning, e.g. [15]. Most of the studies in this area have mainly focused on developing analytical and numerical methods for solving different kind of fractional differential equations (FDEs) in science. Recently, several methods have been proposed for this aim and applied to different areas, e.g. [16–23]. The variational iteration method (VIM) is one of these methods, which was introduced by He [24], and applied to FDEs [25]. Momani [26, 27], also used VIM for solving some FDEs both linear and nonlinear. Only then, multistage technique is adapted to the VIM for getting the essential behavior of the differential equation system for large time t. This technique was introduced by Batiha et al. [28] for a class of nonlinear system of ODEs and applied to delay differential equations by Gökdoğan [29]. In recent years, a lot of modifications and developments have been proposed for the variational iteration method. For example, in calculation of the Lagrange multiplier [30–32], by using a local fractional operators [33, 34] and Laplace transform [35].

In this paper, we construct a fractional gradient based system for solving equality constrained optimization problem. The proposed system shows that the steady state solutions $x(t)$ of the system approximate to the optimal solutions x^* of optimization problem on a continuous path as $t \to \infty$. The variational iteration method (VIM) and multistage technique are used for achieving the intended results.

The paper is organized as follows. In Section 2, some basic theory and results, which will be useful subsequently in this paper, are discussed. In Section 3, the MVIM is adapted to the fractional gradient based system for solving optimization problem. The applicability and efficiency of MVIM is illustrated by comparison among VIM and fourth order Runge-Kutta (RK4) method on some test problems, in Section 4. And finally the paper is concluded with a summary in Section 5.

2. Preliminaries

2.1. Optimization problem

Consider the nonlinear programming problem with equality constraints:

$$\min \ f(x) \quad \text{s.t.} \quad x \in X, \tag{ENLP}$$

where the feasible set is assumed to be non-empty and is defined by

$$X = \{x \in \mathbb{R}^n : \ h(x) = 0\},$$

and $f : \mathbb{R}^n \longrightarrow \mathbb{R}$, $h : \mathbb{R}^n \longrightarrow \mathbb{R}^p$ are C^2 functions. The idea of penalty methods is to approximate a constrained optimization problem by an unconstrained optimization problem. A well-known penalty function for the problem (ENLP) is given by

$$F(x,\eta) = f(x) + \eta \frac{1}{\gamma} \sum_{i=1}^{p} (h_i(x))^{\gamma} \tag{1}$$

where $\gamma > 0$ is a constant and $\eta > 0$ is an auxiliary penalty variable. It can be shown that the solutions of the constrained problem (ENLP) are solutions of of the following unconstrained one,

$$\min \ F(x,\eta) \quad \text{s.t.} \quad x \in \mathbb{R}^n. \tag{UP}$$

under some conditions and when $\eta > 0$ is sufficiently large. One of the main results connecting the minimizers of the constrained problem (ENLP) and unconstrained problem (UP) is as follows.

Theorem 1. *[1, pp.404] Let $\{x_k\}$ be a sequence generated by the penalty method. Then any limit point of the sequence is a solution to the constrained problem.*

2.2. Fractional calculus

Now we will give some definitions and properties of the fractional calculus [15]. We begin with the Riemann-Liouville definition of the fractional integral of order $\alpha > 0$.

Definition 1 (Riemann-Liouville Fractional Integral). *The Riemann-Liouville fractional integral operator of order $\alpha > 0$, of a function $f(x)$, is given as*

$$I^{\alpha} f(x) = \frac{1}{\Gamma(\alpha)} \int_0^x (x-t)^{\alpha-1} f(t)dt, \qquad x > 0,$$

where $\Gamma(.)$ is the well-known Euler's gamma function.

Several definitions of a fractional derivative such as Riemann-Liouville, Caputo, Grünwald-Letnikov, Weyl, Marchaud and Riesz have been proposed. In the following section we formulate the problem in the Caputo sense, which is defined as:

Definition 2 (Caputo Fractional Derivative). *The fractional derivative of $f(x)$ in the Caputo sense with $m-1 < \alpha \leqslant m$, $m \in \mathbb{N}$, is defined as*

$$\begin{aligned} {}_cD^{\alpha} \ f(x) &= I^{m-\alpha} D^m f(x) \\ &= \tfrac{1}{\Gamma(m-\alpha)} \int_0^x (x-t)^{m-\alpha-1} f^{(m)}(t)dt, \quad x > 0 \end{aligned}$$

where $f^{(m)}(.)$ is the usual integer m order derivative of function f.

Note that Riemann-Liouville fractional integral and Caputo fractional derivative satisfy following elementary properties:

Lemma 1. *If $f(x) \in C^m[0,\infty)$ and $m-1 < \alpha \leqslant m$, $m \in \mathbb{N}$, then*

$$I^\alpha D^\alpha f(x) = f(x) - \sum_{s=0}^{m-1} f^{(s)}(0^+)\frac{x^s}{s!}, \quad x > 0, \quad (2)$$

and

$$D^\alpha I^\alpha f(x) = f(x). \quad (3)$$

2.3. Variational iteration method

To describe the solution procedure for variational iteration method (VIM), we consider the following general nonlinear differential equation

$$L(u(t)) + N(u(t)) = g(t) \quad (4)$$

where L is a linear operator, N is a nonlinear operator and $g(t)$ is a known analytical function. According to the He's variational iteration method [24,25,36], we can construct a correction functional for (4) as follows,

$$\begin{aligned} &u_{i,k+1}(t) = u_{i,k}(t) \\ &+ \int_{t_0}^{t} \lambda(\tau)\left\{L(u_{i,k}(\tau)) + N(\tilde{u}_{i,k}(\tau)) - g(\tau)\right\} d\tau, \\ &n \geqslant 0, \end{aligned} \quad (5)$$

where λ is a general Lagrange multiplier, which can be identified optimally via variational theory, u_n is the $n-$th approximate solution. Here $\tilde{u}_n$ is considered as a restricted variation which means $\delta\tilde{u}_n = 0$. The accuracy of the result fully depends on the identification of Lagrange multiplier and initial condition u_0. Finally, the exact solution may be obtained as

$$u_i(t) = \lim_{k\to\infty} u_{i,k}(t).$$

3. Fractional gradient based system

Consider the NLP problem with equality constraints defined by (ENLP). Generally, these type of problems are usually solved by transforming to the unconstrained optimization problem (UP). In the next step, some traditional methods or dynamical system approaches are used to get optimal solution of the unconstrained optimization problem.

In this article a fractional gradient based dynamical system approach is handled for obtaining optimal solutions of (ENLP) by the help of MVIM. The fractional derivative is described in the Caputo sense, because the initial conditions have the same physical meanings according to the integer order differential equations. The fractional gradient based approach for solving optimization problems was introduced by Evirgen and Özdemir [37, 38]. Recently, Khader et al. [39–41] used fractional finite difference method and Chebyshev Collocation Method for solving system of FDEs, which are generated by optimization problem.

Utilizing the quadratic penalty function (1) to the equality constrained optimization problem (ENLP) with $\gamma = 2$, the gradient based fractional dynamical system can be described by the following form:

$$\begin{aligned} &{}_cD^\alpha x(t) = -\nabla_x F(x,\eta) \quad &, m-1 < \alpha \leqslant m \\ &x^{(s)}(0) = x_0^{(s)} &, 0 \leqslant s \leqslant m-1 \end{aligned} \quad (6)$$

where $\nabla_x F(x,\eta)$ is the gradient vector of the quadratic penalty function (1) with respect to the $x \in \mathbb{R}^n$.

Definition 3. *A point x_e is called an equilibrium point of (6) if it satisfies the right hand side of the equation (6).*

The gradient based fractional dynamic system (6) can be simplified for the readers' convenience as follows,

$$\begin{aligned} &{}_cD^\alpha x_i(t) = g_i(t,\eta,x_1,x_2,...,x_n), \\ &i = 1,2,...,n. \end{aligned} \quad (7)$$

The stable equilibrium point of the fractional order system (7) is acquired with the MVIM algorithm. The MVIM can be described by some modifications of VIM. To ensure the validity of the approximations of the VIM for large t, we need to treat (5) under arbitrary initial conditions. Therefore, we divide $[t_0, t)$ interval into subinterval of equal length Δt as $[t_0, t_1)$, $[t_1, t_2)$, ..., $[t_{j-1}, t_j = t)$.

The correction functional for the fractional nonlinear differential equations system (7) according to the MVIM can be approximately constructed as

$$x_{i,k+1}(t) = x_{i,k}(t) + \int_{t^*}^{t} \lambda_i(\tau)\left({}_cD^{\alpha}x_{i,k}(\tau) - g_i\left(\tilde{x}_{1,k}(\tau),...,\tilde{x}_{n,k}(\tau)\right)\right)d\tau \tag{8}$$

where t^* is the left end point of each subinterval, λ_i, $i = 1,2,...,n$ are general Lagrange multiplier, which can be identified optimally via variational theory, and $\tilde{x}_1, \tilde{x}_2, ..., \tilde{x}_n$ denote restricted variations that $\delta\tilde{x}_i = 0$.

Taking variation with respect to the independent variable x_i, $i = 1,2,...,n$ with $\delta x_i(t^*) = 0$,

$$\delta x_{i,k+1}(t) = \delta x_{i,k}(t) + \delta\int_{t^*}^{t} \lambda_i(\tau)\left({}_cD^{\alpha}x_{i,k}(\tau) - g_i\left(\tilde{x}_{1,k}(\tau),...,\tilde{x}_{n,k}(\tau)\right)\right)d\tau$$

and consequently we get following stationary conditions:

$$\lambda_i'(\tau)|_{\tau=t} = 0,$$
$$1 + \lambda_i(\tau)|_{\tau=t} = 0, \quad i = 1,2,...,n.$$

Therefore, the Lagrange multipliers can be easily identified as

$$\lambda_i = -1, \quad i = 1,2,...,n. \tag{9}$$

Substituting Lagrange multipliers (9) into the correctional functional (8), we acquire the following MVIM formula

$$x_{i,k+1}(t) = x_{i,k}(t) - \int_{t^*}^{t} \left({}_cD^{\alpha}x_{i,k}(\tau) - g_i\left(\tilde{x}_{1,k}(\tau),...,\tilde{x}_{n,k}(\tau)\right)\right)d\tau, \tag{10}$$

for $i = 1,2,...,n$. If we begin with initial conditions $x_{i,0}(t^*) = x_{i,0}(t_0) = x_i(0)$, the iteration formula of the multistage VIM (10) can be carried out in every subinterval of equal length Δt, and so all solutions $x_{i,k}(t)$, $(i = 1,2,...,n;\ k = 1,2,...)$ are completely determined.

4. Numerical implementation

To illustrate the effectiveness of the MVIM according to the VIM and fourth order Runge-Kutta (RK4) method, some test problems are borrowed from Hock and Schittkowski [42, 43].

Example 1. *Consider the following nonlinear programming problem [43, Problem No: 216],*

$$\begin{aligned} &\textit{minimize} && f(x) = 100\left(x_1^2 - x_2\right)^2 + (x_1 - 1)^2, \\ &\textit{subject to} && h(x) = x_1(x_1 - 4) - 2x_2 + 12 = 0. \end{aligned} \tag{11}$$

Firstly, we convert it to an unconstrained optimization problem with quadratic penalty function (1) for $\gamma = 2$, then we have

$$F(x,\eta) = 100\left(x_1^2 - x_2\right)^2 + (x_1 - 1)^2 + \frac{1}{2}\eta\left(x_1(x_1 - 4) - 2x_2 + 12\right)^2,$$

where $\eta \in \mathbb{R}^+$, $\eta \to \infty$ is an auxiliary penalty variable. The corresponding nonlinear system of FDEs from (6) is defined as

$$\left.\begin{aligned} {}_cD^{\alpha}x_1(t) &= -400(x_1^2 - x_2)x_1 - 2(x_1 - 1) \\ &\quad - \eta(2x_1 - 4)(x_1^2 - 4x_1 - 2x_2 + 12), \\ {}_cD^{\alpha}x_2(t) &= 200(x_1^2 - x_2) \\ &\quad + 2\eta(x_1^2 - 4x_1 - 2x_2 + 12), \\ x_1(0) &= 0,\ x_2(0) = 0, \end{aligned}\right\} \tag{12}$$

where $0 < \alpha \leqslant 1$. By using the MVIM with auxiliary penalty variable $\eta = 800$, step size $\Delta T = 0.00001$ and Lagrange multipliers $\lambda_i = -1$; the terms of the MVIM solutions for fractional order are acquired by

$$x_{i,k+1}(t) = x_{i,k}(t) - \int_{t^*}^{t} \left({}_cD^{\alpha}x_{i,k}(\tau) - g_i\left(\tilde{x}_{1,k}(\tau),...,\tilde{x}_{n,k}(\tau)\right)\right)d\tau,$$

for $i = 1,2$. In the Figure 1 and Table 1, we clearly see that the fractional MVIM approach the optimal solutions of optimization problem (11) faster than the other methods. Furthermore, MVIM requires only one iteration to reach the optimal solutions for fractional dynamical system. Contrary to this, MVIM for integer order dynamical system requires two iterations.

Example 2. *Consider the nonlinear programming problem [42, Problem No: 79],*

$$\begin{aligned} &\textit{minimize} \quad f(x) = (x_1 - 1)^2 + (x_1 - x_2)^2 \\ &\qquad + (x_2 - x_3)^2 + (x_3 - x_4)^4 + (x_4 - x_5)^4, \\ &\textit{subject to} \\ &\qquad h_1(x) = x_1 + x_2^2 + x_3^3 - 2 - 3\sqrt{2} = 0, \\ &\qquad h_2(x) = x_2 - x_3^2 + x_4 + 2 - 2\sqrt{2} = 0, \\ &\qquad h_3(x) = x_1x_5 - 2 = 0. \end{aligned} \tag{13}$$

This is a practical problem whose exact solution is not known, but the expected optimal solution is $x_1^* = 1.191127$, $x_2^* = 1.362603$, $x_3^* = 1.472818$, $x_4^* = 1.635017$, $x_5^* = 1.679081$. *Following the discussion in Section 3, again we set the quadratic penalty function (1) according to the NLP problem (13)*

$$\begin{aligned} F(x,c) &= f(x) + \tfrac{1}{2}\eta \textstyle\sum_{i=1}^{5} (h_i(x))^2 \\ &= (x_1-1)^2 + (x_1-x_2)^2 + (x_2-x_3)^2 \\ &\quad + (x_3-x_4)^4 + (x_4-x_5)^4 \\ &\quad + \tfrac{1}{2}\eta\left(x_1 + x_2^2 + x_3^3 - 2 - 3\sqrt{2}\right)^2 \\ &\quad + \tfrac{1}{2}\eta\left(x_2 - x_3^2 + x_4 + 2 - 2\sqrt{2}\right)^2 \\ &\quad + \tfrac{1}{2}\eta\left(x_1x_5 - 2\right)^2, \end{aligned}$$

where $\eta \in \mathbb{R}^+$ *and* $\eta \to \infty$. *The corresponding nonlinear system of FDEs can be obtained by way of (6) as follows,*

$$\left.\begin{aligned} {}_cD^{\alpha}x_i(t) &= \nabla_{x_i} f(x) + \eta \textstyle\sum_{i=1}^{5} \nabla_{x_i} h(x) h_i(x), \\ x_i(0) &= 2, \ i = 1,2,3,4,5, \end{aligned}\right\} \tag{14}$$

where $0 < \alpha \leqslant 1$ *is order of fractional derivative. Finally, the MVIM algorithm (10) is adapted to the fractional dynamical system (14) with auxiliary penalty variable* $\eta = 600$, *step size* $\Delta T = 0.00001$ *and Lagrange multipliers* $\lambda_i = -1$, $i = 1,2,3,4,5$. *Tables 2-5 show the approximate solutions for optimization problem (13) obtained by different values of* α *by using methods VIM, MVIM and RK4. The MVIM for the dynamical system of integer and non-integer order is obtained very close solutions to the expected approximate solutions. Again, it should be noted that the MVIM for fractional order system is used by one iteration to reach optimal solutions.*

Example 3. *Consider the nonlinear programming problem [43, Problem No: 320],*

$$\begin{aligned} &\textit{minimize} \quad f(x) = (x_1-20)^2 + (x_2+20)^2, \\ &\textit{subject to} \quad h(x) = \frac{x_1^2}{100} + \frac{x_2^2}{4} - 1 = 0. \end{aligned} \tag{15}$$

This is a practical problem and the exact solution is not known, but the expected optimal solution is $x_1^* = 9.395$, $x_2^* = -0.6846$. *Firstly, the quadratic penalty function (1) is used to get unconstrained optimization problem as follows*

$$\begin{aligned} F(x,\eta) &= (x_1-20)^2 + (x_2+20)^2 \\ &\quad + \tfrac{1}{2}\eta\left(\frac{x_1^2}{100} + \frac{x_2^2}{4} - 1\right)^2, \end{aligned}$$

where $\eta \in \mathbb{R}^+$ *and* $\eta \to \infty$ *and so that the nonlinear system of FDEs can be given by*

$$\left.\begin{aligned} {}_cD^{\alpha}x_1(t) &= 2x_1 - 40 \\ &\quad + \eta\left(\tfrac{1}{5000}x_1^3 + \tfrac{1}{200}x_1x_2^2 - \tfrac{1}{50}x_1\right), \\ {}_cD^{\alpha}x_2(t) &= 2x_2 + 40 \\ &\quad + \eta\left(\tfrac{1}{200}x_2x_1^2 + \tfrac{1}{8}x_2^3 - \tfrac{1}{2}x_2\right), \\ x_1(0) &= 0, \ x_2(0) = 0. \end{aligned}\right\} \tag{16}$$

where $0 < \alpha \leqslant 1$ *is order of fractional derivative. The optimal solutions of problem (15) are achieved by using the MVIM iteration formula (10) with auxiliary penalty variable* $\eta = 10^6$, *step size* $\Delta T = 0.5 \times 10^{-6}$ *and Lagrange multipliers* $\lambda_i = -1$, $i = 1,2$. *As we see in the previous examples, the approximate solutions in Table 6 obviously show that the MVIM for fractional order system is more effective than the other methods with low iteration calculation.*

5. Conclusions

The main goal of this work is to create a bridge between two attractive research areas, which are optimization and fractional calculus. In this sense, the intersection point is composed through the instrument of fractional order differential equations (FDEs) system. The system of FDEs is become appropriate to solve the underlying optimization problem by means of optimality conditions.

Furthermore, the variational iteration method (VIM) and multistage strategy are successfully composed to obtain the essential behavior of the system of FDEs, which is generated by nonlinear programming (NLP) problems. The numerical comparisons among the fourth order Runge-Kutta (RK4), the MVIM ($\alpha = 1$ and $\alpha = 0.9$) and VIM ($\alpha = 0.9$) verifies the efficiency of the MVIM as a promising tool for solving NLP problems.

The MVIM yields a very rapid convergent series solution according to the VIM and RK4, and usually a few iterations lead to accurate approximation of the exact solution. Also, the numerical comparisons show that the fractional order gradient based system is more suitable and stable than the integer order dynamical system to get optimal solutions of NLP problems.

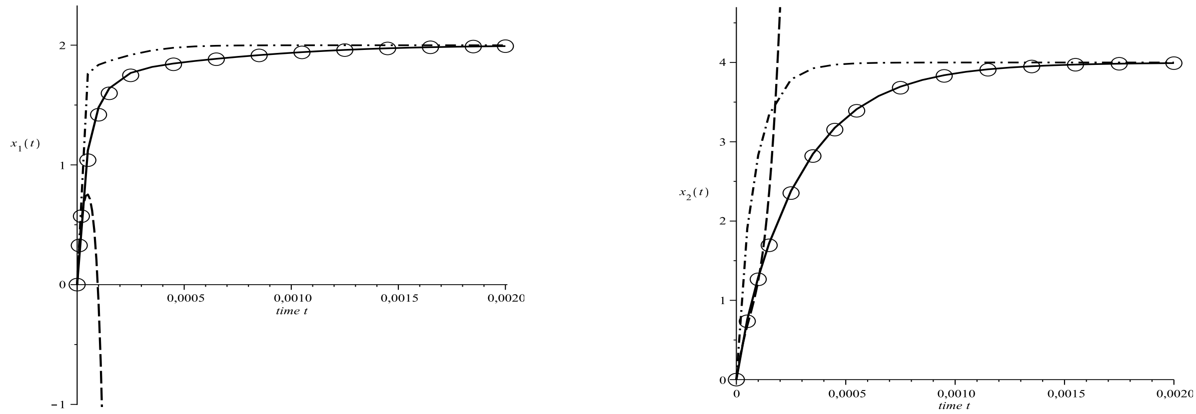

Figure 1. Comparison of $x(t)$ for problem (11). *Dash*: VIM($\Delta T = 0.00001$) for $\alpha = 0.9$, *Dashdot*: MVIM($\Delta T = 0.00001$) for $\alpha = 0.9$, *Solidline*: MVIM($\Delta T = 0.00001$) for $\alpha = 1$, ◯: RK4($\Delta T = 0.00001$) for $\alpha = 1$

Table 1. *Comparison of $x(t)$ for problem (11) between VIM and MVIM with RK4 solutions for different value of α.*

	VIM ($\alpha = 0.9$)		MVIM ($\alpha = 0.9$)		MVIM ($\alpha = 1$)		RK4 ($\alpha = 1$)	
t	$x_1(t)$	$x_2(t)$	$x_1(t)$	$x_2(t)$	$x_1(t)$	$x_2(t)$	$x_1(t)$	$x_2(t)$
0.000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.001	-574009.98	8464.21	1.9991	3.9996	1.9360	3.8628	1.9338	3.8549
0.002	$-0.72E+7$	57043.72	1.9993	3.9998	1.9921	3.9922	1.9916	3.9915
0.003	$-0.31E+8$	172707.57	1.9993	3.9998	1.9987	3.9992	1.9986	3.9992
0.004	$-0.90E+8$	378083.92	1.9993	3.9998	1.9992	3.9998	1.9992	3.9997
0.005	$-0.20E+9$	693522.64	1.9993	3.9998	1.9993	3.9998	1.9993	3.9998

Table 2. *The value of $x(t)$ for problem (13) obtained from VIM ($\alpha = 0.9$).*

	VIM ($\alpha = 0.9$)				
t	$x_1(t)$	$x_2(t)$	$x_3(t)$	$x_4(t)$	$x_5(t)$
0.000	2.000000	2.000000	2.000000	2.000000	2.000000
1.000	-7334.9372	-18838.9751	-60138.6394	518.8163	-2493.4099
2.000	-13689.2090	-35156.5027	-112224.4015	966.4133	-4654.5995
3.000	-19718.7730	-50640.1930	-161648.5441	1391.1378	-6705.3508
4.000	-25546.6995	-65606.0859	-209419.8703	1801.6589	-8687.5220
5.000	-31229.1380	-80198.3714	-255998.6368	2201.9318	-10620.2104
6.000	-36798.2638	-94499.6738	-301648.5816	2594.2228	-12514.3593

Table 3. *The value of $x(t)$ for problem (13) obtained from MVIM ($\alpha = 0.9$).*

	MVIM ($\alpha = 0.9$)				
t	$x_1(t)$	$x_2(t)$	$x_3(t)$	$x_4(t)$	$x_5(t)$
0.000	2.000000	2.000000	2.000000	2.000000	2.000000
1.000	1.222306	1.390861	1.455863	1.557010	1.636218
2.000	1.194457	1.365383	1.471128	1.627228	1.674395
3.000	1.191455	1.362867	1.472646	1.634220	1.678618
4.000	1.191154	1.362620	1.472796	1.634908	1.679042
5.000	1.191125	1.362596	1.472811	1.634976	1.679084
6.000	1.191122	1.362593	1.472812	1.634983	1.679088

Table 4. *The value of $x(t)$ for problem (13) obtained from MVIM* ($\alpha = 1$).

	MVIM ($\alpha = 1$)				
t	$x_1(t)$	$x_2(t)$	$x_3(t)$	$x_4(t)$	$x_5(t)$
0.000	2.000000	2.000000	2.000000	2.000000	2.000000
1.000	1.204109	1.351636	1.475385	1.653569	1.660933
2.000	1.192243	1.360691	1.473436	1.638730	1.677503
3.000	1.190989	1.362112	1.473034	1.636119	1.679274
4.000	1.190965	1.362415	1.472911	1.635451	1.679308
5.000	1.191032	1.362513	1.472860	1.635202	1.679214
6.000	1.191076	1.362554	1.472836	1.635090	1.679153

Table 5. *The value of $x(t)$ for problem (13) obtained from RK4* ($\alpha = 1$).

	RK4 ($\alpha = 1$)				
t	$x_1(t)$	$x_2(t)$	$x_3(t)$	$x_4(t)$	$x_5(t)$
0.000	2.000000	2.000000	2.000000	2.000000	2.000000
1.000	1.201627	1.349464	1.476662	1.659517	1.664366
2.000	1.191021	1.359663	1.474053	1.641579	1.679226
3.000	1.190381	1.361610	1.473337	1.637515	1.680133
4.000	1.190664	1.362168	1.473060	1.636139	1.679733
5.000	1.190884	1.362391	1.472934	1.635542	1.679424
6.000	1.191002	1.362494	1.472872	1.635258	1.679256

Table 6. *Comparison of $x(t)$ for problem (15) between VIM and MVIM with RK4 solutions for different value of α.*

	VIM ($\alpha = 0.9$)		MVIM ($\alpha = 0.9$)		MVIM ($\alpha = 1$)		RK4 ($\alpha = 1$)	
t	$x_1(t)$	$x_2(t)$	$x_1(t)$	$x_2(t)$	$x_1(t)$	$x_2(t)$	$x_1(t)$	$x_2(t)$
0.000	0.000	0.0000	0.000	0.0000	0.000	0.0000	0.000	0.0000
0.050	−23349.61	748.5017	7.937	−1.2166	1.868	−1.9647	1.868	−1.9647
0.100	−410.851.52	6494.9196	9.314	−0.7281	3.491	−1.8741	3.491	−1.8741
0.150	$-0.21E+07$	22548.2227	9.394	−0.6857	4.889	−1.7446	4.889	−1.7446
0.200	$-0.69E+07$	54217.3799	9.396	−0.6846	6.076	−1.5884	6.076	−1.5884
0.250	$-0.17E+08$	106811.3600	9.396	−0.6846	7.062	−1.4160	7.062	−1.4160

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A stochastic mathematical model to locate field hospitals under disruption uncertainty for large-scale disaster preparedness

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Abstract. In this study, we consider field hospital location decisions for emergency treatment points in response to large scale disasters. Specifically, we developed a two-stage stochastic model that determines the number and locations of field hospitals and the allocation of injured victims to these field hospitals. Our model considers the locations as well as the failings of the existing public hospitals while deciding on the location of field hospitals that are anticipated to be opened. The model that we developed is a variant of the P-median location model and it integrates capacity restrictions both on field hospitals that are planned to be opened and the disruptions that occur in existing public hospitals. We conducted experiments to demonstrate how the proposed model can be utilized in practice in a real life problem case scenario. Results show the effects of the failings of existing hospitals, the level of failure probability and the capacity of projected field hospitals to deal with the assessment of any given emergency treatment system's performance. Crucially, it also specifically provides an assessment on the average distance within which a victim needs to be transferred in order to be treated properly and then from this assessment, the proportion of total satisfied demand is then calculated.

Keywords: Stochastic programming; humanitarian logistics; reliable facility location; field hospital; Istanbul.
AMS Classification: 90C11, 90C15.

1. Introduction

Disasters have always been parts of human life and continued to be a steady increase in the number and severity of natural disasters in recent years ([1-2]). Disaster can be defined as any expected or unexpected incident that causes catastrophic injuries to humans' life, or damage to the economy and environment. Past case studies relating to this specific area have shown that effective and well-organized preparation helps in decreasing the catastrophic effect of disasters [3]. Disasters can be natural (such as earthquakes, floods, tsunamis, storms or hurricanes) or man-made (such as terrorist attacks, industrial accidents, or war) ([3-5]). Both types of disasters can cause major economic loss and human fatalities. A recent earthquake occurred on April 2015 in Nepal (with 7.9 magnitude) and caused about 9,000 fatalities. Another example of such a disaster occurred in 2011 with 9.0 magnitude earthquake and tsunami in Japan, which caused around 19,000 fatalities and huge economic loss [6]. The earthquake that hit Haiti in January 2010 caused an estimated 230,000 deaths and 250,000 injuries [7]. Another tsunami occurred in the Indian Ocean on December 26th, 2004, which had a 9.0 magnitude which left in its wake a total of 229,866 people lost: 186,983 officially identified as dead and another 42,883 missing [3]. In relation to man-made disasters, a terrorist attack occurred on September 11, 2001 when terrorists attacked the World Trade Centre in New York, which caused

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the death of 2,750 people and another 2,260 injures. The economic loss suffered as a result of these disasters amounted to trillions of dollars. Economic damage is recoverable while fatalities are not. Therefore, locating the injured people and moving them to the nearest emergency service/hospital for timely treatment is a vital process. Under a disaster scenario it is critical to have available and well-functioning hospitals close to areas where injured people are most densely located.

Even though we cannot forecast a disaster with significant certainty, emergency preparedness is crucial to eliminate or at least minimize potential fatalities ([3], [8]). Succeeding a disaster, hospitals can expect a sudden increase of injured victims which can easily overwhelm and crush a hospitals capacity. One way to be prepared for disasters, such as a bio-terrorist attack or an epidemic, is to have excess capacity as mentioned in [9] and [10]. Since the time and severity of disasters cannot be anticipated, the number of the injured victims is uncertain as highlighted in the author's research study in [12].

The number of injured victims is not the only uncertainty in a disaster. In addition to this, it is uncertain how much disruption will be experienced in hospitals. Therefore, in order to be fully prepared, taking these two uncertainties into account is vital for emergency management systems. Current literature considers the uncertainty in the number of injuries and the uncertainty regarding the disruption of hospitals. However, there is still lack of studies that considers both uncertainties at the same. Therefore, in this study we consider the disruption of hospitals to assist emergency management systems managers.

Despite the importance of field hospitals in mitigating the effects of disasters, there has been a lack of research in the area even there are some valuable studies in the related field ([2] , [5], [7], [12-18]) and some other analysis on capacity planning of hospitals for disaster preparedness ([3], [19-20]). In this paper, we want to separate and distinguish the general humanitarian relief chain from planning field hospitals because medical supplies and treatment are more important than other supplies, especially during the first 72 hours after a disaster occurs. This new area of research provided us with the requisite motivation to seek to determine the optimum number and location for these field hospitals in Zeytinburnu/Istanbul. We aimed to achieve the following objectives:

• To determine the optimal number of field hospitals in Zeytinburnu/Istanbul to satisfy all demand (injured disaster victims) whilst considering existing hospitals and their capacities.
• To determine the optimal locations of field hospitals in order to treat injured people on time.
• Optimal allocation of the demand to the hospitals (both to public hospitals and field hospitals).

This work focuses on developing and analyzing a model for field hospitals' locations and capacity allocation for regions subject to large-scale disasters. While achieving these objectives, we took the failure (disruption) of the existing public hospitals into account. A scenario based two-stage stochastic mathematical model was developed and the results were thereafter presented.

The paper is organized as follows: Section 2 reviews the relevant literature relating to the field hospitals, disaster relief chain and scenario based stochastic programs. Section 3 presents the stochastic P-median mathematical model. Section 4 describes the presented model and results. Analyses are also presented in Section 4. The final section includes conclusions and directions for future researches.

2. Literature review

Disaster literature is somewhat limited when compared with other fields of Operations Research. Nevertheless, the number of studies on disaster has increased in recent years. Altay and Green [21] identified 77 articles that have been published in OR/MS related journals out of a 109 disaster management studies. They stated that 40% of these 109 articles were published between 1990 and 2000, while the remaining articles were published after 2000 [22]. It could be argued, therefore, that more studies need to be done in these topic areas. As in our study, research studies were usually undertaken in some specific disaster regions that have suffered from some type of disaster.

In this literature review section, we analyze facility location problems that are related to disasters. Firstly, we review the logistical problems related to emergency response and disaster management operations. Later we set out a brief analyze of past studies on facility location literature that have touched on areas of research in common with our own proposed model.

Wright et al. [23] published a survey study on models and applications in homeland security.

Their analyses were on emergency preparedness and response, border security, port security, cyber security, and critical infrastructure protection. They proposed location and allocation evacuation models and disaster and response to natural disasters. They also highlighted the apparent lack of research into the whole area of disaster and response. Dekle et al. [24] developed a two-stage model to locate potential disaster recovery centers in the city of Florida. In the first stage of their study, a fixed, total disaster and response coverage area was assessed and determined (ie a 'set covering' problem is solved). They determined the requisite, optimal locations for the future facilities. The coverage of each disaster recovery center was assessed as being within a distance of 20 miles in the first stage. Subsequently, in the second stage, the initial "20 mile" constraints were then relaxed and new locations closest to the original optimal locations were determined and evaluated based on the combined evaluation criteria.

Facility location problem models for medical services relating to large scale disasters such as earthquakes, terrorist attacks, etc. were proposed by [25]. In their study, they reviewed three types of location models: p-median (PMP), p-center (PCP) and set covering (SC), for emergency services. A common formulation was proposed to generalize these three models. In this generalized formulation they presented scenarios and service level requirements. It was determined that each demand point would probably have a different service level requirement under any given scenario. Therefore the service level was calculated and determined by the number and condition of the facilities that served the demand point i.e. the higher the number of facilities and the better the facility conditions there were then the higher the service level one could attract. These two ideas are usually presented in disaster management studies. Balcik and Beamon [1] also proposed a scenario-based model with service levels in a humanitarian relief chain. In this study, they determined the number and the optimal location of the facilities and the amount of supplies stocked at each distribution centers. Their model considered multi commodity types, suppliers with capacity restrictions and a single demand point. Each commodity has a different weight, which shows the critical level of the commodity. Then the total expected demand was maximized by the located distribution centers. They showed the effects of budgetary constraints for both pre and post disaster relief funding separately arising out of the performance of the system. In another study, [26] proposed a facility location model for locating emergency response and distribution centers for the expected earthquake in Istanbul. The model they proposed, which is a two-stage stochastic programming problem, consists of five objectives: the average risk of each distribution center, the cost of opening a new distribution center, the maximum service time for each supply, the total response time and the expected unmet demand. They identified multiple criteria along with the priority level of each objective. A goal programming method was used to solve the proposed problem.

A multi-objective programming methodology for designing relief distribution system was suggested by [27]. Three objectives were featured: minimizing the total cost, minimizing travel time and maximizing the minimal satisfaction. Unlike other studies in the literature, they recommended locating temporary stocking centers due to permanent storage centers tendency to be fully capacitated. To assist with implementing this approach, fuzzy multi-objective linear programming was used. It is different than other studies in terms of the level of the problem it analyzes. In other words, their model is more at operational level.

Unmet daily emergency problems have led in the past to frequent criticism of disaster management models. A PMP on locating fire stations in Barcelona was studied by [28]. A scenario dependent demand and travel time model was developed in this study. The model was constructed considering uncertain parameters i.e. uncertain demand and travel time. Two objectives were sought, namely: the minimization of the maximum travel time per population and maximum regret. The regret was calculated by assessing the difference between optimal travel time and the realized average travel time. Barbarosoglu and Arda [29] proposed a two-stage stochastic programming model with uncertain demand on transportation planning for Istanbul in the event of an expected earthquake. Supply and arc capacities were assumed to be random. The location and the allocation decisions were respectively made in the first and second stages.

One way to measure the effectiveness of a facility location is to determine the average distance travelled [30] and one should make reference using this method to the PMP [31]. Assuming that locations become unreliable when the distance to a demand point increases then this is another method to model the effect of distance as modeled in [32]. Injured victims have different level of survivability defendant upon their injuries. Therefore it essentially follows that the transfer of injured victims to hospital locations should be prioritized based on the survivability times [15]. A study has been done on such prioritizing in [33].

However, essential measures in disaster are not only to minimize the distance travelled but also to maximize the survivability competence of the hospital that the patients are transferred to. Our study as far as we are aware uniquely considers both issues. Since existing public and field hospitals can be count as a facility, we also considered the facility location literature in our study as well. For the most part, the facility location literature assumes that the facilities function is always at a full capacity. This assumption is not realistic for disaster studies whilst recognizing that it is reasonable to make such an assumption for many other different scenarios. There have been various studies in the field of reliable facility location [34].

Next, we review some reliable facility location problem studies that are not necessarily related to disasters but instead are related to our studies in terms of failure/disruption of existing facilities. Facility location decisions are one of the main strategic supply chain decisions and should require noteworthy investment planning spanning over long- term planning horizons, e.g., ranging from 2 to 8 years depending on the business. Given the period of the planning horizon and the level of uncertainty in today's business world, the supply chain designers are now obligated to make an allowance for forestalling and for the planning of uncertain future events in their network design. A significant category of these supply chain uncertainties is the disruption/failure of facilities which affect the supply chain's capability to efficiently fulfill demand [35]. As mentioned earlier, these disruptions/failures can be either natural disasters or man-made (such as terrorist attacks, earthquakes etc.). In many cases, the disruption of a region may spread or migrate through the network and affect other fragments of the supply chain network [36].

Following a disruptive event, there is barely any recourse of action available to modify the supply chain infrastructure quickly [37]. As an alternative, a common recourse of action is to reallocate demand to other existing facilities or organize substitute sources of supply. In both cases, the cost of satisfying customer demand increases e.g., due to higher transportation costs. Over the past decade, consideration of such disruptions disturbing the supply chain network design has received substantial attention from both academics and practitioners.

An exemplary early research in this area can be found in [38]. Authors developed a reliability based formulation called Un-capacitated Facility Location Problem (UFLP) and the PMP. Later, Shen et al. [39] studied a variant of reliable UFLP model and proposed and applied efficient approximation algorithms to URFLP by using the special substructure of the problem. Nonetheless these approximations cannot be employed to the general class of facility location problems such as Capacitated Reliable Facility Location Problems (CRFLP).

In practice, capacity decisions are considered together with location decisions. Further, the capacity of facilities usually cannot be modified right after the event of a disruption. Following a facility failure, demands can be reassigned to other facilities only if these facilities have enough available capacity. Therefore CRFLPs are more complex than their un-capacitated counterparts [39] and the studies considering CRFLPs are limited. Snyder and Ülker [40] studied the CRFLP and developed an algorithm based on Sample Average Approximation (SAA) embedded with Lagrangean relaxation. Gade [41] employed the SAA method in combination with a dual decomposition method to solve CRFLP. Later, Aydin and Murat [34] applied Particle Swarm Optimization (PSO) based SAA to solve the same type of CRFLP faster.

3. Problem statement and methodology

As stated in the JICA report [42] Istanbul expects an earthquake in the near future. JICA provided four scenarios for the earthquake. In all four scenarios the magnitude that will most probably occur will be near to or over 7.0 on the Richter scale. An earthquake with this Richter scale recording will cause a huge number of deaths and injuries to people. This can be assessed from earlier earthquakes. For examples studies in ([3-4], [6-7]) provide valuable analyzes on earthquakes. Succeeding a disaster, a hospital emergency room might expect a rapid flow of injured people that can certainly crush hospital capacities [3], because treatment centers and hospitals are the very first places that injured people will run to after a disaster.

From now on in this study, we will refer to people who need medical treatment after a disaster as ‘victims’.

The capacity of hospitals should, essentially, be sufficient to treat all injured victims. Also the distance of these hospitals to demand points is of equal importance. In our study, we determined the optimal number and location of field hospitals in a district of Istanbul-Zeytinburnu to minimize the distance that victims needed to travel. Zeytinburnu is thought to be one of the most risky earthquake places to live according to the [42]. There are six existing hospitals available in Zeytinburnu. Two of them are public hospitals while the others are privately owned. Besides these hospitals, possible locations were identified as suitable locate field hospitals in the event of a disaster. All these locations are public schools. Using the schools as distribution or emergency centers (field hospitals) was proposed in the [22] and [42]. Also the public schools were considered as possible locations for relief centers [43]. The Report also highlighted that it would not be useful to found a large number of facilities that stayed idle until a disaster occurs. Instead, the report concluded that it would be more effective to operate the existing public schools following a disaster. Thinking along the same lines we proposed to set up field hospitals once the disaster had occurred, in our simulated earthquake disaster, in the same way as the report suggested. Not that, in this study, we aim to determine the locations of field hospitals considering the disruption of field hospitals and not embedding the set up time of field hospitals, which is suggested as a future work.

The field hospitals would thus serve as temporarily located hospitals-field hospitals following a disaster. The main assumption in this study is that the existing hospitals may be disrupted. In our study scenario we assessed that this could happen in many ways, such as by the disruption of fallen buildings or damaged roads. We also considered that the hospitals (both existing and field hospitals) would be capacitated. Furthermore, we made an assumption that the hospitals would be identical in terms of services carried out within the hospitals. Here, we want to highlight another future work, which is restricting the assumption and considering the different capabilities of hospitals. Lastly, we assumed that the field hospitals would survive after the earthquake because these field hospitals are planned to be set up following a disaster and are selected among the schools that are resistant to the earthquakes.

3.1. Data collection

The data we used in this study was gathered from the websites of the Ministry of Health of Turkey [44], the Ministry of National Education of Turkey [45], the Municipality of Zeytinburnu/Istanbul [46] and the [42]. The JICA report provides analyzes for the disaster mitigation study which was compiled at the direction of and under the supervision of the IMM (Istanbul Metropolitan Municipality) and the JICA. Four possible earthquake scenarios in Istanbul were presented in the JICA, i.e. Model A, Model B, Model C and Model D. In the report, the number of victims, buildings and infrastructure damage estimates were provided for each district of Istanbul in Model A and Model C. Model A was identified as the most probable scenario with a magnitude of reading provided for on the Richter scale and Model C was reported as the worst-case scenario and was given a magnitude reading. The fault segment for these two models can be seen in Figure 1 (a) and (b). Each figure shows the entire fault line and the portion of the fault line estimated to be broken for the corresponding scenario.

The JICA [42] also reported that Model A was about km long. This segment starts from the west of the Izmit, where an earthquake occurred in 1999, and ends in Silivri. Model C assumes a simultaneous break of the entire km section of the NAF (North Anatolian Fault) in the Marmara Sea. In this study, we analyzed these two scenarios, separately.

As reported in the JICA [42], there are six existing hospitals and 35 public schools in Zeytinburnu. Each school is considered as a potential location for field hospitals. Coordinates of hospitals and schools were gained from google maps [47]. Figure 2 shows the existing hospitals and the potential school locations on the Zeytinburnu district’s map. On the map, existing hospitals’ locations were shown with “H” and public schools’ locations were shown with circles. Lastly, the triangles represent demand points. Demand points were selected as the center of each neighborhood; these are commonly referred to as “mahalles” in Turkish.

The district based expected number of injured victims are provided as an estimate in JICA [42]. In variance to the JICA [42] and in order to represent the distribution of demand more accurately we used the neighborhoods as the demand points. We identified each neighborhood’s location with a single (x,y)

Figure 1 (a). Fault segment for Model A [42]

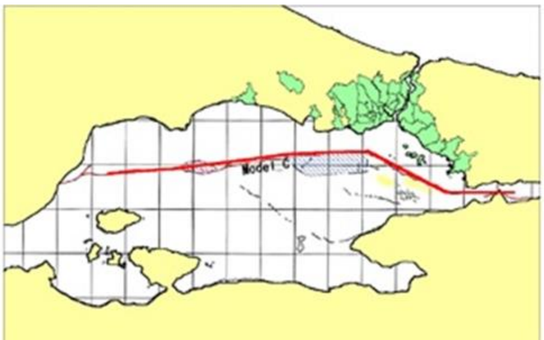


Figure 1 (b). Fault segment for Model C [42]

coordinate. There are neighborhoods in the Zeytinburnu, and the center of each neighborhood was considered as the demand point.

Then, we calculated the expected total number of injured victims for each demand point via the production of population census of each specified neighborhood and the expected percentage of injuries, which is provided in the JICA [42]. When we obtained the numbers we rounded them up to the nearest integer. The population of each neighborhood was gained from the Turkish Statistical Institute [48]. The JICA [42] assumed that 2.80% of the Zeytinburnu's population would be heavily injured if Model A scenario occurs and 3.10 % of the population would be heavily injured if Model C scenario occurs. For instance, the population of Bestelsiz neighborhood was 26,524. Consequently, the number of heavily injured victims in Bestelsiz was calculated as follows: 26,524x0.028= 742.672 ≈ 742. It was assumed in the JICA [42] that the expected total number of injured victims would be three times as the number of heavily injured victims. Therefore the expected total number of victims in Bestelsiz would be 2,228. Lastly, the distance between the two (*x*,*y*) coordinates was calculated by using the Euclidean distance formula.

3.2. Stochastic p-median model

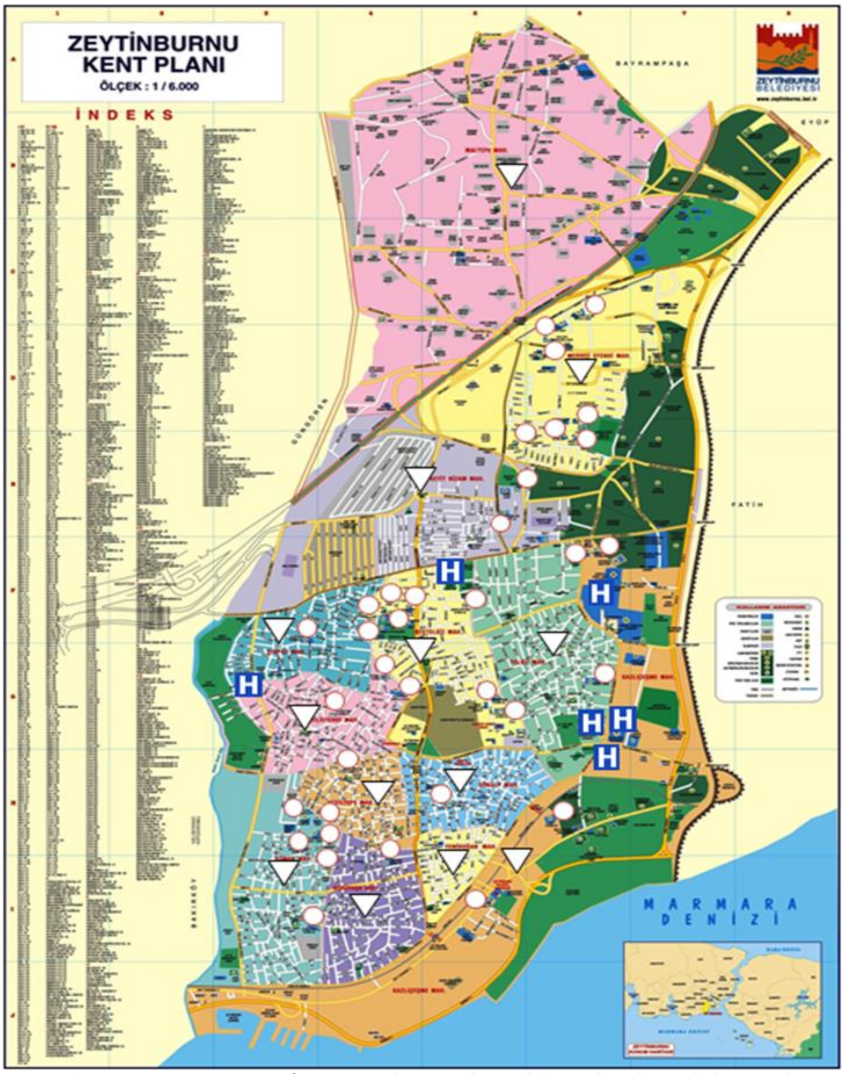


Figure 2. Locations of existing hospitals, public schools and demand points.[46]

We now introduce the following notation which we use throughout the rest of this paper: D denotes the set of demand points (i.e. effected areas in neighborhoods) and H denotes the set of school locations (possible field hospital locations). We let d_j be the demand at neighborhood $j \in D$, and we let f_{ij} denote the distance between the existing or field hospital i and demand point j. We also let H_E denote the set of existing hospitals and H_F denote the set of field hospitals. It was clear that the set of hospitals was $H_E \cup H_F = H$. Each hospital i had a limited capacity and could serve at most, c_i victims. Existing hospitals may have failed during a disaster and may not have been available after the event.

Therefore, victims could not be treated by any of the hospitals when all the other existing hospitals failed, there would not be sufficient capacity at the field hospitals and/or victim treatment at other available hospitals would be prohibited. In such cases, the victim would be assigned to the hospitals in other districts and a large penalty, in terms of distance, would occur. Assigning victims to the hospitals in the other districts could be seen as representing a transportation of victims to hospitals that are not within range. For simplicity, we denoted the last location in H_F as a hospital which was located out of the specified district. Then, $f_{|H_F|j}$ denoted the distance between demand point j and the hospital which was located out of the district. We let Q denoted the number of hospitals that were allowed to be opened (including existing hospitals, $Q \geq |H_E|$). This constraint is added to help decision makers consider their budget while deciding to open field hospitals. This constraint easily can be removed is budget is not an issue for decision makers.

We formulated the problem as a stochastic P-median model. In the first stage, the location decisions for field hospitals were made before random failures of the existing hospitals had occurred (before the event-earthquake occurs). In the second stage, following the existing hospital failures, the victim-hospital assignment decisions were made for each victim on the basis that the existing hospitals had survived or that field hospital were located. The goal was to identify the set of field hospitals to be opened while minimizing the maximum service distance for all the demand points. The service distance for a demand point j was defined as the distances from demand point j to its nearest h_i hospitals.

In the scenario based formulation of the P-median problem, we let s denote a failure scenario and a set of all failure scenarios we denoted as being S, where $s \in S$. We let p_s be the probability when a scenario s occurred and we let $\sum_{s \in S} p_s = 1$. Further we let k_i^s denote whether the hospital i survived (i.e., $k_i^s = 1$, and $k_i^s = 0$ otherwise). For instance, in the case of independent hospital failures, we had $|S| = 2^{|H_E|}$ possible failure scenarios for $|H_E|$ hospitals. Note that our proposed methodology did not require any assumption on independence and distribution for each hospital's failure. Please note that the field hospitals and the hospital that were out of district were perfectly reliable, as abovementioned in detailed.

The binary decision variable y_i specified whether the hospital i was opened or not. Note that $y_i = 1$ where $i \in H_F$. Integer variable x_{ij}^s specified the number of victims that were at demand point j and assigned to hospital i in scenario s. We noted that while the single sourcing assumption was a favored method in practice, it was not restricting for the proposed model.

The scenario-based formulation of two stage stochastic P-median model is as follows:

$$minimize \sum_{s \in S} p_s \left(\sum_{i \in H} \sum_{j \in D} f_{ij} x_{ij}^s \right) \quad (1)$$

$$\sum_{j \in D} x_{ij}^s \leq c_i k_i^s y_i \quad ,\forall i \in H, s \in S \quad (2)$$

$$\sum_{i \in H} x_{ij}^s \geq d_j \quad ,\forall j \in D, s \in S \quad (3)$$

$$\sum_{i \in H_F} y_i = Q \quad (4)$$

$$\sum_{i \in H_E} y_i = |H_E| \quad (5)$$

$$y_i \in \{0,1\} \quad ,\forall i \in H \quad (6)$$

$$x_{ij}^s \geq 0 \; and \; integer \quad ,\forall i \in H, j \in D, s \in S \quad (7)$$

The objective function in (1) finds an optimal facility location solution while minimizes the expected total distance travelled by service victims. The constraints in (2) prevent the assignment of any victim to a hospital that have been failed and ensure that the total demand assigned to the hospital does not exceed hospital's capacity in all scenarios. The constraints in (2) also ensure that a hospital could not function unless it is opened. The constraints in (3) ensure that demand of all affected areas are satisfied in all scenarios. The constraints in (4) guarantee that in total the Q field hospitals function. The constraints in (5) ensure that all existing hospitals are opened (does not matter they are failed or survived, because Constraints in (2) prevent any hospital to serve if it is failed). The constraints in (6) and (7) are integrality constraints.

4. Results

In this section, we provide the results for both un-capacitated and capacitated versions of the P-median model for field hospitals in Zeytinburnu/Istanbul.

We solved both problems optimally by using the deterministic equivalent formulations of the stochastic mathematical models. Models were programmed using MATLAB R2010b and the integer programs were solved by using CPLEX 12.1 (IBM Ilog). The experiments were conducted on a laptop with Intel(R) Core (TM) i7-CPU, a 2.10 GHz processor and a 12.0 GB RAM running on Windows 7 OS. Next, we describe the data in detail.

4.1. Un-capacitated field hospitals and capacitated existing public hospitals

Initially, we analyzed the un-capacitated version of the P-median model. The objective function stayed the same just as in (1). However, we revised the constraints in (2) as follows:

$$\sum_{j\in D} x_{ij}^{s} \leq Mk_i^s y_i \quad ,\forall i \in H, s \in S \qquad (8)$$

where M represent a sufficiently big number. The constraints (3)-(7) stayed the same.

Then, we introduce an artificial hospital to ensure that unsatisfied demand was satisfied by hospital(s) that were located outside the neighborhoods as stated in Section 3.2. The distance between the artificial hospital and all other hospitals was set to $5km$, which was larger than the maximum distance (4.11) between any demand point and the hospitals. The capacities of the existing hospitals were selected based on the data provided by hospital managers. It was note that the total available capacity of the existing hospitals was 31,500 less than expected total demand for both models (i.e., Model A and Model C). The expected total demand for Model A and Model C were estimated as 32,652 and36,151, respectively. (Detailed data can be found in Appendix). As already mentioned in this section the field hospitals were un-capacitated.

In generating the failure scenarios, we assumed that the failure of existing hospitals was independently and identically distributed according to the Bernoulli distribution with probability q_i (i.e. the failure probability of hospital i). In our experiments, we used uniform failure probability (i.e., $q_{i=1,\dots,|H_E|} = q$) and considered the cases $q = \{1.0, 0.5, 0.2, 0.1, 0.0\}$. All field hospitals and artificial hospital were assumed to be perfectly reliable (i.e., $q_{i=1,\dots,|H_F|} = 0$). We noted the case scenario that when $q = 0$ this corresponds to the deterministic P-median problem and when q=1 this corresponds to the case scenario that all existing hospitals fail. The failure scenarios $s \in S$ were generated as follow: We let $H_E^f \subset H_E$ be the set of hospitals that failed, and $H_E^r \equiv H_E \backslash H_E^f$ be the set of hospitals that were reliable (i.e., not failed). We let the hospital failure indicator ($\mathrm{k_i^s}$) be equal to 1 otherwise if $\mathrm{i \in H_E^r}$, then $\mathrm{k_i^s = 0}$. The probability of scenario s was then calculated as $\mathrm{p_s = q^{|H_E^f|}(1-q)^{|H_E^r|}}$. The size of the failure scenario set $|\mathrm{S}|$ assessment$= 64$. The deterministic equivalent formulation was found to have 42 binary variables, $\mathrm{y_i}$, and 34,944 ($\coloneqq |\mathrm{H}| \times |\mathrm{D}| \times |\mathrm{S}|$) integer variables, $\mathrm{x_{ij}^s}$. Similarly, it had constraints (3), (4), (5) and (8) totaling 3,522 $(:= |\mathrm{D}| \times |\mathrm{S}| + 1 + 1 + |\mathrm{H}| \times |\mathrm{S}| = 13 \times 64 + 1 + 1 + 42 \times 64)$ constraints. Note that all datasets used in the paper are available from the authors upon request.

We then presented the results relating to the un-capacitated field hospitals. As mentioned earlier, there were six existing public hospitals in Zeytinburnu. The locations of existing hospitals were fixed and unchangeable. In determining the locations of the field hospitals, the location of existing hospitals were considered. We provided the results for five different failure probabilities relating to the existing hospitals. That is, we considered the following case scenarios: that all the existing hospitals failed (i.e.,$q_i = 1.0$), existing hospitals failed with 50% (i.e.,$q_i = 0.5$), 20% (i.e.,$q_i = 0.2$) and 10% (i.e.,$q_i = 0.1$) probability and finally that all the existing hospitals survived (i.e.,$q_i = 0.0$).

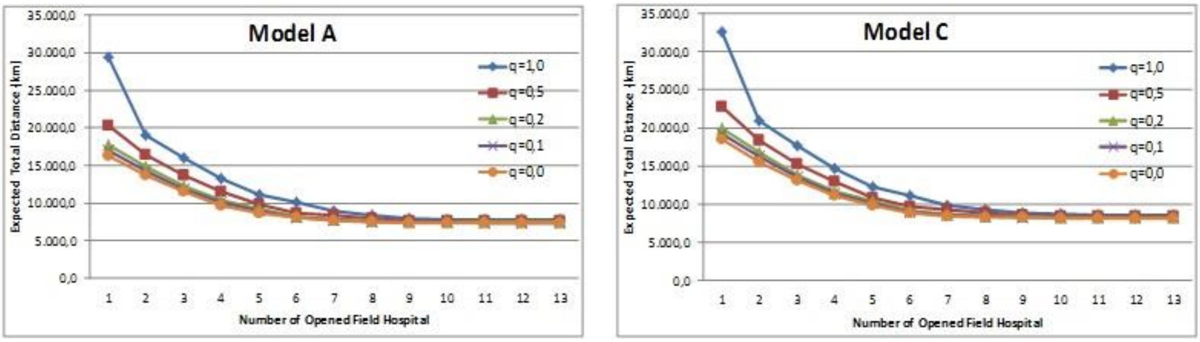
First, we solved the model for both Model A and Model C under all five failure scenarios. The objective function values of these two models (Model A and Model C) are provided in Table 1. The first column in Table 1 shows the number of opened field hospitals (public schools that were going to serve as field hospitals). Column 2 shows the expected total distance for Model A when all existing public hospitals failed (i.e.,$q_i =$ 1.0), and columns $3-6$ show the expected total distance when the public hospitals failed with a probability of $0.5, 0.2$ and 0.1 and then when all public hospitals survived consecutively. Columns $7-11$ show relative results for Model C.

We noted that the solutions in column 2 were the same in the case scenarios where 12 or more field hospitals were opened. In other words, we showed that if the events in Model A occurred and all the public hospitals failed, opening up 12 field hospitals would be sufficient enough to transport victims within a minimum distance. Again, we showed that 12 field hospitals were sufficient if the failure probability was reduced to 0,5, 0,2 and 0,1, as seen in columns 3, 4, and 5. On the other hand, if all public hospitals survived, only 11 field hospitals would be needed to transport all victims within a minimum distance. Since we assessed that 12 field hospitals would be needed for the case scenario where the public hospitals failed with a probability of 1,0, 0.5, 0,2 and 0,1, we deduced that decision makers should consider providing a service of at least 12 field hospitals in order to be able to transport victims within a minimum distance if scenario A occurs.

Table 1. Expected total distance.

Number of Opened Field Hospitals	Model A: Total Distance (m)					Model C: Total Distance (m)				
	Failure Probability					Failure Probability				
	1.0	**0.5**	**0.2**	**0.1**	**0.0**	**1.0**	**0.5**	**0.2**	**0.1**	**0.0**
0	163260.0	96390.8	60289.8	48954.9	37994.4	180755.0	113128.5	76144.1	64367.9	52837.9
1	29399.9	20341.6	17686.2	16970.5	16348.7	32551.6	22858.8	19996.1	19198.6	18512.5
2	18976.5	16464.6	14915.7	14321.9	13756.5	21011.0	18343.3	16821.4	16215.7	15608.7
3	16011.1	13720.7	12294.6	11895.9	11501.5	17727.3	15268.7	13893.8	13487.5	13097.9
4	13259.8	11595.9	10413.2	10064.9	9728.8	14681.1	12921.5	11738.4	11416.8	11077.4
5	11147.1	9839.5	9293.2	8979.2	8684.7	12342.2	10915.0	10400.6	10176.6	9876.7
6	10027.1	8719.5	8243.7	8123.6	8019.1	11102.1	9674.9	9160.4	9031.6	8920.0
7	8954.0	8321.1	7845.4	7725.2	7620.7	9914.0	9233.9	8719.5	8590.6	8479.0
8	8378.6	7954.1	7603.3	7524.9	7462.0	9276.8	8827.5	8451.5	8368.7	8303.3
9	7980.2	7721.1	7544.6	7477.0	7423.9	8835.9	8569.6	8377.2	8305.3	8250.7
10	7834.8	7629.9	7506.5	7438.9	7386.9	8674.9	8462.8	8334.8	8262.9	8208.3
11	7796.7	7591.8	7468.8	7427.1	7375.1	8632.5	8420.4	8293.2	8249.8	8195.2
12	7784.9	7580.0	7457.1	7416.1	7375.1	8619.4	8407.3	8280.0	8249.8	8195.2
13	7784.9	7580.0	7457.1	7416.1	7375.1	8619.4	8407.3	8280.0	8237.6	8195.2
14	7784.9	7580.0	7457.1	7416.1	7375.1	8619.4	8407.3	8280.0	8237.6	8195.2
15	7784.9	7580.0	7457.1	7416.1	7375.1	8619.4	8407.3	8280.0	8237.6	8195.2
16	7784.9	7580.0	7457.1	7416.1	7375.1	8619.4	8407.3	8280.0	8237.6	8195.2
17	7784.9	7580.0	7457.1	7416.1	7375.1	8619.4	8407.3	8280.0	8237.6	8195.2
18	7784.9	7580.0	7457.1	7416.1	7375.1	8619.4	8407.3	8280.0	8237.6	8195.2
19	7784.9	7580.0	7457.1	7416.1	7375.1	8619.4	8407.3	8280.0	8237.6	8195.2
20	7784.9	7580.0	7457.1	7416.1	7375.1	8619.4	8407.3	8280.0	8237.6	8195.2
21	7784.9	7580.0	7457.1	7416.1	7375.1	8619.4	8407.3	8280.0	8237.6	8195.2
22	7784.9	7580.0	7457.1	7416.1	7375.1	8619.4	8407.3	8280.0	8237.6	8195.2
23	7784.9	7580.0	7457.1	7416.1	7375.1	8619.4	8407.3	8280.0	8237.6	8195.2
24	7784.9	7580.0	7457.1	7416.1	7375.1	8619.4	8407.3	8280.0	8237.6	8195.2
25	7784.9	7580.0	7457.1	7416.1	7375.1	8619.4	8407.3	8280.0	8237.6	8195.2
26	7784.9	7580.0	7457.1	7416.1	7375.1	8619.4	8407.3	8280.0	8237.6	8195.2
27	7784.9	7580.0	7457.1	7416.1	7375.1	8619.4	8407.3	8280.0	8237.6	8195.2
28	7784.9	7580.0	7457.1	7416.1	7375.1	8619.4	8407.3	8280.0	8237.6	8195.2
29	7784.9	7580.0	7457.1	7416.1	7375.1	8619.4	8407.3	8280.0	8237.6	8195.2
30	7784.9	7580.0	7457.1	7416.1	7375.1	8619.4	8407.3	8280.0	8237.6	8195.2
31	7784.9	7580.0	7457.1	7416.1	7375.1	8619.4	8407.3	8280.0	8237.6	8195.2
32	7784.9	7580.0	7457.1	7416.1	7375.1	8619.4	8407.3	8280.0	8237.6	8195.2
33	7784.9	7580.0	7457.1	7416.1	7375.1	8619.4	8407.3	8280.0	8237.6	8195.2
34	7784.9	7580.0	7457.1	7416.1	7375.1	8619.4	8407.3	8280.0	8237.6	8195.2
35	7784.9	7580.0	7457.1	7416.1	7375.1	8619.4	8407.3	8280.0	8237.6	8195.2

We assessed that in scenario C, at most 13 hospitals would be needed and that this would occur when the failure probability was 0.1. We concluded that regardless of any of the occurrences in Model A or Model C scenarios. 13 schools would be sufficient enough to serve as field hospitals. It was noted that the improvement in expected total distance was significantly larger for the first few field hospitals than for the others and it was very small after 6-8 field hospitals. We also noted that. no improvement could be attained in expected total traveled distance after a certain number of field hospitals were opened. This state was achieved when all demand points could be assigned to their closest potential hospital out of the 41 locations. Since the field hospitals were un-capacitated. there would be no restriction on the allocation of demand points to assign to their closest field hospital. establishing an additional field hospital could not improve the reduced service distance. We randomly selected 7 as the maximum number of opened field hospitals. This was also meaningful since it was unlikely that a service could be provided to a large number of field hospitals due to the budget constraints that would be imposed in a real life scenario.



(a) (b)

Figure 3. Expected total distances under different failure probabilities for un-capacitated field hospitals.

As was observed in Figure 3 a) and b). the improvement rate in expected total distance reduction got slower when 7 or more field hospitals were opened. in both models. The results were very similar because it was apparent that in only a few neighborhoods reallocation of demand was advantageous. In the majority of the neighborhoods, the schools were sufficiently close to demand points. It was considered that this was also reasonable since there would have been some other public schools that could have been allocated for treatment operations. We concluded that opening only 7 field hospitals would be an acceptable and sufficient number in order to service victims over a reasonable and rational distance.

Table 2. Distance differences between opening 7 and 12 field hospitals.

Number of Opened Field Hospitals	Differences in Distances for 7 and 12 Field Hospitals (km)									
	Model A Failure Probability					Model C Failure Probability				
	1.0	0.5	0.2	0.1	0.0	1.0	0.5	0.2	0.1	0.0
7	11,147	8,321	7,845	7,725	7,621	9,914	9,234	8,719	8,591	8,479
12	7,785	7,580	7,457	7,416	7,375	8,619	8,407	8,280	8,250	8,195
Difference	3,362	741	388	309	246	1.295	827	439	341	284
Average Distance per Victim	0.10	0.02	0.01	0.01	0.01	0.04	0.02	0.01	0.01	0.01

In Table 2 we illustrated that if 12 field hospitals were opened in case of all public hospitals failed. victims would be transported for 3,362 (km) more if only 7 field hospitals were opened. We deduced that if this value was divided by the expected total demand for Model A $(:= 32652)$. each victim would be transported only 0.1 (km) on average. In the cases where the failure probability was reduced to $0.2, 0.1$ and 0.0 each victim would be transported only for 0.01 (km) on average. In Model C, when failure probabilities were $0.2, 0.1$ and 0.0, these values were also 0.01 (km). However, when the failure probability was 1.0 the expected average distance per victim became 0.04. Interestingly. we were able to conclude that opening 7 field hospitals provided a better solution in terms of expected average or total reduced distance. Please note that that distances were divided by $32{,}652$ which was the expected total demand for the Model C.

Table 3. Opened field hospitals under different failure probabilities.

Failure Probability	Open 7 Field Hospitals Model A	Open 7 Field Hospitals Model C	Open 12 Field Hospitals Model A	Open 12 Field Hospitals Model C
1.0	15,18, 19, 20, 26, 27,38	15, 18, 19, 20, 26, 27, 38	12, 15, 17, 18, 19, 20, 22, 23, 26, 27, 36, 38	12, 15, 17, 18, 19, 20, 22, 23, 26, 27, 36, 38
0.5	15, 18, 19, 20, 22, 26, 27	15, 18, 19, 20, 22, 26, 27	12, 15, 17, 18, 19, 20, 22, 23, 26, 27, 36, 38	12, 15, 17, 18, 19, 20, 22, 23, 26, 27, 36, 38
0.2	15, 18, 19, 20, 22, 26, 27	15, 18, 19, 20, 22, 26, 27	12, 15, 17, 18, 19, 20, 22, 23, 26, 27, 36, 38	12, 15, 17, 18, 19, 20, 22, 23, 26, 27, 36, 38
0.1	15,18, 19, 20, 22, 26, 27	15,18, 19, 20, 22, 26, 27	12, 15, 17, 18, 19, 20, 22, 23, 26, 27, 36, 38	12, 15, 17, 18, 19, 20, 22, 23, 25, 26, 27, 36
0.0	15,18, 19, 20, 22, 26, 27	15,18, 19, 20, 22, 26, 27	12, 15, 17, 18, 19, 20, 22, 23, 26, 27, 29, 36	12, 15, 17, 18, 19, 20, 22, 23, 26, 27, 36, 41

Table 4. Average distance per victim for capacitated field hospitals.

Number of Opened Field Hospital	Model A: Average Distance Per Victim										Model C: Average Distance Per Victim									
	Capacity=1000					Capacity=2000					Capacity=1000					Capacity=2000				
	Failure Probability					Failure Probability					Failure Probability					Failure Probability				
	1.0	**0.5**	**0.2**	**0.1**	**0.0**	**1.0**	**0.5**	**0.2**	**0.1**	**0.0**	**1.0**	**0.5**	**0.2**	**0.1**	**0.0**	**1.0**	**0.5**	**0.2**	**0.1**	**0.0**
0	5.00	2.95	1.85	1.50	1.16	5.00	2.95	1.85	1.50	1.16	5.00	3.13	2.11	1.78	1.46	5.00	3.13	2.11	1.78	1.46
1	4.85	2.80	1.70	1.35	1.02	4.70	2.66	1.57	1.24	0.95	4.86	3.00	1.97	1.65	1.33	4.73	2.86	1.84	1.51	1.19
2	4.70	2.66	1.57	1.25	0.95	4.40	2.37	1.33	1.05	0.84	4.73	2.86	1.84	1.52	1.20	4.46	2.60	1.58	1.25	0.93
3	4.55	2.51	1.45	1.15	0.89	4.10	2.08	1.10	0.88	0.74	4.60	2.73	1.71	1.38	1.07	4.19	2.33	1.34	1.05	0.79
4	4.41	2.37	1.33	1.05	0.84	3.81	1.81	0.92	0.75	0.66	4.46	2.60	1.58	1.25	0.93	3.93	2.07	1.13	0.89	0.71
5	4.26	2.23	1.22	0.97	0.79	3.52	1.54	0.77	0.64	0.58	4.33	2.47	1.46	1.14	0.84	3.66	1.82	0.94	0.75	0.64
6	4.11	2.09	1.11	0.88	0.75	3.23	1.30	0.64	0.55	0.51	4.20	2.34	1.35	1.06	0.80	3.40	1.58	0.79	0.64	0.57
7	3.97	1.95	1.01	0.81	0.71	2.94	1.08	0.54	0.47	0.44	4.07	2.21	1.24	0.97	0.75	3.14	1.35	0.66	0.55	0.50
8	3.82	1.81	0.93	0.75	0.66	2.65	0.88	0.46	0.41	0.39	3.94	2.08	1.14	0.90	0.72	2.87	1.14	0.56	0.49	0.45
9	3.68	1.68	0.85	0.70	0.63	2.36	0.71	0.39	0.36	0.34	3.81	1.96	1.04	0.82	0.68	2.61	0.94	0.48	0.42	0.40
10	3.53	1.56	0.78	0.65	0.59	2.07	0.58	0.35	0.32	0.30	3.67	1.83	0.95	0.76	0.65	2.35	0.77	0.41	0.37	0.35
11	3.39	1.43	0.72	0.61	0.56	1.79	0.48	0.32	0.30	0.29	3.54	1.71	0.88	0.71	0.62	2.10	0.64	0.37	0.34	0.32
12	3.24	1.32	0.66	0.57	0.53	1.50	0.40	0.30	0.29	0.27	3.41	1.59	0.81	0.66	0.59	1.84	0.52	0.33	0.31	0.30
13	3.10	1.21	0.62	0.54	0.50	1.22	0.35	0.29	0.28	0.27	3.28	1.48	0.75	0.62	0.56	1.59	0.44	0.31	0.29	0.28
14	2.96	1.10	0.57	0.50	0.47	0.94	0.32	0.28	**0.27**	**0.26**	3.15	1.37	0.70	0.59	0.54	1.33	0.38	0.30	0.29	0.28
15	2.81	1.00	0.53	0.47	0.45	0.67	0.30	**0.27**	0.27	0.26	3.03	1.26	0.64	0.55	0.51	1.08	0.34	0.29	**0.28**	**0.27**
16	2.67	0.91	0.49	0.45	0.42	0.40	0.29	0.27	0.27	0.26	2.90	1.16	0.60	0.52	0.49	0.83	0.31	**0.28**	0.28	0.27
17	2.53	0.83	0.46	0.43	0.40	0.31	**0.28**	0.27	0.27	0.26	2.77	1.07	0.56	0.50	0.47	0.58	0.30	0.28	0.28	0.27
18	2.39	0.76	0.44	0.41	0.39	**0.30**	0.28	0.27	0.27	0.26	2.64	0.98	0.52	0.47	0.45	0.34	**0.29**	0.28	0.28	0.27
19	2.25	0.69	0.42	0.40	0.38	0.30	0.28	0.27	0.27	0.26	2.51	0.90	0.50	0.45	0.43	**0.31**	0.29	0.28	0.28	0.27
20	2.11	0.63	0.41	0.38	0.37	0.30	0.28	0.27	0.27	0.26	2.39	0.83	0.47	0.44	0.41	0.31	0.29	0.28	0.28	0.27
21	1.97	0.59	0.40	0.37	0.36	0.30	0.28	0.27	0.27	0.26	2.26	0.76	0.45	0.42	0.40	0.31	0.29	0.28	0.28	0.27
22	1.83	0.54	0.38	0.37	**0.35**	0.30	0.28	0.27	0.27	0.26	2.13	0.70	0.44	0.41	0.39	0.31	0.29	0.28	0.28	0.27
23	1.69	0.50	0.38	0.36	0.35	0.30	0.28	0.27	0.27	0.26	2.01	0.64	0.42	0.40	0.39	0.31	0.29	0.28	0.28	0.27
24	1.55	0.47	0.37	0.36	0.35	0.30	0.28	0.27	0.27	0.26	1.88	0.60	0.41	0.39	0.38	0.31	0.29	0.28	0.28	0.27
25	1.42	0.44	**0.36**	0.36	0.35	0.30	0.28	0.27	0.27	0.26	1.76	0.56	0.40	0.39	0.38	0.31	0.29	0.28	0.28	0.27
26	1.28	0.42	0.36	**0.35**	0.35	0.30	0.28	0.27	0.27	0.26	1.64	0.52	0.40	0.39	0.38	0.31	0.29	0.28	0.28	0.27
27	1.15	0.41	0.36	0.35	0.35	0.30	0.28	0.27	0.27	0.26	1.52	0.49	**0.39**	**0.38**	0.38	0.31	0.29	0.28	0.28	0.27
28	1.02	0.40	0.36	0.35	0.35	0.30	0.28	0.27	0.27	0.26	1.39	0.47	0.39	0.38	**0.37**	0.31	0.29	0.28	0.28	0.27
29	0.89	0.39	0.36	0.35	0.35	0.30	0.28	0.27	0.27	0.26	1.27	0.45	0.39	0.38	0.37	0.31	0.29	0.28	0.28	0.27
30	0.76	0.39	0.36	0.35	0.35	0.30	0.28	0.27	0.27	0.26	1.16	0.44	0.39	0.38	0.37	0.31	0.29	0.28	0.28	0.27
31	0.63	**0.38**	0.36	0.35	0.35	0.30	0.28	0.27	0.27	0.26	1.04	0.43	0.39	0.38	0.37	0.31	0.29	0.28	0.28	0.27
32	0.52	0.38	0.36	0.35	0.35	0.30	0.28	0.27	0.27	0.26	0.93	0.43	0.39	0.38	0.37	0.31	0.29	0.28	0.28	0.27
33	**0.46**	0.38	0.36	0.35	0.35	0.30	0.28	0.27	0.27	0.26	0.83	0.43	0.39	0.38	0.37	0.31	0.29	0.28	0.28	0.27
34	0.46	0.38	0.36	0.35	0.35	0.30	0.28	0.27	0.27	0.26	0.74	0.43	0.39	0.38	0.37	0.31	0.29	0.28	0.28	0.27
35	0.46	0.38	0.36	0.35	0.35	0.30	0.28	0.27	0.27	0.26	**0.66**	**0.42**	0.39	0.38	0.37	0.31	0.29	0.28	0.28	0.27

In Table 3, we presented the opened field hospitals for seven and twelve field hospital cases under different failure probabilities. In the opening seven field hospitals case scenarios, when the failure probability was 1.0, schools in regions 15,18,19,20,26,27 and 38 were selected for both Model A and Model C. When the failure probability was reduced to 0.5,0.2,0.1 and 0.0 existing hospitals had more chance to survive, then model substituted school in region 38 by 22 and it decided to open schools in regions 15,18,19,20,26 and 27. We determined the effect of such changes in a scenario where twelve field hospitals were opened as well. Surprisingly, we concluded that high demand had the same effect on the decision making if the field hospitals had infinite capacity and the number of opened field hospitals stayed the same. Next, we analyzed the capacitated field hospitals case scenarios.

4.2. Capacitated field and capacitated existing public hospitals

In the previous section, we assumed that the field hospitals were un-capacitated, whereas the existing public hospitals and field hospitals would have been capacitated in real life. In this section, we present the results and analysis where both field and existing hospitals were deemed to be capacitated. In this approach, we assumed that the total capacity of the existing hospitals was the same as in the previous section (31,500). We analyzed the capacitated version of the problem for multiple cases such as low capacity and high capacity and each case tested for 5 different failure scenarios.

The values in the Table 4 show the average distances per victim. The averages that have been taken represent the division of expected total distances dependent upon demand. The first column in Table 4 shows the number of opened field hospitals. Columns 2-6 show the average distances per victim in Model A for 5 different failure scenarios when the capacity of the field hospitals was 1,000. Columns 7 to 11 show the results, when the capacities of the field hospitals were increased to 2,000. Likewise, columns 12 to 16 and 17 to 21 show the average distance per victim in Model C for 5 different failure scenarios when the capacities of the field hospitals were equal to 1,000 and 2,000, consecutively.

As expected, the average serving distance decreased as the demand decreased. The locations of the field hospitals were selected as close as possible to the most populated neighborhoods. In comparison with the un-capacitated case scenario, the requisite number of field hospitals was higher in order to provide a service within the same distance range. For instance, in Model A when all existing hospitals failed the minimum average distance that could be achieved was (km) with 33 field hospitals. However, the minimum average distance that could be achieved in the un-capacitated case scenario was 0.24 (km) and with only 9 field hospitals. This comparison is valid for all the other cases as predicted. Further, the average serving distance decreased as the failure probability decreased in both the capacitated and un-capacitated cases because as the failure probability is decreased more existing hospitals are survived and more capacity is became available to serve victims.

If the capacities of the field hospitals were equal to 1,000, the total available capacity would be 35,000 and if the capacities were equal to 2,000, the total available capacity would be 70,000. The total demand was always less than the total available capacity in Model A. However, the total demand was higher than the total available capacity in Model C when the capacities of the field hospitals were restricted to 1,000. Obviously, the total demand would be always less than the total available capacity in un-capacitated cases. This is because in capacitated cases, the selected locations of the field hospitals were farther away than in the un-capacitated cases. The capacitated model could not achieve as lower an average distance per victim as in the case of the un-capacitated model. In other words, in capacitated cases, the model does not allow for an allocation of victims to the closest field or existing hospitals because of the capacity constraints. Therefore as a result, higher than expected serving distance would occur.

As seen in Figure 4 a) 33 field hospitals were needed to achieve a minimum average distance per victim when all existing hospitals failed and 31,25,26 and 22 field hospitals were needed when the failure probabilities reduced to 0.5,0.2,0.1 and 0.0 (all existing hospitals survived), respectively. When capacities increased to 2,000, (Figure 4b) the number of needed field hospitals decreased to 18,17,15,14, and 14. Even though, the total demand and the minimum average distance that could be achieved is higher in model C, there was really little difference between Model A and Model C when the capacities of the field hospitals were equal to 2,000 this was because the capacity of the field hospitals was doubled while the demand increased only by 0.3%-which represents a figure of only 3,499 more victims.

We went on to analyze the effect of the different failure probabilities and field hospital capacities on expected total unsatisfied demand. We presented the results in Table 5. The first observation we gained was that when the failure probability was very high or high, i.e., q=1.0,0.5, and the capacities of the field hospitals were equal to 1,000, full satisfaction of demand could not be achieved even if all 35 field hospitals were opened, in Model C.

As mentioned earlier, these circumstances occurred when the total capacity of the field hospitals was less than the total demand. In another words, some of the victims needed to be transported to the hospitals located on the other regions because of the lack of capacity. Therefore, in this event it should have been suggested to decision makers to set up higher capacitated field hospitals. Secondly, at least 11 field hospitals for Model A and 13 for Model C would be needed to satisfy demand even if the failure probability was very low, i.e., q=0.1, and the capacities of the field hospitals was equal to 2,000. This may be a reference point for how failure of the existing hospitals effects satisfying the demand.

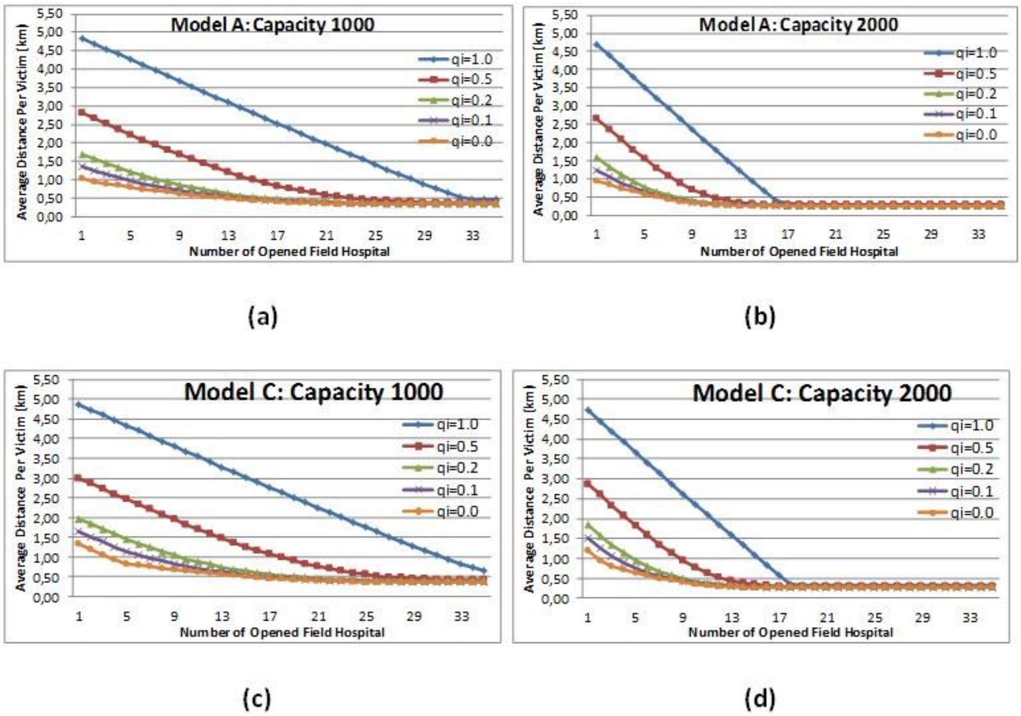


Figure 4. Average distance under different failure probabilities for capacitated field hospitals.

Consequently, decision makers or managers may in the future be better off giving greater attention to the infrastructure of the existing public hospitals as well as and strengthening the buildings. We want to state out that if public hospitals are projected to fail then its whole capacity is unusable. Here, we would like to point out a future study, which will be the next step of this study and consider partial failure of the existing public hospitals. Lastly, it was observed that the demand satisfaction rate increased with a smaller number of field hospitals when the failure probability of the existing hospitals decreased. We therefore deduced from this that this further observation supported our second observation.

5. Conclusions and future studies

The next predicted earthquake is anticipated to cause major havoc to many regions of Istanbul. The anticipated earthquake severely threatens human life and health, not to mention the substructure and it even threatens the economy of Turkey as a whole. The Government and the Istanbul Metropolitan Municipality have taken preparatory action, both for pre-disaster and post disaster scenarios, against possible earthquakes to mitigate the effects of such a disaster. Locating the disaster relief facilities is one of the crucial tasks to strategically prepare for this event. Life-saving decisions on the location of hospitals, such as field hospitals, essentially must rank as one of the most important managerial issues.

Table 5. Percentage of unsatisfied demand for Model A and Model C under different failure probabilities and field hospital capacities.

	Model A: Percentage of Unsatisfied Demand										**Model C: Percentage of Unsatisfied Demand**									
	Capacity=1000					**Capacity=2000**					**Capacity=1000**					**Capacity=2000**				
	Failure Probability					**Failure Probability**					**Failure Probability**					**Failure Probability**				
Number of Opened Field Hospital	**1.0**	**0.5**	**0.2**	**0.1**	**0.0**	**1.0**	**0.5**	**0.2**	**0.1**	**0.0**	**1.0**	**0.5**	**0.2**	**0.1**	**0.0**	**1.0**	**0.5**	**0.2**	**0.1**	**0.0**
0	100.00	51.76	22.82	13.18	3.53	100.00	51.76	22.82	13.18	3.53	100.00	56.43	30.29	21.58	12.87	100.00	56.43	30.29	21.58	12.87
1	96.94	48.70	19.76	10.11	0.47	93.87	45.68	17.38	8.43	**0.00**	97.23	53.67	27.53	18.81	10.10	94.47	50.90	24.76	16.05	7.33
2	93.87	45.68	17.38	8.43	**0.00**	87.75	39.67	12.93	5.62	0.00	94.47	50.90	24.76	16.05	7.33	88.94	45.37	19.23	10.51	1.80
3	90.81	42.66	15.12	7.00	0.00	81.62	33.73	8.81	3.11	0.00	91.70	48.13	21.99	13.28	4.57	83.40	39.89	14.67	6.96	**0.00**
4	87.75	39.67	12.93	5.62	0.00	75.50	28.10	5.98	1.76	0.00	88.94	45.37	19.23	10.51	1.80	77.87	34.48	10.75	4.51	0.00
5	84.69	36.70	10.87	4.37	0.00	69.37	22.80	3.85	0.94	0.00	86.17	42.62	16.71	8.26	**0.00**	72.34	29.17	7.22	2.42	0.00
6	81.62	33.73	8.81	3.11	0.00	63.25	17.87	2.29	0.46	0.00	83.40	39.89	14.67	6.96	0.00	66.81	24.13	4.83	1.35	0.00
7	78.56	30.89	7.26	2.32	0.00	57.12	13.50	1.27	0.20	0.00	80.64	37.17	12.63	5.67	0.00	61.27	19.45	3.09	0.73	0.00
8	75.50	28.10	5.98	1.76	0.00	51.00	9.88	0.71	0.09	0.00	77.87	34.48	10.75	4.51	0.00	55.74	15.13	1.82	0.35	0.00
9	72.44	25.38	4.77	1.25	0.00	44.87	6.86	0.37	0.04	0.00	75.10	31.81	8.89	3.38	0.00	50.21	11.30	0.99	0.14	0.00
10	69.37	22.80	3.85	0.94	0.00	38.75	4.41	0.15	0.01	0.00	72.34	29.17	7.22	2.42	0.00	44.68	8.16	0.54	0.07	0.00
11	66.31	20.27	3.00	0.67	0.00	32.62	2.72	0.07	**0.00**	0.00	69.57	26.64	5.96	1.83	0.00	39.14	5.59	0.27	0.03	0.00
12	63.25	17.87	2.29	0.46	0.00	26.50	1.56	0.03	0.00	0.00	66.81	24.13	4.83	1.35	0.00	33.61	3.58	0.11	0.01	0.00
13	60.19	15.65	1.76	0.32	0.00	20.37	0.79	0.01	0.00	0.00	64.04	21.75	3.87	0.98	0.00	28.08	2.14	0.05	**0.00**	0.00
14	57.12	13.50	1.27	0.20	0.00	14.25	0.33	**0.00**	0.00	0.00	61.27	19.45	3.09	0.73	0.00	22.55	1.19	0.02	0.00	0.00
15	54.06	11.61	0.95	0.13	0.00	8.12	0.13	0.00	0.00	0.00	58.51	17.20	2.37	0.50	0.00	17.01	0.58	0.01	0.00	0.00
16	51.00	9.88	0.71	0.09	0.00	2.00	0.03	0.00	0.00	0.00	55.74	15.13	1.82	0.35	0.00	11.48	0.25	**0.00**	0.00	0.00
17	47.94	8.25	0.51	0.06	0.00	**0.00**	**0.00**	0.00	0.00	0.00	52.98	13.14	1.36	0.23	0.00	5.95	0.09	0.00	0.00	0.00
18	44.87	6.86	0.37	0.04	0.00	0.00	0.00	0.00	0.00	0.00	50.21	11.30	0.99	0.14	0.00	0.42	0.01	0.00	0.00	0.00
19	41.81	5.53	0.24	0.02	0.00	0.00	0.00	0.00	0.00	0.00	47.44	9.70	0.75	0.10	0.00	**0.00**	**0.00**	0.00	0.00	0.00
20	38.75	4.41	0.15	0.01	0.00	0.00	0.00	0.00	0.00	0.00	44.68	8.16	0.54	0.07	0.00	0.00	0.00	0.00	0.00	0.00
21	35.69	3.53	0.11	0.01	0.00	0.00	0.00	0.00	0.00	0.00	41.91	6.80	0.39	0.04	0.00	0.00	0.00	0.00	0.00	0.00
22	32.62	2.72	0.07	**0.00**	0.00	0.00	0.00	0.00	0.00	0.00	39.14	5.59	0.27	0.03	0.00	0.00	0.00	0.00	0.00	0.00
23	29.56	2.09	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	36.38	4.46	0.17	0.01	0.00	0.00	0.00	0.00	0.00	0.00
24	26.50	1.56	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	33.61	3.58	0.11	0.01	0.00	0.00	0.00	0.00	0.00	0.00
25	23.44	1.09	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	30.85	2.80	0.08	**0.00**	0.00	0.00	0.00	0.00	0.00	0.00
26	20.37	0.79	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	28.08	2.14	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00
27	17.31	0.51	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	25.31	1.65	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00
28	14.25	0.33	**0.00**	0.00	0.00	0.00	0.00	0.00	0.00	0.00	22.55	1.19	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00
29	11.18	0.23	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	19.78	0.84	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
30	8.12	0.13	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	17.01	0.58	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
31	5.06	0.08	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	14.25	0.36	**0.00**	0.00	0.00	0.00	0.00	0.00	0.00	0.00
32	2.00	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	11.48	0.25	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
33	**0.00**	**0.00**	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	8.72	0.16	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
34	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	5.95	0.09	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
35	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	3.18	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

In this study, we developed models and solutions for the problem of locating field hospitals in Zeytinburnu/Istanbul in the event of a major earthquake hitting this region. Using our representative, we divided the hospitals into two categories: existing public hospitals and field hospitals. Existing public hospitals are the hospitals that are already functioning and that are supposed to function or fail with a probability in a post-disaster scenario.

On the other hand, field hospitals are defined as schools that will function as emergency centers or as hospital post-disaster. However, the decision making about the location of these field hospitals must be done prior to the occurrence of any such disaster. Therefore we set out to determine the possible and preferable locations of the field hospitals whilst collaterally also considering the existing public hospitals functionality after a disaster. We also considered, within our analysis, optimizing the number of such field hospitals needed in the event of a failure of the existing hospitals. We developed a two stage stochastic P-median model and then identified the number, location and size of the field hospitals in Zeytinburnu/Istanbul. The district of Zeytinburnu has multiple neighborhoods. Each neighborhood has different sizes of population and the locations of these neighborhoods were taken as the demand points whilst we were constructing our solution model to determine the field hospitals locations.

We constructed and provided solutions to numerous case scenario models for this purpose and after analyzing the results further provided alternate, more efficient solution improvements. Two different earthquake scenarios, called Model A and Model C and provided by [42], are analyzed separately in detailed.

In the first case, we considered un-capacitated field hospitals and analyzed how to minimize the expected total distance to them. The marginal development of establishing an additional field hospital reduced abruptly after the first few field hospitals. We observed that seven field hospitals would be satisfactory for both Model A and Model C scenarios. While analyzing Model A, it was realized that nine field hospitals could reasonably serve the victims with an average distance of 0.24 km even if all the existing hospitals failed. Generally, the average serving distances were found to be between 0.23 km and 0.30 km, even with only a few field hospitals in the un-capacitated field hospitals case scenarios. However, more field hospitals were needed for capacitated cases. If capacities of the field hospitals were low (1,000 km), then we found that these ranges were never reached. In Model C, the same ranges of expected total distance as was the case with Model A could be achieved for un-capacitated field hospital cases. The expected total distance when comparing locating either 7 or alternatively 12 field hospitals had a significant different resulting effect in Model A and Model C. Under high failure probabilities locating 7 field hospitals in Model C provided more advantages than in locating 7 field hospitals in Model A. For example, in the 1.0 failure probability case the difference in the expected total distance between locating 7 field hospitals and 12 field hospitals was 3,362 km in Model A while it was as low as 1,295 km in Model C. However we demonstrated that an opposing, opposite analysis could be obtained if the failure probability was lower. In the second case, we consider capacitated field hospitals and the whole question of how to minimize the expected total distance as well as calculating average distances per victim. We found that there was no marginal development requirement for establishing an additional field hospital in both Models A or C when the capacity of the field hospitals was 1,000. As increasing number of opened field hospitals improved average distance reductions but we concluded that more than 35 field hospitals would be needed, [which would be very costly], to achieve the requisite average level of distance reduction achieved in the un-capacitated case scenario. The average distance levels that were achieved in the un-capacitated case with up to 7 field hospitals could be achieved with more than 20 field hospitals in capacitated cases.

The selected numbers of field hospitals under the various case scenarios were found to be sufficient. However, the result of introducing capacity limits to field hospitals caused the need for a higher number of field hospitals in order to gain the desired level of average distances. Since the un-capacitated field hospitals are not practical in real life and the capacity expansion of field hospitals makes a high impact on serving victims within a minimum distance a higher level of capacity is desirable.

In our study we analyzed the effects of available earthquake scenarios on existing hospitals. In our opinion the associated damage estimates of earthquake scenarios on roads, substructure, and network could usefully be incorporated into the models. Furthermore, in this study, if public hospitals are projected to fail then its whole capacity is unusable. Another future study, which will be the next step of this study, may be considering partial failure of the existing public hospitals and categorizing type of victims (as in [2]) and the service type that a hospital serve because all public hospitals do not serve the same types treatments. Also for a larger number of case scenarios heuristic methods, such as SAA (Sample Average Approximation [49]) and GA (Genetic Algorithm), etc., could usefully be applied. We believe that our study provides valuable, contributory information for the benefit of the aforementioned decision makers.

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Appendix

Demand for Model A and Model C of neighborhoods in Zeytinburnu/Istanbul

		Model A			Model C		
	Population	**Heavily Injured**	**Injured**	**Total**	**Heavily Injured**	**Injured**	**Total**
Beştelsiz	26,524	743	2,228	2,971	822	2,467	3,289
Çırpıcı	29,946	838	2,515	3,354	928	2,785	3,713
Gökalp	20,978	587	1,762	2,350	650	1,951	2,601
Kazlıçeşme	1,289	36	108	144	40	120	160
Maltepe	153	4	13	17	5	14	19
Merkezefendi	22,413	628	1,883	2,510	695	2,084	2,779
Nuripaşa	27,885	781	2,342	3,123	864	2,593	3,458
Sümer	37,565	1,052	3,155	4,207	1,165	3,494	4,658
Telsiz	38,742	1,085	3,254	4,339	1,201	3,603	4,804
Yenidoğan	10,709	300	900	1,199	332	996	1,328
Yeşiltepe	23,026	645	1,934	2,579	714	2,141	2,855
Seyitnizam	23,405	655	1,966	2,621	726	2,177	2,902
Veliefendi	28,914	810	2,429	3,238	896	2,689	3,585
Total	**291,549**	**8,163**	**24,490**	**32,652**	**9,038**	**27,114**	**36,151**

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Using 2-Opt based evolution strategy for travelling salesman problem

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Abstract. Harmony search algorithm that matches the (μ+1) evolution strategy, is a heuristic method simulated by the process of music improvisation. In this paper, a harmony search algorithm is directly used for the travelling salesman problem. Instead of conventional selection operators such as roulette wheel, the transformation of real number values of harmony search algorithm to order index of vertex representation and improvement of solutions are obtained by using the 2-Opt local search algorithm. Then, the obtained algorithm is tested on two different parameter groups of TSPLIB. The proposed method is compared with classical 2-Opt which randomly started at each step and best known solutions of test instances from TSPLIB. It is seen that the proposed algorithm offers valuable solutions.

Keywords: Travelling salesman problems; TSP; harmony search; HS; (μ+1) evolution strategy; 2-Opt; TSPLIB.
AMS Classification: 68T20; 90-08

1. Introduction

The travelling salesman problem (TSP) is one of the most popular combinatorial optimization problems in complexity theory [1]. TSP for minimizing the tour length is quite difficult to solve and classified as NP-Hard, it will be time consuming to solve larger instances. However, TSP is used in many theoretical and practical applications such as manufacturing planning, logistics, and electronics manufacturing. Due to the nature of TSP, obtaining the optimal solution is not possible in polynomial time if solved via integer programming. Also, it is known that the solution time extends exponentially as the problem size grows. Therefore, as an alternative solution approach, the meta-heuristics are commonly used to determine near optimal solutions in acceptable solution times [2-8].

In the related literature, many known meta-heuristics were used to solve TSPs for minimizing the tour lengths. For instance, Freisleben and Merz [9] presented an algorithm by using genetic algorithm (GA) to find near-optimal solution for a set of symmetric and asymmetric TSP instances and obtained high quality solutions in a reasonable time. Chowdhury et al. [10] also used GA for solving a flow-shop scheduling problem to minimize makespan via finding optimal order of cities. The simulated annealing (SA) algorithm is also used for TSP by Wang and Tian [11] in which an improved SA is employed. Meta-heuristics approach is generally used to solve the problem in reasonable time if the problem size increases. For large TSPs, Fiechter [12] used a parallel tabu search algorithm. Similarly, different types of ant colony algorithm are used for the TSP [13-15]. Also, Wang et al. [16] developed swap operator and swap sequence in order to use particle swarm optimization (PSO) for TSP.

Recently, with the progresses in computational sciences, the new meta-heuristics methods have

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been developed and used for solving combinatorial problems. Some of them are cuckoo search algorithm [17-21], firefly algorithm [22-24] and harmony search (HS) algorithm [25-28]. In Table 1, the main literature is chronologically summarized.

In our study, the harmony search algorithm was used as the core algorithm to solve TSPs. HS is first proposed by Geem et al. [5]. Weyland [29] proved that HS is theoretically a special case of an evolution strategy known as (μ +1) evolution strategy [30]. In Geem et al [5], the 20-cities TSP, constraint optimization problem, and water network pipeline design are solved. For 20-cities TSP, neighbouring city-going and city-inverting operators were designed. Their operators were used to find the closest city that will be visited next and to produce a new path on feasible nodes, respectively. Geem et al [5] did not give the details of the discrete structure. However, later Geem [31] detailed the HS for TSP that uses stochastic derivative for discrete variables

Table 1. Meta-heuristic studies on TSP

Meta-heuristics	***Literature directly on TSP***
Genetic Algorithm (GA)	Freisleben & Merz (1996), Chowdhury et al. (2013)
Simulated Annealing (SA)	Wang & Tian (2013),
Tabu Search (TS)	Fiechter (1994),
Ant Colony Optimization (ACO)	Stüzle & Hoss (1997), Randall & Montgomery (2003), Chu et al. (2004)
Particle Swarm Optimization (PSO)	Wang et al. (2003),
Cuckoo Search (CS)	Yang & Deb (2009), Ouyang et al. (2013), Ouaarab et al. (2013), Ouaarab et al. (2014)
Firefly Algorithm (FA)	Yang (2010), Jati & Suyanto (2011), Kumbharana & Pandey (2013a)
Harmony Search (HS)	Geem et al. (2001), Wang et al. (2010), Pan et al. (2011), Huang & Peng (2013), Yuan et al. (2013), Weyland (2015)
Hybrid Studies	Pang et al. (2004) (PSO&Fuzzy); Thamilselvan & Balasubramanie (2009) (GA&TS); Kaveh & Talatahari (2009) (PSO, ACO&HS); Yan et al. (2011); Chen & Chien (2011) (GA&ACO); Chen & Chien (2011) (GA,SA,PSO&ACO); Kumbharana & Pandey (2013b) (GA,SA&ACO); Yun et al. (2013) (HS&ACO)

In addition, we directly use the index values of the normally distributed harmony numbers in our method. In this process, besides the use of meta-heuristics algorithms, the hybrid approaches involving the hybridization of two or more heuristics were applied in order to eliminate the weakness of single meta-heuristics for solving the large scale TSP optimization problems [32-39]. On the other hand, HS is directly adapted for TSP.

In TSP, the main goal is to find the shortest closed tour that visits each city once and exactly once in a given list with the best route. There are tour construction methods such as the nearest neighbor, greedy, insertion heuristics, Christofides method. After the tour has been generated by any tour construction heuristics, it is improved with tour improvement heuristics such as 2-Opt, 3-Opt, k-Opt, Lin-Kernighan, Tabu-Search, Simulated Annealing, Genetic Algorithms etc. The 2-Opt approach is a well-known method used for this purpose. The 2-Opt is a simple local search algorithm and it was first proposed by Croes [2] for TSP. It swaps edges in a tour for shortening the total tour length.

The HS algorithm, a special case of evolution strategy which is called (μ+1) evolution strategy, is a meta-heuristic optimization method that inspired by the mimics of the improvisation ability of musicians. Using HS algorithm, the musical instruments are played with discrete notes under the musicians' experience and their improvisation ability randomly. The musical harmony, aesthetic standard, pitches of instruments and the improvisation process are design parameters of HS algorithm. HS works with the harmony size (HMS), the harmony considering rate and the pitch adjusting rate as optimization operators [5, 31].

A brief overview of TSP from the literature especially on meta-heuristics is surveyed in this section. The rest of the paper is organized as follows: the proposed method in which the HS algorithm with its continuous structure is directly used for the TSP is presented in Section 2. With the proposed algorithm, the transformation mechanism for HS to solve the TSP is obtained by using 2-Opt local search algorithm. Then, in Section 3, the obtained algorithm is tested on two different parameter groups of TSPLIB.

2. Proposed algorithm: 2-Opt based harmony search algorithm

The proposed algorithm that is called 2-Opt Based Harmony Search Algorithm (2-Opt_cHS) combines the algorithms of 2-Opt and the HS. The first advantage of the proposed algorithm is to convert the real numbers into index values for solving combinatorial optimization problem such as TSP. Thus, the modified algorithm provides to solve discrete optimization problems.

In general, for TSP problems, roulette wheel selection is used in evolution strategies for the transformation between randomly generated real numbers of heuristic solutions and ordered numbers of combinatorial problem solutions. In this paper, HS algorithm with its continuous structure is directly used for the travelling salesman problem. The transformation of real numbers of continuous HS algorithm to integer numbers of discrete form is obtained by using 2-Opt local search algorithm which is used to define a function from continuous to discrete functions and vice versa. The pseudo code of the HS algorithm is given as Algorithm 1 in Table 2 [29] and the proposed algorithm is given as Algorithm 2 in Table 3.

Table 2. The pseudo code of the harmony search algorithm [29]

Algorithm 1: The Harmony Search Algorithm	
1:	Initialize the harmony memory with HMS randomly generated solutions
2:	**repeat**
3:	create a new solution in the following way
4:	**for** all decision variables **do**
5:	with probability HMCR use a value of one of the solutions in the harmony memory (selected uniform random numbers) and additionally change this value slightly with probability PAR
6:	otherwise (with probability 1-HMCR) use a random value for this decision variable
7:	**end for**
8:	**if** the new solution is better than the worst solution in the harmony memory **then**
9:	replace the worst solution by the new one
10:	**end if**
11:	**until** the maximum number of iterations has been reached
12:	**return** the best solution in the harmony memory

According to Algorithm 2, firstly, the objective function is generated with real number arrays for initial harmonics. And then, its limits and bandwidths, and the values are defined as parameters. The 2-Opt algorithm is used for designing discrete variables and is defined with the step size (v) and application parameter (opt). By using v, the 2-Opt application is used once in each v steps.

The use of v parameter is the design idea of this study as using 2-Opt at each step increases the simulation time and decreases the effect of using HS algorithm. When a solution is obtained after HS with 2-Opt procedure, it will be evaluated using the fitness function. Respecting to HMS and fitness of new solution, the new solution may be inserted in harmony memory or not. Eventually, the termination criteria can be defined as a problem dependent number of iterations or as reaching to a specific quality of solution. In this study, the algorithm will stop when maximum number of iterations is met, and otherwise the while loop case will be repeated for each iteration.

A sample TSP solution taken from TSPLIB, known as Burma14, is demonstrated in Table 2. As can be seen in Table 2, IH and NH are real Harmony numbers. By using their index values in HOI, the route information is obtained and then improved by using 2-Opt. The proposed algorithm with respect to the pseudo-code in

Algorithm 2 by taking maximum number of iteration as 5 is given. The 2-Opt is used instead of the initial solution and the 3rd iteration. At iteration 4, the optimal solution for Burma14 is obtained as 3323. In Figure 1, the route improvement at some steps are visualized for Burma14.

Table 3. The pseudo code of the proposed algorithm: 2-Opt based harmony search

Algorithm 2: The Proposed Algorithm: 2-Opt Based Harmony Search	
1:	Initialize the harmony memory with HMS randomly generated solutions
	Get Harmony Ordered Indexes
	Get 2-Opt Ordered Indexes
2:	**repeat**
3:	create a new solution in the following way
4:	**for** all decision variables **do**
5:	with probability HMCR use a value of one of the solutions
	in the harmony memory (selected **normal** random numbers)
	and additionally change this value slightly with probability PAR
6:	otherwise (with probability 1-HMCR) use a random value for this decision variable
	if (at each step size *v*) for the 2-Opt is provided
	Get Harmony Ordered Indexes
	Get 2-Opt Ordered Indexes
	else
	Get Harmony Ordered Indexes
	end if
7:	**end for**
8:	**if** the new solution is better than the worst solution in the harmony memory **then**
9:	replace the worst solution by the new one
10:	**end if**
11:	**until** the maximum number of iterations has been reached
12:	**return** the best solution in the harmony memory

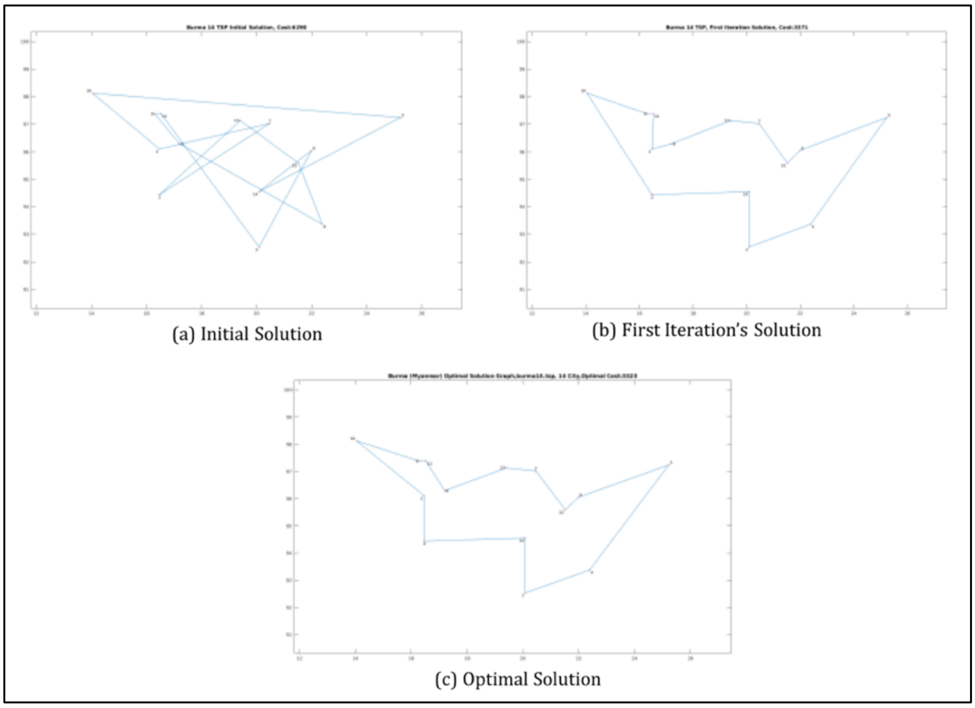

Figure 1. Burma14 solution graphs in the proposed algorithm iterations

Table 4. The proposed algorithms solution steps for Burma14

Level	Algorithms	2	3	4	5	6	7	8	9	10	11	12	13	14	Cost
Initial Solution	**IH**	2.3033	-0.1139	0.7853	-2.6272	-0.4115	4.6309	0.4681	0.2114	-2.6841	-0.111	1.2406	1.7914	-1.0448	
	HOI	10	5	14	6	3	11	9	8	4	12	13	2	7	
2opt		2-Opt Not Applied													6290
1	**NH**	-2.6455	-2.651	-1.0227	-0.3836	-0.1139	-0.158	0.2518	0.4681	0.7592	1.2074	2.1269	2.3215	4.5906	
	HOI	3	2	4	5	7	6	8	9	10	11	12	13	14	
2opt	**2-Opt OI**	8	13	7	12	6	5	4	3	14	2	10	9	11	3371
2	**NH**	0.2518	2.3688	-0.1647	2.0943	-0.1139	-0.3919	-1.0296	-2.6904	4.5745	4.2033	0.7515	0.5164	1.2113	
	HOI	9	8	7	4	6	2	13	12	14	5	3	11	10	
2opt	**2-Opt OI**	2	3	4	5	6	12	14	7	13	8	11	9	10	3448
3	**NH**	0.2146	-3.2888	-0.1647	2.0904	-3.436	0.7206	1.2203	-0.4167	0.5488	2.3025	4.244	-2.6809	4.5407	
	HOI	6	3	13	9	4	2	10	7	8	5	11	12	14	
2opt		2-Opt Not Applied													7927
4	**NH**	0.2617	2.3827	-0.1616	2.0943	-0.1314	0.7515	1.1767	-0.3962	0.5164	-1.0119	4.1601	-2.6559	4.5252	
	HOI	13	11	9	4	6	2	10	7	8	5	3	12	14	
2opt	**2-Opt OI**	2	14	3	4	5	6	12	7	13	8	11	9	10	3323
5	**NH**	0.2617	4.5213	2.3827	-0.1395	2.105	-0.0897	4.1389	0.7562	-2.638	1.1976	-1.0169	-0.3962	0.5506	
	HOI	10	12	13	5	7	2	14	9	11	6	4	8	3	
2opt	**2-Opt OI**	2	14	3	4	5	6	12	7	13	8	11	9	10	3323
	Best Solution	**2**	**14**	**3**	**4**	**5**	**6**	**12**	**7**	**13**	**8**	**11**	**9**	**10**	**3323**

IH: Initial Harmonies, NH: New Harmonies, HOI: Harmony Ordered Indexes, 2-Opt OI: 2-Opt Ordered Indexes

3. Computational results

In this section, some benchmark problem sets from TSPLIB95 [40] such as *eil51, berlin52, st70, pr76, eil76, kroA100, kroB100, eil101, bier127, chr130, ch150, kroA150, kroB200 and lin318* are considered. As the simulation platform, i7 CPU and 4 GB RAM hardware and MATLAB® 8.2 software package are used. Also, some functions of the Matlog: Logistics Engineering Matlab Toolbox [41] are used. For the simulations, two different parameter sets are chosen that are given as two cases in Table 5.

Table 5. The parameter settings

Parameters	***Case 1***	***Case 2***
HMS	20	1
r_{accept}	0.6	0.95
r_{pa}	0.7	0.7
v	4	3

For Case 1, the parameter values are taken as the most frequently used ones in the literature. For Case 2, on the other hand, the parameters are obtained by trial and error and especially, in order to shorten the simulation time, and as a result HMS is chosen as 1. By trial and error, the acceptable value of v is taken as 3. For Case 1, the iteration is limited as 3600s or best known solution (BKS) whereas for Case 2, 500s or BKS is used. For both cases, each test instance is executed 100 times and the simulation results are analysed in Table 6 and Table 7 for both cases of Table 5 using the mentioned problem sets of TSPLIB.

In Table 6 and Table 7, #Opt/Run is the number of BKS values obtained in total of 100 runs, BKS is the best known solution, $BSol_j$ and $WSol_j$ are the obtained best and worst solutions of 100 runs of the jth instance, respectively. ASolj is the average of Soli (i=1…100) for jth instance and can be given as

$$ASol_j = \frac{\sum_{i=1}^{100} Sol_i}{100} \qquad i=1..100, \quad j=1..14 \qquad (1)$$

where Sol_i (i=1…100) is each solution of 100 runs. ADevj and BDevj are the percentage deviations of the $ASol_j$ and $BSol_j$ from BKSj, respectively, and can be given as

$$ADev_j = \frac{|BKS_j - ASol_j|}{BKS_j} \times 100, \qquad j=1..14 \qquad (2)$$

$$BDev_j = \frac{|BKS_j - BSol_j|}{BKS_j} \times 100, \qquad j=1..14 \qquad (3)$$

It can be seen from Table 6 and Table 7 that the $ADev_j$ solutions of Case 2 are all better than Case 1 whereas $BDev_j$ (j=14) of Lin318 is only slightly worse for Case 2. When #Opt/Run values are taken into consideration, one can see that the parameter set of Case 2 is better in finding the BKS values in total simulations.

Table 6. Summary of computational results for the proposed method for Case 1

#	Name	BKS	BSol	WSol	ASol	ADev	BDev	#Opt/Run
1	***eil51***	426.00	426.00	429	426.95	0.22	0.00	33/100
2	***berlin52***	7542.00	7542.00	7542	7542	0.00	0.00	100/100
3	***st70***	675.00	675.00	679	675.79	0.12	0.00	43/100
4	***pr76***	108159.00	108159.00	109161	108538.43	0.35	0.00	3/100
5	***eil76***	538.00	539.00	552	546.9	1.65	0.19	0/100
6	***kroA100***	21282.00	21282.00	21495	21345.38	0.30	0.00	8/100
7	***kroB100***	22141.00	22179.00	22522	22368,68	1.03	0.17	0/100
8	***eil101***	629.00	635.00	654	646.71	2.82	0.95	0/100
9	***bier127***	118282.00	118936.00	121730	120206.68	1.63	0.55	0/100
10	***ch130***	6110.00	6139.00	6309	6244.38	2.20	0.47	0/100
11	***ch150***	6528.00	6598.00	6796	6700.02	2.64	1.07	0/100
12	***kroA150***	26524.00	26846.00	27538	27188.09	2.50	1.21	0/100
13	***kroA200***	29368.00	29693.00	30594	30146.91	2.65	1.11	0/100
14	***lin318***	42029.00	43113.00	44418	43881.24	4.41	2.58	0/100

Table 7. Summary of computational results for the proposed method for Case 2

#	Name	BKS	BSol	WSol	ASol	ADev	BDev	#Opt/Run
1	***eil51***	426.00	426.00	428.00	426.07	0.02	0.00	94/100
2	***berlin52***	7542.00	7542.00	7542.00	7542.00	0.00	0.00	100/100
3	***st70***	675.00	675.00	675.00	675.00	0.00	0.00	100/100
4	***pr76***	108159.00	108159.00	108701.00	108324.39	0.15	0.00	5/100
5	***eil76***	538.00	538.00	546.00	542.46	0.83	0.00	1/100
6	***kroA100***	21282.00	21282.00	21319.00	21293.08	0.05	0.00	34/100
7	***kroB100***	22141.00	22141.00	22356.00	22259.81	0.54	0.00	1/100
8	***eil101***	629.00	634.00	647.00	641.74	2.03	0.79	0/100
9	***bier127***	118282.00	118724.00	120241.00	119527.81	1.05	0.37	0/100
10	***ch130***	6110.00	6133.00	6245.00	6192.18	1.35	0.38	0/100
11	***ch150***	6528.00	6556.00	6719.00	6644.63	1.79	0.43	0/100
12	***kroA150***	26524.00	26690.00	27161.00	26981.45	1.72	0.63	0/100
13	***kroA200***	29368.00	29622.00	30144.00	29896.52	1.80	0.86	0/100
14	***lin318***	42029.00	43153.00	44281.00	43764.46	4.13	2.67	0/100

In addition, the solution times of each case are given in Table 8. Instead of berlin52 (i=2) Case 2 has better simulation times in average. This is an expected situation as HMS value is 1 for Case 2. Fig.2 is given in order to show the results $ADev_j$ and $BDev_j$ respectively. When the deviations for both cases are investigated from Fig.2, it can be seen that ADev$_j$ has more deviation than BDev$_j$.

Table 8. Simulation times for test instances of Case 1 and Case 2

		Case 1 - Time (seconds)			***Case 2 - Time (seconds)***		
#	***Name***	***Best***	***Worst***	***Avg***	***Best***	***Worst***	***Avg***
1	***eil51***	12.14	3600.16	975.68	16.08	503.50	421.35
2	***berlin52***	2.60	344.55	72.35	5.70	451.47	101.48
3	***st70***	2.63	2453.73	430.54	13.16	523.59	383.36
4	***pr76***	145.21	3602.20	3479.21	33.24	523.67	503.19
5	***eil76***	1464.98	3601.34	3579.00	500.00	524.40	507.15
6	***kroA100***	61.60	3606.30	2960.44	38.56	549.28	496.07
7	***kroB100***	2490.69	3605.85	3590.16	500.01	563.28	524.03
8	***eil101***	3600.01	3603.44	3600.76	500.42	543.10	515.96
9	***bier127***	3600.00	3604.34	3601.52	500.14	567.12	534.31
10	***ch130***	3600.02	3604.38	3601.77	501.76	609.22	545.59
11	***ch150***	3600.05	3617.40	3604.29	501.55	681.52	622.67
12	***kroA150***	3600.06	3616.69	3603.84	502.44	708.62	571.90
13	***kroA200***	3600.20	3626.39	3611.84	504.01	1137.15	728.29
14	***lin318***	3600.24	3825.49	3671.98	2046.31	2466.96	2208.23

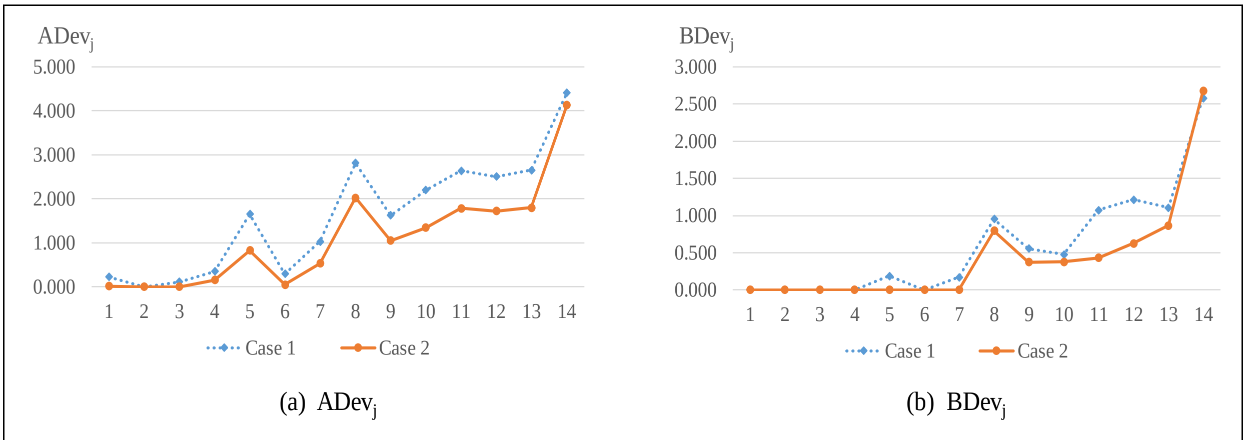

Figure 2. The results of *ADev$_j$* and *BDev$_j$* for Case 1 and Case 2

In order to show the main contribution of this study, the results of 2-Opt_cHS algorithm is compared with classical 2-Opt solutions. The comparison results are given in Table 9 where it is seen that an improvement is obtained in simulation performances by using the proposed method. The classical 2-Opt is simulated by randomly generating a new route information and then applying only the 2-Opt algorithm at each step.

The performances in Table 9 are also evaluated in Table 10 as an indicator of the solution quality using the *ADev$_j$*, *BDev$_j$* and Opt/Run parameters between 2opt_cHS and classical 2opt algorithms.

Table 9. Solutions for test instances of *2opt_cHS* and *classical 2opt*

			2opt_cHS			*Classical 2-opt*		
#	*Name*	*BKS*	*BSol*	*WSol*	*ASol*	*BSol*	*WSol*	*ASol*
1	***eil51***	426.00	426.00	428.00	426.07	431.00	479.00	449.51
2	***berlin52***	7542.00	7542.00	7542.00	7542.00	7542.00	8821.00	8196.08
3	***st70***	675.00	675.00	675.00	675.00	678.00	774.00	712.13
4	***pr76***	108159.00	108159.00	108701.00	108324.40	108943.00	119484.00	112415.10
5	***eil76***	538.00	538.00	546.00	542.46	548.00	605.00	573.18
6	***kroA100***	21282.00	21282.00	21319.00	21293.08	21367.00	24069.00	22312.73
7	***kroB100***	22141.00	22141.00	22356.00	22259.81	22389.00	25201.00	23407.38
8	***eil101***	629.00	634.00	647.00	641.74	659.00	697.00	676.96
9	***bier127***	118282.00	118724.00	120241.00	119527.80	119408.00	133490.00	126352.70
10	***ch130***	6110.00	6133.00	6245.00	6192.18	6200.00	6909.00	6497.10
11	***ch150***	6528.00	6556.00	6719.00	6644.63	6717.00	7342.00	7019.57
12	***kroA150***	26524.00	26690.00	27161.00	26981.45	27155.00	29305.00	28369.89
13	***kroA200***	29368.00	29622.00	30144.00	29896.52	29856.00	32406.00	31204.24
14	***lin318***	42029.00	43153.00	44281.00	43764.46	43814.00	46295.00	44991.41

According to Table 10, it can be seen from the BKS results of the proposed 2-Opt-cHS algorithm, ADevj and BDevj average deviations are -1.10 and -0.44, respectively. On the other hand, for the classical 2-Opt algorithm, ADevj and BDevj average deviations are -6.38 and -1.72, respectively. Therefore, it is seen that by using 2-Opt together with HS, the performance is improved with respect to classical 2-Opt algorithm. Also, the proposed 2-Opt_cHS algorithm can reach BKS values of seven test instances. However, for the classical 2-Opt algorithm, BKS value of one test instance (berlin52) is obtained. Thus, less deviation values and higher #Opt/Run values of 2-Opt_CHS show the advantage obtained using the proposed method. Thus, it is seen that using only classical 2-Opt at each step, optimal solutions cannot be obtained in general. This is an indicator that the proposed method has an improvement by applying the faster HS at each step and slower 2-Opt at some steps.

Table 11 is designed to compare our solutions with the results obtained from literature. It can be observed that all the average solution performances of meta-heuristics including the basic Discrete Cuckoo Search (DCS) have worse results than proposed 2-Opt_cHS. On the other hand, improved DCS of their study and proposed 2-Opt_cHS in our study, have similar performances and both are better than the other meta-heuristics mentioned in their study.

4. Conclusions and further research

The travelling salesman problems are mostly studied in the class of NP-Hard problems. In order to solve these problems many techniques and solution approaches are designed in the literature. In this paper, a harmony search algorithm is directly used as a solution method. The transformation of real numbers of continuous harmony search algorithm to integer numbers of discrete form is obtained by using index values and the 2-Opt local search algorithm. As computational test instances the problem sets of eil51, berlin52, st70, pr76, eil76, kroA100, kroB100, eil101, bier127, ch130, ch150, kroA150, kroB200 and lin318 from TSPLIB are selected and two different cases are designed for experimental study. The results have shown that acceptable solutions can be obtained with the given algorithm.

The results of the proposed method are compared with conventional 2-Opt algorithm and also with other meta-heuristics. Consequently, it is shown that by using the proposed 2-Opt_cHS algorithm useful results could be obtained. The proposed method can be used for all TSP variants such as production planning, electronic manufacturing, and logistics.

Table 10. Relatively comparison for the solutions of *2opt_cHS* and *classical 2opt*

		2opt_cHS			*2-opt*		
#	*Name*	*ADev*	*BDev*	*#Opt/Run*	*ADev*	*BDev*	*#Opt/Run*
1	***eil51***	-0.02	0.00	94/100	-5.52	-1.17	0/100
2	***berlin52***	0.00	0.00	100/100	-8.67	0.00	1/100
3	***st70***	0.00	0.00	100/100	-5.50	-0.44	0/100
4	***pr76***	-0.15	0.00	5/100	-3.94	-0.72	0/100
5	***eil76***	-0.83	0.00	1/100	-6.54	-1.86	0/100
6	***kroA100***	-0.05	0.00	34/100	-4.84	-0.40	0/100
7	***kroB100***	-0.54	0.00	1/100	-5.72	-1.12	0/100
8	***eil101***	-2.03	-0.79	0/100	-7.62	-4.77	0/100
9	***bier127***	-1.05	-0.37	0/100	-6.82	-0.95	0/100
10	***ch130***	-1.35	-0.38	0/100	-6.34	-1.47	0/100
11	***ch150***	-1.79	-0.43	0/100	-7.53	-2.90	0/100
12	***kroA150***	-1.72	-0.63	0/100	-6.96	-2.38	0/100
13	***kroA200***	-1.80	-0.86	0/100	-6.25	-1.66	0/100
14	***lin318***	-4.13	-2.67	0/100	-7.05	-4.25	0/100
	Avg	-1.10	-0.44		-6.38	-1.72	

Table 11. Comparison of ASol results of the proposed method with different meta-heuristics from the literature

	Compared Test Instances				
Solution Methods	**Eil51**	**Berlin52**	**St70**	**Eil76**	**KroA100**
BKS	426.00	7542.00	675.00	538.00	21282.00
Proposed 2-Opt_cHS	426.07	7542.00	675.00	542.50	21293.10
Basic DCS [20]	439.00	7836.40	696.90	565.70	22419.90
Improved DCS [21]	426.00	7542.00	675.00	538.00	21282.00

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Optimization of recirculating laminar air flow in operating room air conditioning systems

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Abstract. The laminar flow air conditioning system with 100% fresh air is used in almost all operating rooms without discrimination in Turkey. The laminar flow device which works with 100% fresh air should be absolutely used in Type 1A operating rooms. However, it is not mandatory to use 100% fresh air for Type 1B defined as places performed simpler operation. Compared with recirculating laminar flow, energy needs of the laminar flow with 100 % fresh air has been emerged about 40% more than re-circulated air flow. Therefore, when a recirculating laminar flow device is operated instead of laminar flow system with 100% fresh air in the Type 1B operating room, annual energy consumption will be reduced. In this study, in an operating room with recirculating laminar flow, optimal conditions have been investigated in order to obtain laminar flow form by analyzing velocity distributions at various supply velocities by using computational fluid dynamics method (CFD).

Keywords: Laminar flow; numerical modeling; CFD; recirculating air; operating room.
AMS Classification: 60G12

1. Introduction

Operating rooms (ORs) are among the most demanding healthcare work areas. Therefore, the infection risks are quite high in these areas. In this sense, because the surgical site infection has negative effect on recuperation time or patient mortality and cost of healthcare services, forming of bacteria colony that caused infection should be under control. Because of reasons aforementioned above, the selection of an appropriate air conditioning system is the first requirement to preserve air quality including remove airborne bacteria, chemicals like waste medical gases used for anesthesia and disinfection and other particles such as skin squames shed by operation team and odors from ORs. The quality of air in an operating room is essentially assessed with regard to how effective the air distribution strategy is in minimizing the possibility of risks mentioned above. In addition, the OR air conditioning system should provide thermal comfort for surgical team to facilitate their demanding work during operations. For this purpose, environmental factors such as temperature, humidity and air velocity should be steerable through various methods.

In practice, achieving of zero contamination in an OR is impossible. But it should be kept at minimum levels. Air contamination less than 10 colony-forming units (cfu)/m^3 is

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internationally accepted as definition of ultra clean air [1]. In practice in Turkey, DIN 1946-4 standard is generally taken into consideration. According to DIN 1946-4 standard, the ranges of temperature, relative humidity and air velocity parameters mentioned in this standard should be under control at a range of 19–26 ºC, 30–60% and 0.23-0.30 m/s, respectively [2]. At 1200 mm height from floor, desired the air velocity is at the range of 0.23-0.25 m/s (Figure1).

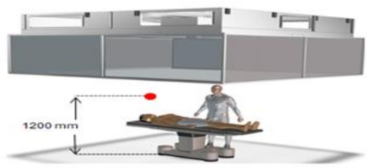


Figure 1. Operating room

In literature, there are a lot of studies on OR air conditioning. Woloszyn et al. (2004) studied diagonal air-distribution system for operating rooms experimentally and by using computational fluid dynamics (CFD) modeling. They revealed that the contaminant distribution depended strongly on existing obstacles such as medical equipment and operation team [3]. Comparing of mixed and laminar airflow systems and the influence of human factors were studied by Andersson et al. (2014). This study was achieved experimentally in a Swedish orthopedic center. Their study shows that laminar airflow air conditioning system used in operating rooms offer high-quality air throughout surgery, with very low levels of (cfu)/m3 close to the surgical wound [4]. Baskan et al. investigated scalar modes for a periodic laminar flow experimentally and computational used finite element method (FEM) [5]. There are a lot of various studies on ORs such as indoor thermal conditions in hospital operating rooms [6,7], impact of different-sized laminar air flow on bacterial counts in the operating room during surgery [8], impact of airflow systems on bacterial burden [9,10] and economical assessment of air-conditioning systems [11].

The air conditioning system in an OR should be carefully designed to ensure thermal comfort conditions and also without infection risks mentioned above. The inlet velocity of air supplied to mixing chamber, the placement of filters and the structure of air grille are highly effective to achieve desired velocity and temperature distributions. The usage of vertical filter has adverse effect on laminar air flow to be achieved. Due to turbulence and the formation of dead volumes in the mixing chamber, a homogeneous air distribution can not be obtained at grille outlet (Figure 2).

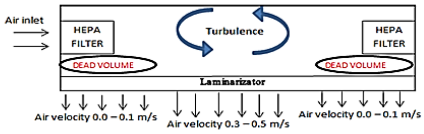


Figure 2. OR Air-ceiling unit with vertical HEPA filter

The use of horizontal filters helping to stabilize the turbulence in the mixing chamber facilitates to obtain a uniform air distribution in air ceiling output (Figure 3).

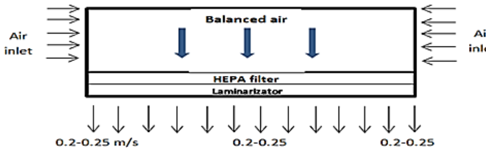


Figure 3. OR Air-ceiling unit with horizontal HEPA filter

This study focuses on a detailed analysis of the air distribution in an operating room under the laminar airflow condition by using CFD analysis method. In an operating room with recirculating laminar flow and horizontal HEPA filter, optimal conditions have been investigated in order to obtain laminar flow form by analyzing velocity and temperature distributions at various supply velocities by using CFD method.

2. Materials and method

2.1. Realizable k-ε turbulence model

There are various turbulence models such as k-epsilon (k-ε) models (e.g. Standard k-ε model, Realisable k-epsilon model, Re-Normalisation Group (RNG) k-ε model), Spalart-Allmaras Model, k-w models, v2-f model, Reynolds stress equation model and Large eddy simulation (LES) model [12]. In this study, realizable k-ε turbulence model is used because of the most common model used in CFD to simulate mean flow characteristics. The original impetus for the k-epsilon model was to improve the mixing-length model, as well as to find an alternative to algebraically prescribing turbulent length scales in moderate to high complexity flows [12].

The first transported variable determines the energy in the turbulence and it is called turbulent kinetic energy (k). The second transported variable is the turbulent dissipation (ε) which determines the rate of dissipation of the turbulent kinetic energy. In realizable k-ε model, turbulent kinetic energy (k) and its the rate of dissipation (ε) together with turbulent viscosity and turbulent conductivity were stated as follow [12].

$$\frac{\partial}{\partial t}(\rho k)+\frac{\partial}{\partial x_i}(\rho k u_i)=\frac{\partial}{\partial x_j}\left[\left(\mu+\frac{\mu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right]+G_k+G_b-\rho\varepsilon-Y_M+S_k \quad (1)$$

$$\frac{\partial}{\partial t}(\rho \varepsilon)+\frac{\partial}{\partial x_i}(\rho \varepsilon u_i)=\frac{\partial}{\partial x_j}\left[\left(\mu+\frac{\mu_t}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial x_j}\right]+\rho C_1 S\varepsilon-\rho C_2\frac{\varepsilon^2}{k+\sqrt{v\varepsilon}}+C_{1\varepsilon}\frac{\varepsilon}{k}C_{3\varepsilon}G_b+S_\epsilon \quad (2)$$

where k is turbulent kinetic energy (m^2/s^2), ϵ is the rate of dissipation of the turbulent kinetic energy (m^2/s^3), ρ is density (kg/m^3), μ is dynamic viscosity (Pa.s), μ_t is turbulent viscosity (Pa.s), $\sigma_k$andσ_ϵ are turbulence Prandtl constants, $C_{1\epsilon}$andC_2 are model constants for k-ε turbulence model, $G_b$is turbulence depending on buoyant force, $S_k$andS_ε are source terms defined by the user and Y_M is the effect of compressibility on turbulence.

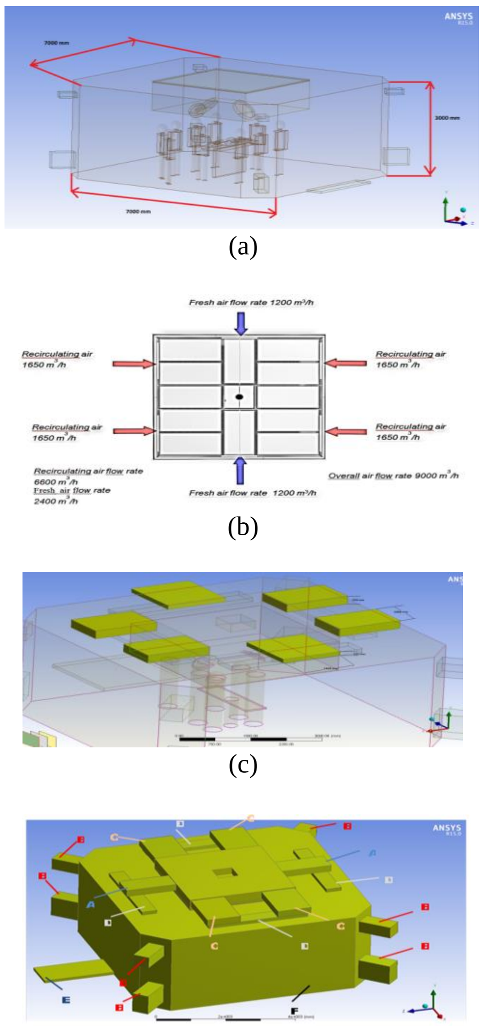
$C_{\epsilon 1}$ = 1.44, C_2 = 1.92, σ_k = 1.0, σ_ϵ = 1.2 [12].

2.2. Boundary conditions

Although an intensive structure of mesh provides a better numerical accuracy since physical accuracy of calculation is limited to physical accuracy of model, unrelated details are avoided. In order to achieve high quality numerical mesh, quality ratio of elements is desired to be in the range of 0.1-1 [13]. If this ratio is close to 1, it indicates high quality of the used components. Automatic mesh method was not used to increase the mesh quality and the element number in the critical domains. In geometry, there are 7783036 mesh elements and 2723271 nodes. Maximum skewness and minimum orthogonal values are equal to 0.79 and 0.24, respectively. In addition, it is assumed that HEPA filters and laminarizator have a porous structure. CFD analysis details are given in Table 1. In analysis; supplying air is fed from the laminar flow air ceiling unit in size with 3200x3200 mm towards the OR in size with 7000x7000x3000 mm (Figure 4). Exhaust air at 2200 m^3/h is sucked from four corners via suction grilles. 2400 m^3/h of fresh air treated by air conditioner is fed into a laminar flow unit. 6600 m^3/h of re-circulated air is sucked from grilles placed at the ceiling and then it is blown into the laminar flow air ceiling unit.

Table 1. Solution methods and fluent adjustments

Fluent adjustments	Solution Methods
Precision: Double Precision	Scheme: Simple
Viscous Model: Standard k-epsilon	Gradient: Green-Gauss Node Based
Inlet Turbulent Intensity: %5	Pressure: Second Order
Inlet Turbulent Viscosity Ratio: % 5	Momentum: Second Order Upwind
Outlet Turbulent Intensity: %5	Turbulent Kinetic Energy: First Order Upwind
Outlet Turbulent Viscosity Ratio : % 5	Turbulent Dissipation Ratio: First Order Upwind
	Energy: Second Order Upwind



(a) (b) (c) (d)

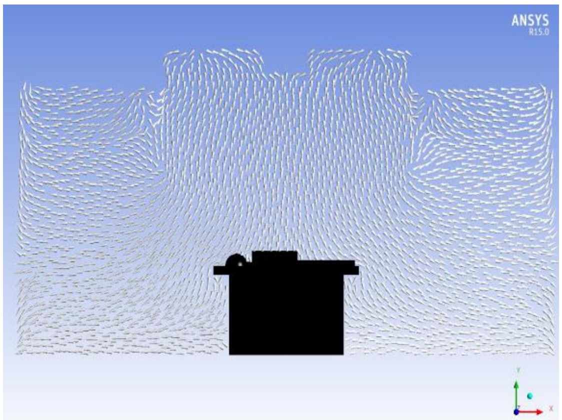
Inlet Velocity = A, Outlet Velocity = B, Recirculating air Inlet = C, Recirculating air Outlet = D
Outlet Pressure = E, Wall = F

Figure 4. Dimensions (a), air ceiling unit (b), grille locations and description (c and d) of analysed OR model

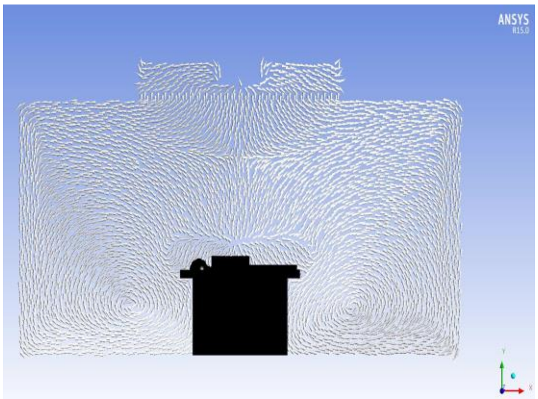
Firstly, numerous analyses have been made to determine the location and area of the suction grilles. In this regard, the analysis which is nearest to desired air distribution has been selected and presented evaluation about it. Each recirculating exit area for analysis is at 1.35 m^2.

3. Results

The effect of crystal curtain usage can be seen in Fig.5. When the crystal curtain which helps direct the laminar flow air towards the surgical area is not used, instabilities and high turbulent intensity are seen. As an improvement on desired air distribution, the design of crystal curtain includes a physical barrier which surrounds the surgical zone on all four sides. In this respect, the analysis was carried out for air ceiling model with crystal curtain. Besides, because laminar air ceiling dimensions are 3200x3200 mm, sterile laminar air flow covers not only operating table but also surgery team and medical equipment. Accordingly, area of high hygienic environment in the ORs is increased.



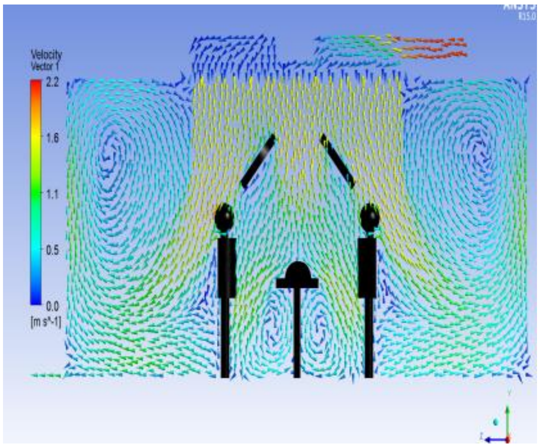
(a)



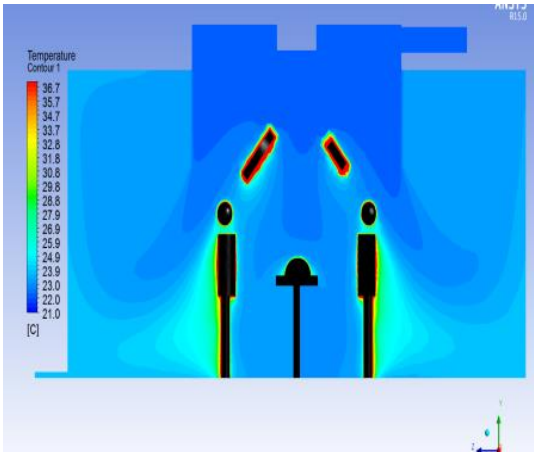
(b)

Figure 5. Comparison of air ceilings with (a) and without (b) crystal curtain unit

Following this identification, air velocity distributions in OR were investigated at different air inlet velocities. For example, for 2.2 m/s air inlet velocity, turbulent regions occur in the ORs and also, air velocities exceed the range of velocity mentioned in the standard (Figure 6a). As can be seen from Figure 6, the turbulent is formed in the upper regions out of crystal curtain, meanwhile, air velocities are changing at 0.5-1.1 m/s ranges. Air velocities in exit of crystal curtain are at 1.5-1.8 m/s ranges and these values exceed 6 times more than the desired value. Temperature values vary in the range of 21-27 ºC for the OR (Figure 6b).



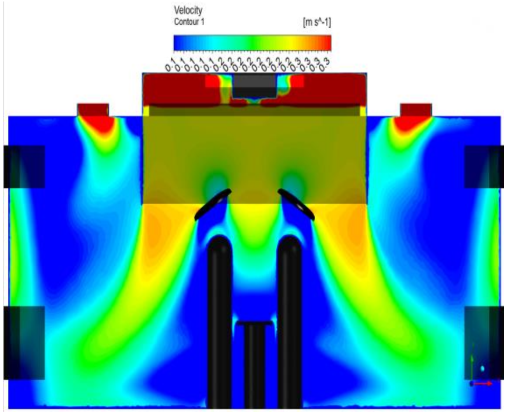
(a)



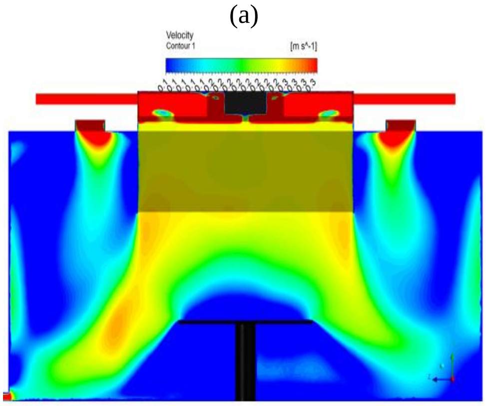
(b)

Figure 6. Air velocity (a) and temperature (b) distributions in OR

In the result of optimization studies performed for 0.5 m/s air inlet velocity to the mixing chamber, the range of 0.23-0.30 m/s mentioned in standard could be achieved (Figure 7). But air inlet velocity will change according to air ceiling dimensions and needed air flow rates.



(a)



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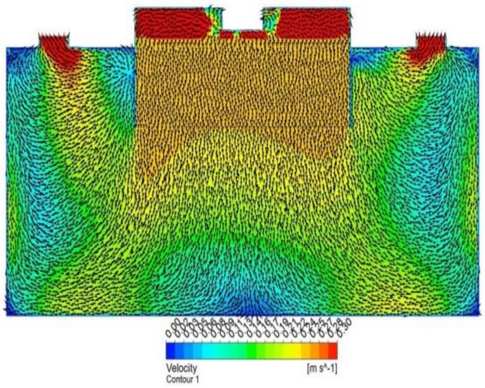
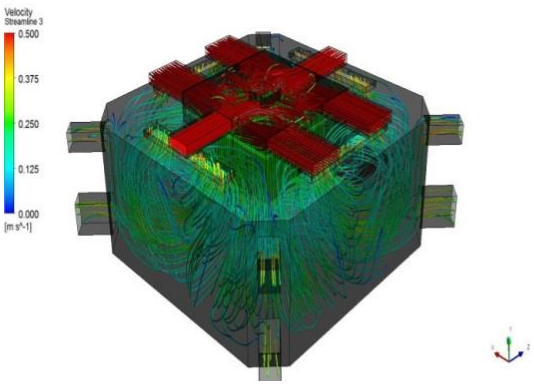


(c)



(d)

Figure 7. Optimized air velocity distribution for OR, (a) 2D xy coordinates (b) 2D zy coordinates (c) velocity vectors and (d) 3D air velocity distribution

4. Conclusion

The crystal curtain provides a barrier between the clean zone and the less clean zone. The CFD models clearly indicate that the directional flow from the laminarizator creates a customized air flow dynamic in the room. There is a significant increase in impact area for the OR. The analysis results are also evident that the equipment in the room causes turbulence that should be inhibited. At the laminarizator outlet, if air velocity is kept at 0.3 m/s or lower, turbulence will not occur. The majority of existing OR air conditioning systems are not convenient in terms of air mass flow rate, laminar flow air ceiling applications and dead volume forming. In this sense, the existing systems should be modified based on DIN 1946-4.

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Portfolio selection problem: a comparison of fuzzy goal programming and linear physical programming

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Abstract. Investors have limited budget and they try to maximize their return with minimum risk. Therefore this study aims to deal with the portfolio selection problem. In the study two criteria are considered which are expected return, and risk. In this respect, linear physical programming (LPP) technique is applied on Bist 100 stocks to be able to find out the optimum portfolio. The analysis covers the period from April 2009 to March 2015. This period is divided into two; April 2009-March 2014 and April 2014 – March 2015. April 2009-March 2014 period is used as data to find an optimal solution. April 2014-March 2015 period is used to test the real performance of portfolios. The performance of the obtained portfolio is compared with that obtained from fuzzy goal programming (FGP). Then the performances of both method, LPP and FGP, are compared with BIST 100 in terms of their Sharpe Indexes. The findings reveal that LPP for portfolio selection problem is a good alternative to FGP.

Keywords: Portfolio selection problem; linear physical programming; fuzzy goal programming.
AMS Classification: 90C70, 90C05

1. Introduction

The purpose of investors is to maximize the total return of their investments while considering the risk factor. In order to minimize the risk, a portfolio concept has arisen. Investing funds into a portfolio instead of one asset may be less risky because poor performance of one investment instrument can be easily balanced with good performance of another investment instrument.

In order to maximize the return of assets in their portfolio, investors need to manage the portfolio efficiently [1]. Portfolio management can be defined as the allocation of the funds between the securities for ensuring the maximum return and minimum risk [2]. In real world, the ambiguity exists because of uncertainty and the lack of efficient information. Therefore, portfolio selection problem is a challenging problem for researchers. And various studies have been done so far about portfolio selection problem.

Modern portfolio optimization studies began with the work of Markowitz in the 1950's. Markowitz suggested a mean-variance model. Markowitz studied how to ensure a portfolio that includes stocks with maximum return at a given level of risk [3]. Markowitz portfolio optimization model was the source of inspiration to many studies and was theoretically mostly known model, but the model was criticized for the need of gathering accurate information and the large number of calculations [4, 5].

Several authors have tried to develop Markowitz' modern portfolio theory. Sharpe [6] proposed to estimate the total risk of market

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instead of estimating the risk of each stock with simple regression model. Later on modern portfolio theory was elaborated with Sharpe [7], Lintler [8], Ross [9], Huberman [10] by proposing capital asset pricing and the multifactor arbitrage pricing models. Sharpe [11] and Lintler [12] developed Capital Asset Pricing Model (CAPM). In this model different from modern portfolio theory investors have the opportunity to invest in risk-free assets. And this theory was evolved with arbitrage pricing theory which was proposed by Ross [13] and extended by Huberman [14]

In later years, there were studies that try to transform the quadratic problems into linear problems such as [15], [16], [17] and [18]. One of the most popular of them was mean absolute deviation model. Konno and Yamazki [19] proposed an alternative model to quadratic models called as mean absolute deviation model. In this model, they accepted absolute deviation as the risk measure instead of the standard deviation. Many researchers try to extend portfolio selection problem by using linear models such as maximum model [20] minimax model [21, 22].

In real world, uncertainty exists for determining the expected return and expected risk of stocks, therefore some researchers have devoted considerable efforts to deal with the vague aspirations of a decision maker using fuzzy set theory. When the information about the objectives is naturally vague, Fuzzy goal programming (FGP) approach lets the involvement of decision makers (DMs) to the determination process of imprecise aspiration levels for the goals. FGP have already been applied to the portfolio selection problem by Parra et al. [23] and Alinezhad et al. [24].

In order to obtain the optimum stocks for portfolio, this study proposes to use linear physical programming (LPP) approach. In LPP approach, DM can take in account different goals and determine these goals in different desirability levels such as ideal desirable, tolerable, undesirable, highly desirable and unacceptable. The major advantage of linear physical programming is its capability of taking into account of numerous constraints, numerous goals and considering the preference range for the goals [25]. To show the effectiveness of the use of LPP in portfolio selection problem, FGP was also applied to the problem and the results of the both methods are compared.

The remainder of this paper is organized as follows. The second section briefly explains the linear physical programming method. Mathematical modelling of portfolio selection problem will be presented in the third section. In the fourth part, a real life portfolio selection problem will be solved under two conflicting objectives: maximum return and minimum risk possible. Finally conclusion and further research will be discussed..

2. Linear physical programming

LPP is a multi-objective optimization method that proposes specific algorithms for obtaining the weights of multiple objectives and use these weights in the optimization process to obtain optimal results [26].

Different from goal programming and fuzzy goal programming techniques that have already been applied to portfolio optimization problem, LPP uses the satisfaction levels (such as desirable, tolerable, undesirable, highly undesirable, or unacceptable) at which a particular goal (i.e. expected return) to obtain the weights and reach optimal results. Mainly, LPP distinguishes itself from the other techniques by removing the decision maker from the weight determination process [27].

Weight determination is one of the essential steps of multi-objective optimization which has inherently the challenge in determining the correct weights. A traditionally preprocessing constant weight determination may lead to bias in some cases [28]. In the LPP, it is not needed to set the weights of objectives in priori. Differently, LPP determines the weights in a systematic approach with the integration of the solution phase to find optimal results. Weight determination procedure uses the one versus others criterion rule (OVO rule) and contains a little complicated arithmetic. Therefore it needs to use a computer program to obtain the weights. The details of weight determination procedure can be found in [28]

In the LPP, DMs use four different classes named as soft classes to express their preferences according to each objective function. The most frequently used class functions, Class-1S (Smaller is Better) and Class-2S (Larger is Better), can be shown in figure 1. The decision variables, the *q*th objective function, the class function that will be minimized for the *q*th objective function are denoted by x and $g_q(x)$, Z_q respectively. In the figure 1, $g_q(x)$ is on the horizontal axis, Z_q is on the vertical axis. As it can be deduced from the figure, the smaller value of a class function improves the satisfaction level of the goal. Therefore, it is desired to obtain the value of the class function as zero. Besides soft classes, the constraints that must be satisfied without any deviation is defined as *Hard Classes*. Each soft class function is a part of the weighted aggregated objective function of LPP that is wanted to be minimized. The weights of Soft Class functions are determined by LPP weight algorithm [28].

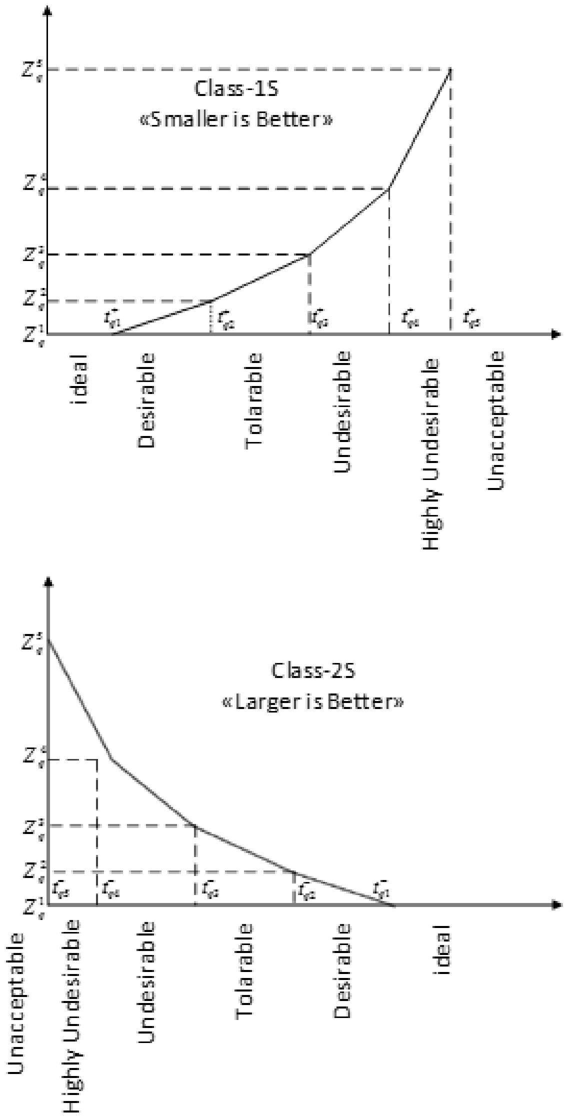

Figure 1. Smaller is Better and Larger is Better soft class functions [27]

Step 1. Selection of the appropriate soft and hard classes for each criterion,
Step 2. Determination of the target values that can be defined as the limits of the ranges of different degrees of desirability (i.e. t_{qs}^-, t_{qs}^+).
Step 3. Determination of the weights for each criterion by using the weight algorithm.
Step 4. Solving the following LP problem:

$$Min = \sum_{q=1}^{n_{sc}} \sum_{s=2}^{5} \left(w_{qs}^- d_{qs}^- + w_{qs}^+ d_{qs}^+ \right) \quad (1)$$

$$g_q - d_{qs}^+ \leq t_{q(s-1)}^+;$$

$$d_{qs}^+ \geq 0; g_q \leq t_{q5}^+, s = 2, \ldots, 5 \; \forall q \; in \; 1S \quad (2)$$

$$g_q + d_{qs}^- \geq t_{q(s-1)}^-;$$

$$d_{qs}^- \geq 0; g_q \geq t_{q5}^-, s = 2, \ldots, 5 \; \forall q \; in \; 2S \quad (3)$$

and *hard constraints*

where, s denotes a range, d_{qs}^+ and d_{qs}^- are deviational variables, n_{sc} denotes the number of soft classes, t_{qs} is the limit of different ranges, and, w_{qs} denotes the weight of range s in goal q. As it can be seen from Figure 1, there are five ranges that differs six degrees of desirability form ideal to unacceptable.

3. Model construction for portfolio selection problem

In a portfolio selection problem, it is assumed that there are N stocks from M sectors and K indexes to be selected for satisfying decision maker's objectives. The selected objectives are as follows;

Expected Rate of Return: The expected rate of return measures the return of each stock. The price of the stock *x* at time *t* is subtracted from the price of stock *x* at time (*t*-1) then divided by the price of stock x at time (*t*-1)

Risk: The standard deviation of the expected rate of return of each stock

The system parameters and assumptions are given in below.

- i stock type $i = 1,2,..n$.
- j sector type $j=1,2,..m$.
- k Stock indexes $k=1,2..,k$.
- X_i the ratio of stock i.

The mathematical representation of the objective functions are shown as below:

Objective 1: Maximization of the Expected Rate of Return

$$Z_1 = \sum_{i=1}^{n} (r_i * X_i) \quad (4)$$

Where r_i denotes the expected rate of return of the Stock i over the planning period.

Objective 2: Minimization of the Risk

$$Z_2 = \sum_{i=1}^{n} (\sigma_i * X_i) \quad (5)$$

Where σ_i represents the standard deviation of the expected rate of return of each stock over the planning period.

The constraints of the portfolio selection problem are represented below;

Constraint 1: The following formula ensures that the total weights of the stocks must equal to 1.

$$\sum_{i=1}^{n} X_i = 1 \quad (6)$$

Constraint 2: Beyond the objective of minimizing expected risk of portfolio, it is important to avoid allocating all resources to the small number of stocks which operates in the same sector. In order to diversify the portfolio, at least four different sectors must be included in the portfolio selected. In other words, the weights of each sector must be at most 25 %.

$$\sum_{i \in IE_1} X_i \leq 0.25, \forall j \quad (7)$$

Where SE_j represents the set of stocks which belong to the j^{th} sector.

Constraint 3: In order to ensure the long-term profitability and to maximize the possibility of success in the long run, the model proposes to invest at least 50 % or more on the firms in Bist 50 index.

$$\sum_{i \in IE_1} X_i \geq 0.5\left(\sum_{i \in IE_1} X_i + \cdots + \sum_{i \in IE_k} X_i\right) \quad (8)$$

Where IE_k represent the set of stocks which belong to the *kth* Bist index.

Moreover in the model lower and upper bound for each stock was decided as 0≤xj ≤0.1 in order to ensure diversity.

$$0 \leq X_i \leq 0{,}1 \quad (9)$$

3.1. Linear physical programming model for portfolio selection problem

To maximize the expected rate of return of the selected portfolio, the first goal is defined as Class-2S type (i.e. "Larger is Better).

$$\sum_{i=1}^{n}(r_i * X_i) + d_{1S}^{-} \geq t_{1(s-1)}^{-}; d_{1S}^{-} \geq 0;$$
$$\sum_{i=1}^{n}(r_i * X_i) \geq t_{15}^{-};\ s = 2, \ldots, 5 \quad (10)$$

The second goal is for portfolio risk measurement which is represented by Class-1S type (i.e. " Smaller is Better").

$$\sum_{i=1}^{n}(\sigma_i * X_i) - d_{2S}^{+} \leq t_{2(s-1)}^{+}; d_{2S}^{+} \geq 0;$$
$$\sum_{i=1}^{n}(\sigma_i * X_i) \leq t_{25}^{+};\ s = 2, \ldots, 5 \quad (11)$$

Then, the LPP model can be constructed as follows:

$$Min = \sum_{q=1}^{2}\sum_{s=2}^{5}\left(w_{qs}^{-} d_{qs}^{-} + w_{qs}^{+} d_{qs}^{+}\right) \quad (12)$$

Subject to (6) - (11).

3.2. Fuzzy goal programming model for portfolio selection problem

Since there are two conflicting objectives which force decision maker to accept trade-off values in the final decision, the problem can also be modelled by using fuzzy goal programming which can handle the ambiguity of the decision making process as follows:

Objective 1: The expected rate of return

$$\sum_{i=1}^{n}(r_i * X_i) \succ \widetilde{Z_1} \quad (13)$$

Where $\tilde{Z}_1$ represents the desirable achievement value for the expected rate of return objective. The symbol " $\succ$ " denotes the statement of "approximately greater than or equal to". The fuzzy goal can be expressed as a triangular membership function $\mu(Z_1)$ with tolerance limits for the goal (Z_1^L, Z_1^U) as follows:

$$\mu(Z_1) = \begin{cases} 1 & if\ Z_1^U \leq Z_1 \\ \frac{Z_1 - Z_1^L}{Z_1^U - Z_1^L} & if\ Z_1^L \leq Z_1 \leq Z_1^U \\ 0 & Z_1 \leq Z_1^L \end{cases} \quad (14)$$

The membership function for fuzzy expected rate of return goal is shown as in the Figure 2.

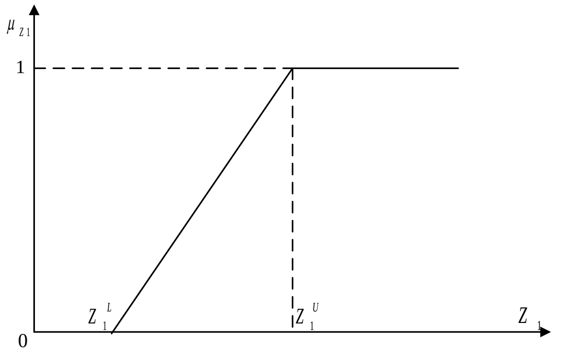

Figure 2. Membership function of fuzzy expected rate of return goal

Objective 2: The Risk

$$\sum_{i=1}^{n}(\sigma_i * X_i) \prec \widetilde{Z_2} \quad (15)$$

Where $\tilde{Z}_2$ represents the desirable achievement value for the risk. The symbol "$\prec$ " means that the objective function should be "approximately less than or equal to" the predefined limits. The fuzzy goal can be expressed as a triangular membership function $\mu(Z_2)$ with two parameters (Z_2^L, Z_2^U) as follows:

$$\mu(Z_2) = \begin{cases} 1 & if\ Z_2 \ \leq Z_2^L \\ \frac{Z_2^U - Z_2}{Z_2^U - Z_2^L} & if\ Z_2^L \leq Z_2 \ \leq Z_2^U \\ 0 & Z_2 \ \geq Z_2^U \end{cases} \quad (16)$$

The membership function for fuzzy risk goal is shown as in the Figure 3.

The lower and upper tolerance limits (i.e. Z^L, Z^U aspiration levels) are determined by constructing a pay-off table which contains the solutions of two single objective problems. In the solution methodology, the problem is solved separately with expected rate of return and risk objectives, respectively. Then the best and worst values are determined and used as the aspiration levels for the fuzzy goals.

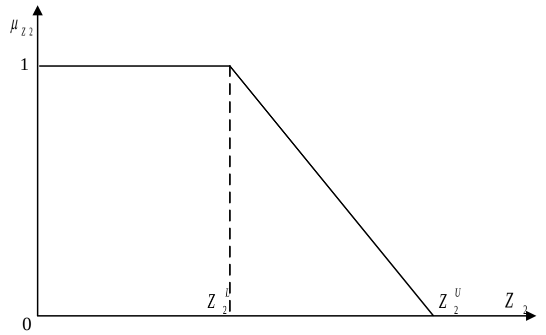

Figure 3. Membership function of fuzzy risk goal

After constructing fuzzy membership functions for the goals, fuzzy goal programming model can be presented as follows [29]:

$$Max = \lambda \quad (17)$$
$$\mu(Z_1) \geq \lambda$$
$$\mu(Z_2) \geq \lambda$$
Subject to (6) - (9)

Where λ denotes overall achievement level of fuzzy goals.

4. A portfolio selection model with the help of linear physical programming

In order to show the effectiveness of LPP on portfolio optimization problem, a real-life experimental study was performed by selecting stocks operating in Borsa İstanbul. The performance of the obtained portfolio is compared with that obtained from fuzzy goal programming (FGP). Then the performances of both methods, LPP and FGP are compared with BIST 100 in terms of their Sharpe Indexes. The details about numerical analysis can be found in the following subsections.

4.1. Results for linear physical programming model

In the model we consider two criteria: the expected return of stocks and risk. The sample consists of 89 companies that traded continuously at BIST 100 between April 2009 - March 2014. The observation period is April 2009- March 2015. This period is divided into two; April 2009-March 2014 and April 2014 – March 2015. April 2009-March 2014 period is used as data to find an optimal portfolio. April 2014-March 2015 period is used to test the real performance of selected portfolios. The expected rate of return values are calculated by using the closing prices at the beginning of each month for each stock. The data are gathered on monthly basis for April 2009 – March 2014 period. The number of observations gathered was 60.

The physical programming represents different desirability degrees for each criteria. These desirability degrees are expressed by using six types of ranges which are ideal, desirable, tolerable, undesirable, highly undesirable and unacceptable [30]. Table 2 represents the target values for expected rate of return and risk. Generally, decision makers estimate the target values based on their knowledge and experience. In the paper, the interval target values are also estimated, however, a payoff table (see Table 1) which contains the solutions of 2-single objective problem is constructed to estimate the max. and min. limits of these target values which are also used in constructing the FGP membership functions.

Table 1. Corresponding pay-off table

Objectives	Expected Rate of Return	Risk
Maximize Expected Rate of Return	4.059	16.326
Minimize Risk	2.148	10.347

Table 2. Target values for criteria's

	Expected Rate Of Return	Risk
Ideal	>4.0590	<10.340
Desirable	4.059-3.247	10.340-12.408
Tolerable	3.247-2.760	12.408-13.648
Undesirable	2.760-2.354	13.648-14.688
Highly Undesirable	2.354-2.140	14.680-16.320
Unacceptable	<2.14	>16.320

Once the class function is defined according to the target values, the LPP weight algorithm was used to calculate the weights presented below.

$$w_{12}^{+} = 0.1232, w_{13}^{+} = 0.2258,$$
$$w_{14}^{+} = 0.3626\ w_{15}^{+} = 1.5842$$
$$w_{22}^{-} = 0.0484, w_{23}^{-} = 0.0887,$$
$$w_{24}^{-} = 0.1411\ w_{25}^{-} = 0.0229$$

By solving the LLP mathematical model, 12 stocks were selected for our portfolio. Table 3 presents the stocks notations, their expected rate of returns, the risks for April 2009 – March 2014 period, Bist index classification and their sectors. And the last column shows the proportions of each stock in the portfolio for optimal solution.

Table 3. Selected stocks for portfolio with the help of linear physical programming

Notation	Expected Return	Risk	Bist Index	Proportions
DOAS E	3.5010	15.3098	50	0.0814
NTTUR E	1.4591	9.0580	100	0.0686
TCELL E	0.6816	6.3395	30	0.1000
CCOLA E	3.4262	8.5122	50	0.1000
TTRAK E	5.2838	11.6482	100	0.1000
ULKER E	3.9302	10.8275	30	0.0500
NTHOL E	3.8334	10.8504	100	0.1000
TAVHL E	3.3310	9.8978	30	0.1000
YAZIC E	2.4507	8.8134	100	0.0500
ASELS E	3.3459	16.3101	30	0.1000
LOGO E	4.1332	13.4042	100	0.1000
NETAS E	2.7919	23.2095	100	0.0500

4.2. Results for fuzzy goal programming model

In order to compare the performance of LPP, we have also solved the problem with FGP. The lower and upper tolerance limits are determined as in Table 1 by constructing a pay-off table which contains the solutions of 2-single objective problem. These max-min limits guarantee the feasibility of each fuzzy goal in the solution. Figure 4 and 5 shows the membership functions for satisfaction levels of the expected return and risk goals, respectively.

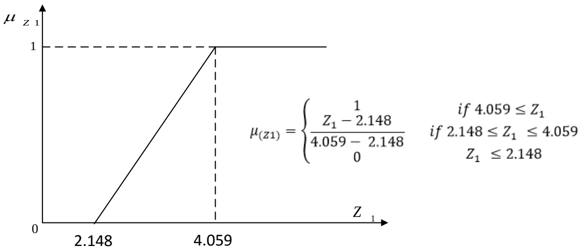

Figure 4. The membership function of expected return goal

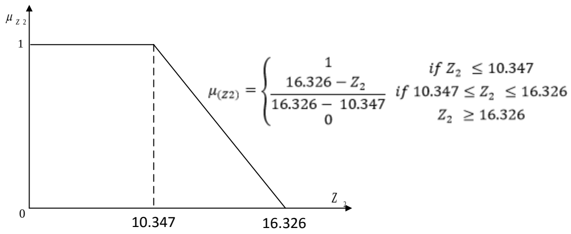

Figure 5. The membership function of risk goal

After applying FGP, the following results were obtained.

Table 4. Selected stocks for portfolio with the help of fuzzy goal programming

Notation	Expected Return	Risk	Bist Index	Proportions
DOAS E	3.5010	15.3098	50	0.1000
NTTUR E	1.4591	9.0580	100	0.0500
TCELL E	0.6816	6.3395	30	0.1000
CCOLA E	3.4262	8.5122	50	0.1000
TTRAK E	5.2838	11.6482	100	0.1000
ULKER E	3.9302	10.8275	30	0.0500
NTHOL E	3.8334	10.8504	100	0.1000
TAVHL E	3.3310	9.8978	30	0.1000
VKGYO E	5.7724	19.8805	100	0.0434
YAZIC E	2.4507	8.8134	100	0.0066
ASELS E	3.3459	16.3101	50	0.1000
LOGO E	4.1332	13.4042	100	0.1000
NETAS E	2.7919	23.2095	100	0.0500

Table 4 presents the stocks, their notation, their expected rate of return and the risk for April 2009 – March 2014 period when the problem is solved with the help of fuzzy goal programming. And the last column shows the weights of each stock in the portfolio for optimal solution.

The overall results of both LPP and FGP models are provided in Table 5. The results show that the portfolio returns obtained from LPP model is fewer than those obtained from the FGP model. However, the risk obtained from LPP is fewer than the one obtained from FGP model. Although both FGP and LPP models provide compromise solutions, the piecewise linear goal functions and multiple target values of LPP model allow to generate the different sets of Pareto optimal solutions.

Table 5. Results for fuzzy goal programming and linear physical programming

Objective	FGP	LPP
Expected Rate of Return	3.4292	3.247
Risk	12.3025	11.706

4.3. The performance of portfolio for control period

In order to test the performance of our portfolio, we use the control period. The control period is April 2014 – March 2015. The performance of our portfolio performance will be compared with the present market. We assume that the investor invests his/her fund in the selected portfolio determined by linear physical programming in April 2014 and hold this portfolio for 12 months till March 2015.

BIST 100 index is selected to represent the market. Firstly the return and risk are calculated for both BIST 100 and the selected portfolio on monthly basis. In order to compare the performance of selected portfolio and the index more vigorously, Sharpe index [31] was used. The success of the portfolio will be evaluated by comparing the Sharpe index of market and Sharpe index of selected portfolio. Higher Sharpe Index is better. So if the Sharpe Index value of the portfolio is higher than the Sharpe Index value of the market, the performance of portfolio will be better than the market. Sharpe index considers both risk and return at the same time. Sharpe index is calculated as follows;

$$S = (rp - rf)/\partial\, rp \qquad (17)$$

rp = The average return of portfolio for a given period,

rf = The average risk free interest rate (usually state bond or treasury bond interest rates are accepted) for a given period,

$\partial\, rp$= the standard deviation of portfolio for a given period (represents the risk criteria).

Table 6. Sharpe index value of Bist 100 and the portfolios selected with the help of LPP and FGP

	BIST 100 Index	LPP	FGP
Expected Rate of Return	% 2.4	% 4.33	%3.90
Risk	% 6.48	% 8.41	%8.96
Sharpe Index	0.26	0.43	0.36

Table 6 represents the expected rate of return, risk and Sharpe Index value of both BIST 100 and the portfolio obtained by LPP and FGP. Sharpe Index of BIST 100 and the portfolio are calculated for April 2014 and March 2015. The risk-free interest rate is taken as the average Treasury bond interest rate for April 2014 – March 2015 period (%0,7220). The findings reveal that the performance of the portfolio obtained by LPP was better than both the performance of BIST 100 Index and the portfolio obtained by FGP in terms of sharpe index.

5. Conclusion

The success of portfolio selection problem can only be ensured by successful selection of stocks. In this paper, a new technique for portfolio optimization problem with the aid of LPP is presented. The main purpose of the study is to find an optimum portfolio that maximizes the return which at the same time minimizes the risk. We compared the performance of the portfolios obtained by LPP and FGP approaches with the present market (BIST 100 Index) for the control period within April 2014 – March 2015 in terms of Sharpe index. The results revealed that LPP has potential to help the investors to find the efficient portfolio as much as possible. Finally, this study is thought to make contribution to literature by introducing the LPP for portfolio selection problems. Further research may consider more criteria and more constraints.

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Modeling of higher order systems using artificial bee colony algorithm

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Abstract: In this work, modeling of the higher order systems based on the use of the artificial bee colony (ABC) algorithm were examined. Proposed model parameters for the sample systems in the literature were obtained by using the algorithm, and its performance was presented comparatively with the other methods. Simulation results show that the ABC algorithm based system modeling approach can be used as an efficient and powerful method for higher order systems.

Keywords: Artificial bee colony algorithm; system modeling; parameter optimization
AMS Classification: 68T01, 93C05, 68W99, 93A30, 68U20

1. Introduction

In design of a control system, the use of an accurate model structure has a great significance to obtain the correct characteristic values of the system to be controlled. A mathematical definition procedure to characterize the system behavior by using input-output data of the system can be called as the system modeling [1-6]. Determination of the unknown parameters in the system model is usually performed by using an appropriate adaptation algorithm. In order to obtain a good model description, the elimination of the error value between the model and actual outputs by the algorithm is aimed [1, 4].

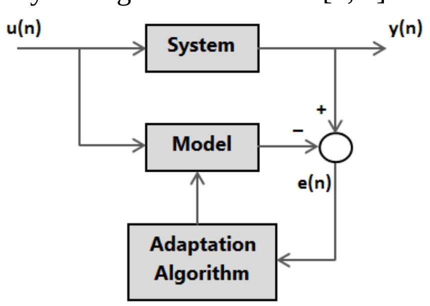

Figure 1. Basic block diagram for system modeling process

Basic block diagram of system modeling process is presented in Figure 1 [7]. In this figure, u(n), y(n) and e(n) are input, output and error signals, respectively.

Studies in recent years clearly prove that the artificial intelligence based algorithms can be widely and successfully used in the system modeling problems. Genetic algorithm (GA), differential evolution algorithm (DEA), clonal selection algorithm (CSA), particle swarm optimization (PSO) and artificial bee colony algorithm (ABC) are often used in the modeling of the systems [3-17]. In one of these studies presented by Zorlu and Ozer [3], a CSA is used in identification of nonlinear systems. In the study of Coelho and Pessoa [4], the authors use the GA in the nonlinear model identification process of an experimental ball-and-tube system. Luitel and Venayagamoorthy use the PSO approach for the modeling of the nonlinear systems [8]. In the study of Senberber and Bagis, the model parameters for time delay systems are estimated by using ABC algorithm [9]. In another study presented by Zorlu and Ozer [10], the parameter values of the proposed model

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structures for Box-Jenkins and a sample bilinear system problems are obtained by DEA and GA. Nam and Powers use the recursive least squares (RLS) method in the identification of the Volterra systems [11]. Performance of the ABC algorithm for time-delay model structure in the modeling of the unstable systems is examined by Senberber and Bagis [12]. A PSO based system modeling approach is presented by Deng [13]. In a study given by Bagis, the results of a system modeling approach based on the use of PSO method are presented in the definition of the higher order oscillatory systems by lower order models [14]. Ming and Dazi use new Luus-Jaakol algorithm (NLJ) for the identification of fractional systems in their work [15]. Least mean squares (LMS) algorithm is used for rapid system identification in the study given by Yu et al. [16]. In a detailed study conducted by the Senberber, different model structures have been proposed for the modeling of various systems with different characteristics and performance of the ABC algorithm in determination of the parameter values of these models have been presented as compared with the DEA and GA [17].
The main purpose of this paper is to examine the performance of the ABC algorithm in the modeling of higher order systems. For this reason, lower order model structures for the transfer functions of the various higher order systems given in the literature are taken into account. The parameter values of the proposed models are determined by using ABC algorithm. The simulation results are presented comparatively with the DEA, GA, and the model structures of the other methods given in the literature.
In the second section, the algorithms used in this study have been introduced briefly. The results of simulation studies obtained from different algorithms are given in the third section. The values of the model parameters, ISE error value, and computation times for the model parameters are presented in the tables in this section. Furthermore unit step responses of the systems and model results obtained by algorithms are also graphically given in this section. The conclusions of the study are presented in the final section.

2. The algorithms used for comparison

2.1 Artificial bee colony (ABC) algorithm

Artificial bee colony(ABC) algorithm utilizing the foraging behavior of honey bees was developed by Karaboğa in 2005 [17-25]. The basis of the algorithm is dependent on the proposed model by Tereshko and Loengarov to describe the foraging behavior of honey bee colonies [20]. This model has three main components; employed bees, unemployed bees and food sources [21]. Unemployed bees are divided into two groups, which are scout and onlooker.
Process steps of ABC algorithm can be given as follows [17-22]:

i) Scout bees randomly start the food searching process without any guidance.
ii) Nectar is moved to the hive from discovered food source by scout bees.
iii) Employed bees bring food to the hive, go back to the source or nectar source information is transferred to scout bees in hive via "information dance.
iv) Employed bee which consumes food, starts to work as a scout bee again and it searches the new food sources.
v) Information of food sources depending on the quality and frequency of the dance from employed bees is obtained by scout bees in the hive, and they are canalized to the food sources.

Considering this information, the basic steps of the ABC algorithm can be specified as the following [17, 21]:

1) *Produce an initial source*
2) *Send the employed bees to the food sources and determine the amount of nectar.*
3) *Calculate the selection probabilities of the sources by scout bees.*
4) *Selection of food sources by scouts bees according to the probability values.*
5) *Abandonment criteria: production of limit and scout bee*
6) *Number of Cycles: maximum number of cycles*
7) *Repeat the cycle until the stopping criteria is reached and return to step 2.*

ABC algorithm, that is a method based on swarm intelligence, quite closely simulates the actual foraging behavior of the bees. Main important features of the algorithm are that it has quite simple and flexible structure with few control parameters. Especially, the variations in the two important parameters such as colony size and limit value directly affect the quality of the solution. While global search is done by the scout bees, local search is done by the employed bees and onlooker bees. These searches are carried out in parallel. A general statement produced by employed and onlooker bees can be given in the following form. This definition specifies the location of new food sources.

$$v_{ij} = x_{ij} + \phi_{ij}(x_{ij} - x_{kj}) \quad (1)$$

Assuming SN is the number of employed or onlooker bees, ABC algorithm starts to work with a random initial population P consisting of SN solutions. In Equation (1), k ($k \in \{1,2,\ldots,SN\}$) a randomly chosen index that has to be different from i and j ($j \in \{1,2,\ldots,D\}$) are randomly selected indice, and φ_{ij} is a random value in the interval [-1,1]. In here, D is the number of optimization parameters.

The i[th] solution set in the population is defined by X_i. Thus, the size of variable 'i' is limited to the population or colony size. On the other hand, in Equation (1), the parameters of X_{ij} and X_{kj} can be considered as the current and neighbour solutions, respectively. And, a possible new solution is specified by the parameter of V_{ij} .

Quality value of each solution in the population is calculated by the employed bee of the related solution. When determining the new possible food source, a greedy selection method is used. This method makes the choice between the new source and the previous source in the memory.

If a position in the algorithm does not improve enough during the predetermined cycles, this means that the food source specified by the position is abandoned. This cycle size is an important and critical control parameter for the algorithm and it is called as limit. The value of this parameter is generally determined in proportion to the colony size and the number of parameters optimized. In this study, the limit parameter is defined according to the statement of [(colonies size x number of parameter) / 2]. Therefore, when the colony size for a sample system having a model with 5 parameters is selected as 10, the value of limit parameter is obtained as 25 ([(10x5) / 2]).

2.2 Differential evolution algorithm (DEA)

Differential evolution algorithm is a population-based heuristic algorithm to optimize the functions developed by Storn and Price [23, 24]. DEA introduced for solving numerical optimization problems is used the differences between solutions in the production of the new solutions during the simultaneous search at many points.

In the DEA benefiting from some main operators such as crossover, mutation, and selection to improve the solutions, the advancement in the solution quality is based on the use of mutation operator rather than crossover operation contrary to GA. The basis of this process is that the differences between the vectors defining possible solutions are multiplied by some coefficients such as scaling factor (F). Basic steps of the algorithm can be summarized as follows:

1. *Create the initial population*
2. *Evaluate the population members*
3. *Perform the following steps until the terminating criterion is achieved*
   - ***3.a.*** *Mutation*
   - ***3.b.*** *Crossover*
   - ***3.c.*** *Evaluation*
   - ***3.d.*** *Selection*

In the mutation process, the variation of the solutions is provided by using the differences between the population elements which are randomly selected. This difference is multiplied by a coefficient called as the scaling factor, and it is added to another population member. This situation for i[th] element of the population can be defined as follows:

$$v_i^{(G+1)} = x_a^{(G)} + F(x_b^{(G)} - x_c^{(G)}) \quad (2)$$

Where, V is the mutation vector, G is the generation, Xa, Xb, Xc are different members of the population, and F is the scaling factor. In this study, in order to produce a new mutation vector Vi, the element of Xa is assumed as Xbest which is the best individual with high quality in the population.

Then, a recombination or crossover operation is applied to the population. In this paper, the binomial type crossover operation is used in the algorithm. According to this procedure, the new solution element $u_{i,j,G}$ of the population is obtained as follows:

$$u_{i,j,G+1} = \begin{cases} v_{i,j,G+1} & \text{if } rand_j(0,1) < CR \text{ or } j = j_{rand} \\ x_{i,j,G} & \text{other} \end{cases} \quad (3)$$

Where, $V_{i,j,G+1}$ is the mutant vector, randj is a uniform random number in the interval of (0,1), CR is the crossover rate ($CR \in [0, 1]$), j =1, 2, .., D is an integer number, and j_{rand} is a random number in [1,D].

The selection process defines a criterion to propagate the fittest elements in the new generation for new produced solution vector. The performances of the trial vector $U_{i,G+1}$ and its parent $X_{i,G}$ are compared and the better one is selected according to their fitness values *f( )*. The selection process can be expressed as follows:

$$x_{i,G+1} = \begin{cases} u_{i,G+1} & if \;\; f\left(u_{i,G+1}\right) \leq f\left(x_{i,G}\right) \\ x_{i,G} & other \end{cases} \tag{4}$$

DEA is a fast, easy to use, simple approach that can provide effective convergence with low computational cost for complex optimization problems. Operation of the DEA is continued by improving the individuals in the population at each iteration, and when the desired solution or predetermined iterations are reached, search of the algorithm is terminated.

2.3 Genetic algorithm (GA)

The foundations of genetic algorithm (GA), which is used for solving various optimization problems in many field of the engineering in today, were firstly introduced by J. Holland [25-30]. Basic principle of the GA that is a population-based heuristic algorithm is relied on the two keynotes: (i) protection of the best, and (ii) obtaining the new individuals with better quality by utilizing the old solutions generated. For this purpose, initially, algorithm starts to the operation by the evaluation of a population which consists of a certain number of possible solutions (chromosome). Then, GA tries to improve the solution quality by performing operations of reproduction, crossover and mutation in the population. The main aim in these processes depending on the crossover and mutation rates is to achieve the high quality solutions instead of the poor quality solutions. Basic steps of the algorithm can be summarized as follows:

1. *Create the initial population*
2. *Determine the fitness (quality) values of the population elements*
3. *Apply the mating, crossover and mutation processes*
4. *Create a new population, determine the individual with most appropriate solution*
5. *If the stopping criterion is satisfied stop, otherwise return to step 2.*

The main important element of the GA is the crossover operator which is used to explore the search space for high quality new solutions. In this paper, single point crossover operator is employed as shown in Figure 2. In this process, crossover point is randomly determined, and then, the solution parts of the parents are mutually moved. Thus, new possible solutions for new population are obtained. On the other hand, the main principle in the mutation is the randomly alteration of the genes on the chromosome. In this operation, according to mutation rate which is in the interval of [0-1], some genes on the chromosome are randomly determined, and the values of these elements are changed into a different value. A typical mutation process is given in Figure 3.

crossover point

Parent 1	→	1	1	0	1	1	**0**	**0**	**1**	**0**	**0**
Parent 2	→	1	0	0	1	0	1	1	0	0	0
New Individual 1	→	1	1	0	1	1	1	1	0	0	0
New Individual 2	→	1	0	0	1	0	**0**	**0**	**1**	**0**	**0**

Figure 2. Single point crossover operation

Individual	→	1	1	0	1	1	1	1	0	0	0
					↓						
Mutant Individual	→	1	1	0	**0**	1	1	1	0	0	0

Figure 3. Typical mutation operation

Predictably, some critical decisions such as the coding of the solutions, crossover type, crossover and mutation rates directly affect the performance of the GA that can provide fast solutions depending on the population size used.

3. Simulation results

In this study, model structures for some examples of high-order systems in the literature are proposed, and the parameters of these model structures are obtained by using the ABC algorithm. This work is also repeated by using DEA and GA, and then, the results are comparatively presented. Four different high-order systems, which are previously given in the literature, are used in this study (Equations 5-8) [14, 31-39]. Time delay does not exist in the systems defined as G1 [31], G2 [14, 32-36], G3 [37, 38], and G4 [39]. The proposed model structure for these systems is stated as Gm in Eq.9. The number of parameters that must be obtained by the algorithms is 5 for the model considered.

Control parameters of algorithms that are used to obtain the proposed model parameters are presented in Table 1. The studies were repeated for 30 times at least in the model, and the control parameters in the studies with the values of the best model parameters are given in this table. In the simulations, Matlab program package and Intel Core 2 Duo E7500 2.93 GHz computer were used [40-42].

$$G1(s)=\frac{35s^7+1086s^6+13285s^5+82402s^4+278376s^3+511812s^2+482964s+194480}{s^8+21s^7+220s^6+1558s^5+7669s^4+24469s^3+46350s^2+45952s+17760} \quad (5)$$

$$G2(s)=\frac{18s^7+514s^6+5982s^5+36382s^4+122664s^3+222088s^2+185760s+40320}{s^8+36s^7+546s^6+4536s^5+22449s^4+67284s^3+118124s^2+109584s+40320} \quad (6)$$

$$G3(s)=\frac{6s^4+50s^3+196s^2+418s+434}{s^6+12s^5+71s^4+256s^3+575s^2+804s+585} \quad (7)$$

$$G4(s)=\frac{25.2s^2+21.2s+3}{s^5+16.58s^4+25.41s^3+17.18s^2+11.70s+1} \quad (8)$$

$$Gm(s)=\frac{ps+q}{as^2+bs+c} \quad (9)$$

The parameter values obtained by the algorithms in the proposed model for different systems are presented in Table 2. The information of the ISE (integrated squared error) error value, computation time for the algorithms and standard deviation are also given in the table. Unit step responses of the model obtained by the algorithms and iteration-error variations during the optimizations for all processes are collectively presented in Figure 4. Step response curves for G1, G2, G3, and G4 processes are illustrated in Figures 4(a), (c), (e), and (g). The iteration-error changes belonging to these curves are stated in Figures 4(b), (d), (f), and (h). Because it is sufficient to measure the model performance, the time intervals in the simulations are used as [0-6] sec for G1, G2, and G3 processes. This time period for G4 process is in the interval of [0-30] sec.

Table 1. Control parameters of the algorithms used in this study

Control Parameter		DEA	GA	ABC
Population (or colony) Size		10	10	10
Crossover Rate		0.9	0.9	-
Mutation Rate		-	0.3	-
Scaling Factor (F)		0.8	-	-
Combination Factor		0.8	-	-
Limit		-	-	25
Number of Generation (or iteration)	G1	300	300	300
	G2, G3, G4	100	100	100
Parameter Search Interval		[0-100]		

It can be clearly seen from Table 2 that the DEA based models have the lowest ISA error value nearly for all systems. For G1, G2, and G4 processes in the case of using DEA, the lowest error values such as 0.724, 4.090x10-4, and 3.290 as compared with other algorithms are obtained. Similarly, a very low ISE value as 7.52x10-4 can be achieved for G3 process by DEA again. On the other hand, it should be momentously noted that the error performance of the ABC algorithm is impressively good for all models. The error value obtained based on the use of ABC algorithm for G2, G3, and G4 is found almost equal to the DEA. The error value in the model of G1 is achieved at the low values as 1.224. Thus, it would not be wrong to argue that the error performances of the DEA and ABC algorithms are close to each other for all models examined. However, the modeling performance of the GA lies distinctly behind the other algorithms.

On the other hand, it can be clearly seen from Table 2 that an interesting feature of the ABC algorithm is noted in the table: computation time of the ABC algorithm for model parameters is dramatically longer than the other algorithms. This time none of the systems in the ABC approach is close to other algorithms, and moreover, it is three or four times more of the calculation costs in the other algorithms for the processes. For the modeling of the G1 and G2 processes, the computation times in DEA algorithm is obtained as 44 sec and 39 sec, at these times ABC algorithm is approximately noted as 135 sec and 184 sec, respectively.

Table 2. Parameter values of the models based on the algorithms used for different systems

System	Alg.	Model Parameters					Error (ISE)	Time (sec)	Standard Deviation
		p	q	a	b	c			
G1	DEA	10.079	93.631	0.249	0.448	8.763	0.724	44.349	0
	GA	99.711	71.669	0.949	8.776	6.819	29.540	42.658	12.349
	ABC	8.177	98.915	0.248	0.476	9.208	1.224	135.076	7.569
G2	DEA	68.779	20.419	4.021	28.050	20.728	4.090×10^{-4}	39.224	0
	GA	77.036	33.659	5.216	30.830	30.569	2.307×10^{-2}	43.590	5.610×10^{-2}
	ABC	19.697	5.889	1.152	8.024	5.957	4.150×10^{-4}	184.357	1.151×10^{-2}
G3	DEA	4.8102	58.988	13.293	39.450	79.828	7.52×10^{-4}	22.196	0.003509
	GA	50.613	46.124	54.565	63.249	79.427	42.81×10^{-4}	18.169	0.0015
	ABC	1.471	45.132	9.345	27.064	60.561	0.71×10^{-4}	45.420	0.003415
G4	DEA	3.91×10^{-4}	61.3966	32.3769	20.7600	22.0936	3.290	13.391	2.452×10^{-2}
	GA	9.6599	77.334	49.423	27.828	27.934	3.595	13.452	8.729×10^{-1}
	ABC	1.00×10^{-6}	88.899	46.422	31.015	31.956	3.291	30.440	2.632×10^{-2}

Modeling times of the G3 process for DEA, GA, and ABC methods occur as 22sec, 18sec, and 45sec, respectively again. In the modeling of G4, while the computation time is about 13 sec for the other algorithms, this time it is 30 sec for ABC algorithm. This situation is no doubt a negative feature that should not be ignored for the ABC algorithm.

From the results given in Table 2, it can be obviously show detected that the standard deviation values of the DEA indicate a striking accuracy to achieve the same ISE values. In the modeling of the other systems except for G1, the ABC algorithm exhibits satisfying standard deviation values. Although the order of the systems and the number of numerator-denumarator components in the processes of G1 and G2 is same, it is an interesting fact that while a high consistency is achieved in the modeling of G2 by the ABC, this consistency level in the modeling of G1 is undesirably low.

From the step responses of the models in Figure 4, it can be seen that there is a good agreement among the ABC, DEA, and actual process outputs in general. Admittedly, the proposed model structure is not sufficiently successful to define the process of G4. Howbeit, in order to see the power of the ABC algorithm, is an appropriate example. On the other hand, the modeling performance of the GA is quite behind the other algorithms in accordance with the error values given in Table 2, and thus, it is not successful in terms of the agreement of the model-actual outputs observed in the Figure 4.

As mentioned previously, from the results in Table 2, the DEA exhibits a better performance with less computational effort than those of ABC algorithm in terms of the modeling error. At this point, an interesting and exciting result is clearly observed when the curves of iteration-error change in Figure 4 are examined: the ABC algorithm can reduce the error at the remarkably lower iteration numbers than the other algorithms. For example, for all of the processes, while the ISE error is dramatically eliminated in the first 10 iterations by the ABC algorithm, DEA and GA need the use of high iteration numbers for the optimization. For a considerable reduction in the ISE error, 50 iterations approximately are spent by the DEA.

In the G2 and G3 processes, this algorithm does not come close to the error value achieved by the ABC during the 20 iterations at least. The similar situation is also present for the modeling of G4 process. For this process, the remarkable error reductions in the first 10 iterations of the ABC algorithm are hardly achieved at about 50 iterations by the DEA. It can be said that the error reduction performance of the ABC algorithm is highly impressive according to other methods.

To objectively evaluate the performances of the algorithms, the error values of the different approaches in the literature are presented in Table 3 as compared with the error values of the models optimized. In this table, the model structures that can be used for each process are shown as more clearly. From the values in the Table 3, it is possible to say that the error values of the ABC based model are much better than those of the other methods stated in the literature.

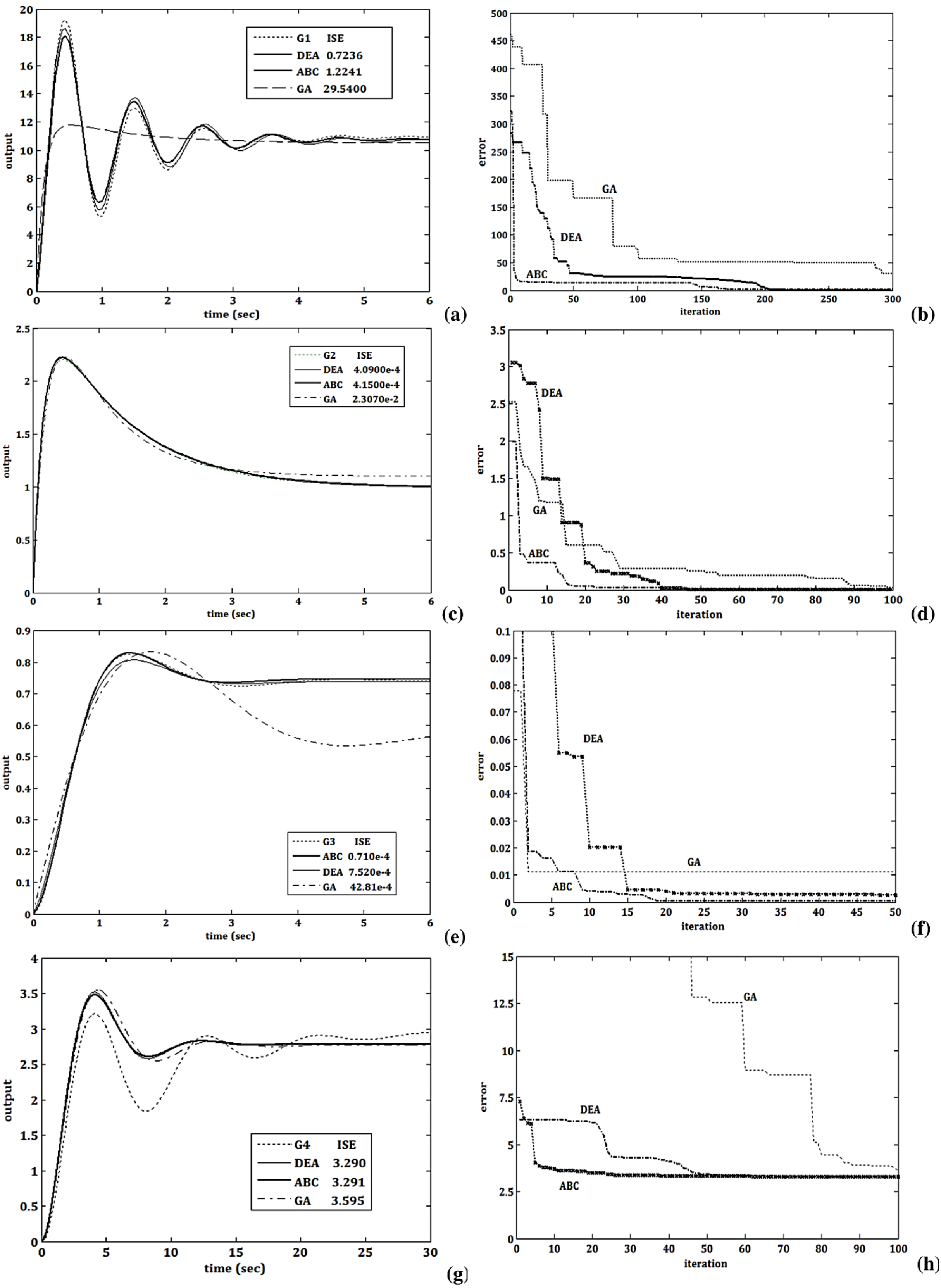

Figure 4. Unit step responses and iteration-error variation curves of the models obtained by the algorithms for each process

Table 3. Proposed model structures in the literature for the systems used

System	Method/Model/Error	System	Method/Model/Error
G1	**Manigandan [31]** $\frac{35s+401.21}{s^2+1.436s+36.63}$ ISE: 2.5021	G2	**Lucas [32]** $\frac{18s+5.603}{s^2+7.415s+5.603}$ ISE: 2.000×10^{-3}
	DEA $\frac{10.079s+93.631}{0.249s^2+0.448s+8.763}$ **ISE: 0.724**		**Krajewski [32]** $\frac{17.23s+6.375}{s^2+7s+6}$ ISE: 1.800×10^{-1}
	GA $\frac{99.711s+71.669}{0.949s^2+8.776s+6.189}$ ISE: 29.54		**Mukherjee [32]** $\frac{16.89s+5.279}{s^2+6.85s+5.279}$ ISE: 7.081×10^{-4}
	ABC $\frac{8.177s+98.915}{0.248s^2+0.476s+9.208}$ **ISE: 1.224**		**Shamash [33]** $\frac{6.779s+2}{s^2+3s+2}$ ISE: 7.3183
G3	**Parmar[37]** $\frac{5.275}{s^2+3.051s+7.109}$ ISE: 4.174×10^{-3}		**Prasad [34]** $\frac{17.986s+500}{s^2+13.25s+500}$ ISE: 18.430
	Layer [38] $\frac{6}{s^2+3.66s+7.78}$ ISE: 3.363×10^{-3}		**Mittal [35]** $\frac{7.091s+1.991}{s^2+3s+2}$ ISE: 6.9159
	DEA $\frac{4.8102s+58.988}{13.293s^2+39.45s+79.828}$ ISE: 7.52×10^{-4}		**Parmar [36]** $\frac{24.144s+8}{s^2+9s+8}$ ISE: 1.792
	GA $\frac{50.613s+46.124}{54.565s^2+63.249s+79.427}$ ISE: 4.281×10^{-3}		**Bagis [14]** $\frac{4.256s+1.263}{0.248s^2+1.735s+1.282}$ ISE: 4.176×10^{-4}
	ABC $\frac{1.471s+45.132}{9.345s^2+27.064s+60.561}$ **ISE: 0.71×10^{-4}**		**DEA** $\frac{68.779s+20.419}{4.021s^2+28.05s+20.728}$ **ISE: 4.090×10^{-4}**
G4	**DEA** $\frac{3.9059.10^{-4}s+61.3966}{32.3769s^2+20.76s+22.0936}$ **ISE: 3.290**		**GA** $\frac{77.036s+33.659}{5.216s^2+30.83s+30.569}$ ISE: 2.307×10^{-2}
	GA $\frac{9.6599s+77.334}{49.423s^2+27.828s+27.934}$ ISE: 3.595		**ABC** $\frac{19.697s+5.889}{1.152s^2+8.024s+5.957}$ **ISE: 4.150×10^{-4}**
	ABC $\frac{1.00.10^{-6}s+88.899}{46.422s^2+31.015s+31.956}$ **ISE: 3.291**		

4. Conclusions

In this study, an investigation about the definition of the higher-order systems by the lower order models is presented. For this aim, transfer functions of some higher-order systems in the literature are taken into consideration, and several second-order models with 5 parameters are proposed to describe these systems. Determination of the model parameters with the smallest error value is expected from the ABC algorithm. In order to provide a comparison with the literature, ISE error index is used as the performance criteria. Simulation studies are also repeated by using DEA and GA, and the results obtained are comparatively presented in the tables. Other model structures in the literature and error values are also included in the tables.

From the simulation results, the general evaluations given below can be done:

a. When considering the unit step input behavior, appropriate approaches through the use of second order low-order models to describe the higher order systems can be obtained. It is clear that, if different characteristic values of time and frequency domains are taken into consideration, the abilities to define the processes of these models can be improved and their accuracy properties can be increased.
b. It is important for us to investigate the system modeling performance of the ABC algorithm that presents the efficient results for numerical optimization problems especially. Error reduction performance of the ABC algorithm is satisfactory in the determination of model parameters. According to simulation results, it is possible to obtain the lower values of the error in very low iterations by using this approach.
c. In order to achieve the low error values, long computation time of the ABC algorithm as compared with the other algorithms is a negative feature of the method. For a good time cost-performance relationship, some improvements and time-recovering adjustments in the program used for the operation are clearly required.
d. When compared to the other algorithms in this study and the error values of the other models in the literature, it is possible to say that the modeling performance of the ABC algorithm is in a successful level. The performance results presented by the algorithm proved that ABC algorithm can be safely and effectively used in the modeling and controlling of more complex and more oscillatory systems for various purposes.

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New complex exact travelling wave solutions for the generalized-Zakharov equation with complex structures

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Abstract. In this paper, we apply the sine-Gordon expansion method which is one of the powerful methods to the generalized-Zakharov equation with complex structure. This algorithm yields new complex hyperbolic function solutions to the generalized-Zakharov equation with complex structure. Wolfram Mathematica 9 has been used throughout the paper for plotting two- and three-dimensional surface of travelling wave solutions obtained.

Keywords: The sine-Gordon expansion method; generalized-Zakharov equation with complex structure; complex hyperbolic function solution; dark soliton solutions.
AMS Classification: 35Axx; 35Cxx; 34Mxx

1. Introduction

The new complex exact travelling wave solutions of nonlinear partial differential equations plays an important role in various fields such as engineering, plasma physics, solid state physics, optical fibers, quantum field theory, hydrodynamics, fluid dynamics and applied sciences. They submit to the literature new reviews in terms of better understanding of mathematical models of physical problems. Especially various type travelling wavesolutions such as dark, complex, elliptic, Jacobi elliptic, exponential, rational, hyperbolic and trigonometric function solutions means that they have new properties of physical problems. In the process many powerful methods such as sumudu transform method, Riccati-Bernoulli sub-ODE method, G'/G-expansion method, Exp-function method, Fitted finite difference method, extended jacobi elliptic function expansion method, modified simple equation method and Generalized Bernoulli Sub-ODE method, functional variable method, variational iteration method, improved Bernoulli sub-equation function method, Laplace-variationaliteration method, finite difference method, generalized Kudryashov method and so on have been used to find new solutions of nonlinear evolution equations [1-14,27-50]. In the rest of this paper, we present the general properties of the sine-Gordon expansion method(SGEM) in comprehensive manner in section 2. In section 3, we obtain the complex travelling wavesolutions to the generalized- Zakharov equation with complex structure which reads as following [15]:

$$\begin{aligned} & iu_t + u_{xx} - 2a|u|^2 u + 2uv = 0, \\ & v_{tt} - v_{xx} + \left(|u|^2\right)_{xx} = 0, \end{aligned} \tag{1}$$

where a is real constants and non-zero. In the last section of manuscript, a comprahensive conclusion has been submitted by mentioning significant properties of $u(x,t)$ and $v(x,t)$.

Shi Jin, P. A. Markowich and C. Zheng have applied the time-splitting spectral method for obtaining numerical solutions of Eq.(1) [24]. Yuhuai Sun et al. have considered the first integral method for finding exact explicit solutions of Eq.(1) [25]. Malomed B. et al. have investigated

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the Dynamics of Solitary Waves of Eq.(1) [26].

2. General facts of the SGEM

Let's consider the following sine-Gordon equation [16-18, 51];

$$u_{xx}-u_{tt}=m^2\sin(u), \tag{2}$$

where $u=u(x,t)$, and m is real constant. When we apply the wave transform $\xi=\mu(x-ct)$ to Eq.(2), we obtain the nonlinear ordinary differential equation (NODE) as following;

$$U''=\frac{m^2}{\mu^2(1-c^2)}\sin(U), \tag{3}$$

where $U=U(\xi)$, and, ξ is the amplitude of the travelling wave, c is the velocity of the travelling wave. If we reconsider Eq.(3), we can write in the full simplified version as following;

$$\left[\left(\frac{U}{2}\right)'\right]^2=\frac{m^2}{\mu^2(1-c^2)}\sin^2\left(\frac{U}{2}\right)+K, \tag{4}$$

where K is the integration constant. When we resubmit as $K=0,\ w(\xi)=\frac{U}{2}$, and $a^2=\frac{m^2}{\mu^2(1-c^2)}$ in Eq.(4), we can obtain following equation;

$$w'=a\sin(w). \tag{5}$$

If we put as $a=1$ in Eq.(5)($a=1$, for convenience [16]), we can obtain following equation;

$$w'=\sin(w). \tag{6}$$

If we solve Eq.(6) by using separation of variables, we find the following two significant equations;

$$\sin(w)=\sin(w(\xi))=\left.\frac{2pe^{\xi}}{p^2e^{2\xi}+1}\right|_{p=1}, \quad =\sec h(\xi), \tag{7}$$

or

$$\cos(w)=\cos(w(\xi))=\left.\frac{p^2e^{2\xi}-1}{p^2e^{2\xi}+1}\right|_{p=1}, \quad =\tanh(\xi), \tag{8}$$

where p is the integral constant and non-zero. For obtaining the solution of following nonlinear partial differential equation;

$$P(u,u_x,u_t,\cdots)=0, \tag{9}$$

let's consider as

$$U(\xi)=\sum_{i=1}^{n}\tanh^{i-1}(\xi)[B_i\sec h(\xi)+A_i\tanh(\xi)]+A_0. \tag{10}$$

We can rewrite Eq.(10) according to Eqs.(7,8) as following;

$$U(w)=\sum_{i=1}^{n}\cos^{i-1}(w)[B_i\sin(w)+A_i\cos(w)]+A_0. \tag{11}$$

Under the terms of homogenous balance technique, we can determine the values of n under the terms of $NODE$. Let the coefficients of $\sin^i(w)\cos^j(w)$ all be zero, it yields a system of equations. Solving this system by using Wolfram Mathematica 9 give the values of A_i,B_i,μ,c. Finally, substituting the values of A_i,B_i,μ,c in Eq.(10), we can find the new travelling wave solutions to the Eq.(9).

3. Implementations of proposed method

In this subsection of this paper, we provide some experimental results to illustrate the performance of the travelling wave algorithm proposed.

Example: We consider the traveling wave transformation defined by

$$u(x,t)=e^{i\theta}U(\xi),\ \theta=\alpha x+\beta t, \quad v(x,t)=V(\xi),\ \xi=x-2\alpha t, \tag{12}$$

where α,β are real constant and non-zero. When we can apply Eq.(12) to the Eq.(1), we can find the following $NODE$ under the some simplifications [15];

$$V(\xi)=\frac{c-U^2(\xi)}{4\alpha^2-1}, \tag{13}$$

where c is second integration constant and the first one is taken to zero. Considering Eq.(13), we rewrite the following ODE [15];

$$RU''+SU+TU^3=0, \tag{14}$$

where $R=-1,\ S=\beta+\alpha^2-\frac{2c}{4\alpha^2-1}$, $T=2\left(a+\frac{1}{4\alpha^2-1}\right)$. When we reconsider the Eq.(11) for homogenous balance method between U'' and U^3, we obtain the value of n as following;

$$n=1. \tag{15}$$

If we put Eq.(15) in Eq.(11), we obtain follows;

$$U(w)=B_1\sin(w)+A_1\cos(w)+A_0, \quad (16)$$

$$U'(w)=B_1\cos(w)\sin(w)-A_1\sin^2(w), \quad (17)$$

$$U''(w)=B_1\left[\cos^2(w)\sin(w)-\sin^3(w)\right] -2A_1\sin^2(w)\cos(w). \quad (18)$$

Substituting Eqs.(16,18) in Eq.(14) by using Wolfram Mathematica 9, we can obtain following equation;

$$\begin{aligned} &SA_0+TA_0^3+S\cos(w)A_1+3TA_0^2A_1\cos(w)\\ &-2RA_1\cos(w)\sin^2(w)+3TA_1^2A_0\cos^2(w)\\ &+TA_1^3\cos^3(w)+SB_1\sin(w)+TB_1^3\sin^3(w)\\ &+B_1R\sin(w)\cos^2(w)-RB_1\sin^3(w)\\ &+3TA_0^2B_1\sin(w)+6TA_0A_1B_1\sin(w)\cos(w)\\ &+3TA_1^2B_1\sin(w)\cos^2(w)+3TB_1^2A_0\sin^2(w)\\ &+3TB_1^2A_1\cos(w)\sin^2(w)=0. \end{aligned} \quad (19)$$

When we equal to zero all the same power of trigonometric terms, we find the following equations;

$$\begin{aligned} &Cons\tan t: SA_0+TA_0^3+3TA_1^2A_0=0,\\ &\sin(w): SB_1+3TA_0^2B_1=0,\\ &\cos(w): SA_1-2RA_1+3TA_0^2A_1+3TB_1^2A_1=0,\\ &\sin^2(w): 3TB_1^2A_0-3TA_1^2A_0=0,\\ &\sin(w)\cos(w): 6TA_0A_1B_1=0,\\ &\cos^2(w)\sin(w): B_1R+3TA_1^2B_1=0,\\ &\sin^3(w): TB_1^3-RB_1=0,\\ &\cos^3(w): 2RA_1+TA_1^3-3TB_1^2A_1=0. \end{aligned} \quad (20)$$

Solving the system of equations Eq.(20) yields the following coefficients:

$$A_0=0, A_1=\frac{-\sqrt{-1+4\alpha^2}}{\sqrt{1+a\left(-1+4\alpha^2\right)}}, \quad \beta=\frac{2+2c-7\alpha^2-4\alpha^4}{-1+4\alpha^2}, B_1=0. \quad (21)$$

$$A_0=0, A_1=\frac{\sqrt{-1+4\alpha^2}}{\sqrt{1+a\left(-1+4\alpha^2\right)}}, \quad \beta=\frac{2+2c-7\alpha^2-4\alpha^4}{-1+4\alpha^2}, B_1=0. \quad (22)$$

$$A_0=0, A_1=A_1, B_1=0, \quad c=\frac{1}{2}\left(-1+4\alpha^2\right)\left(2+\alpha^2+\beta\right), \quad a=\frac{1}{1-4\alpha^2}+\frac{1}{A_1^2}. \quad (23)$$

Substituting Eq.(21) coefficients in Eq.(12) along with Eq.(16) for $u(x,t)$ and in Eq.(13) for $v(x,t)$, we obtain the complex hyperbolic function solution to the Eq.(1) as following;

$$u_1(x,t)=re^{i(\alpha x+wt)}\tanh(x-2t\alpha), \quad (24)$$

$$v_1(x,t)=\frac{c}{-1+4\alpha^2}-\frac{1}{\vartheta}e^{2i(\alpha x+wt)}\tanh^2(x-2t\alpha),$$

where

$$r=\frac{-\sqrt{-1+4\alpha^2}}{\sqrt{1+a\left(-1+4\alpha^2\right)}}, w=\frac{2+2c-7\alpha^2-4\alpha^4}{-1+4\alpha^2},$$

$$\vartheta=1+a\left(-1+4\alpha^2\right).$$

When we consider the Eq.(22) coefficients in Eq.(12) along with Eq.(16) for $u(x,t)$ and in Eq.(13) for $v(x,t)$, we find another complex hyperbolic function solution to the Eq.(1) as following;

$$u_2(x,t)=pe^{i(\alpha x+kt)}\tanh(x-2\alpha t), \quad (25)$$

$$v_2(x,t)=\frac{c}{-1+4\alpha^2}-\frac{e^{2i(\alpha x+kt)}}{\varpi}\tanh^2(x-2\alpha t),$$

where

$$p=\frac{\sqrt{-1+4\alpha^2}}{\sqrt{1+a\left(-1+4\alpha^2\right)}}, \varpi=1+a\left(-1+4\alpha^2\right),$$

$$k=\frac{2+2c-7\alpha^2-4\alpha^4}{-1+4\alpha^2}.$$

Substituting the Eq.(23) coefficients in Eq.(12)

along with Eq.(16) for $u(x,t)$ and in Eq.(13) for $v(x,t)$, we find another hyperbolic function solution to the Eq.(1) as following;

$$u_3(x,t)=A_1 e^{i(\alpha x+\beta t)}\tanh(x-2\alpha t),$$
$$v_3(x,t)=\frac{\nu-2e^{2i(\alpha x+\beta t)}A_1^2\tanh^2(x-2\alpha t)}{-2+8\alpha^2}, \quad (26)$$

where $\nu=(-1+4\alpha^2)(2+\alpha^2+\beta)$.

4. Tables and Figures

In this subsection of paper, we have plotted two- and three-dimensional surfaces of travelling wave solutions obtained in this paper under the suitable values of parameters by using SGEM as follows.

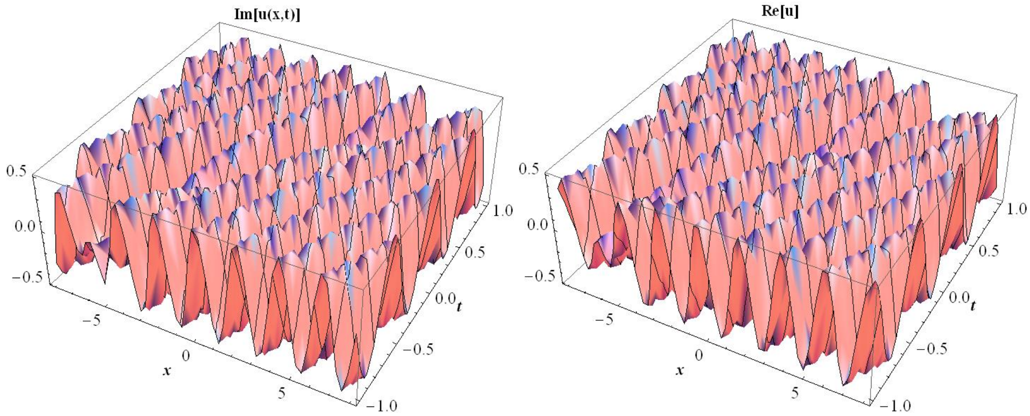

Figure 1. The 3D surfaces of u_1 of Eq.(24) under the terms of considering the values $c=5,\ a=4, \alpha=3, -8<x<8, -1<t<1$.

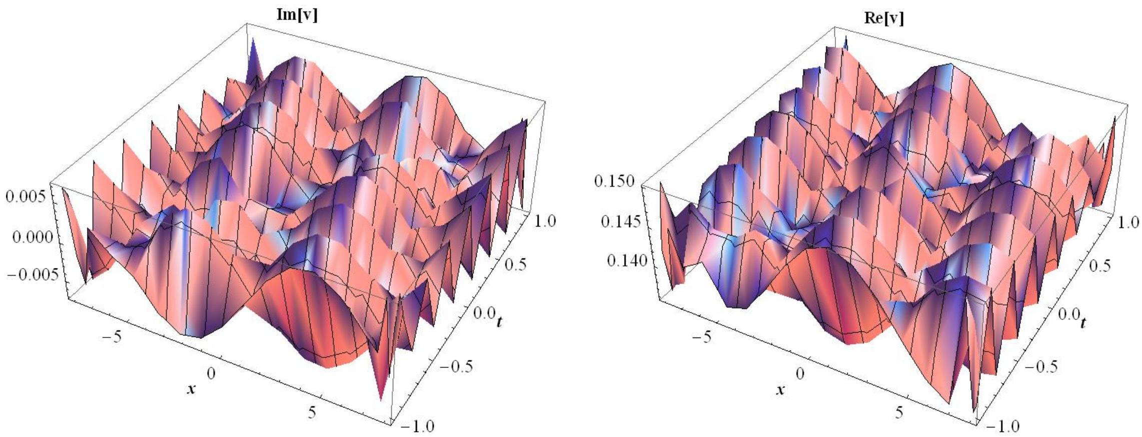

Figure 2. The 3D surfaces of v_1 of Eq.(24) under the terms of considering the values $c=5,\ a=4, \alpha=3, -8<x<8, -1<t<1$.

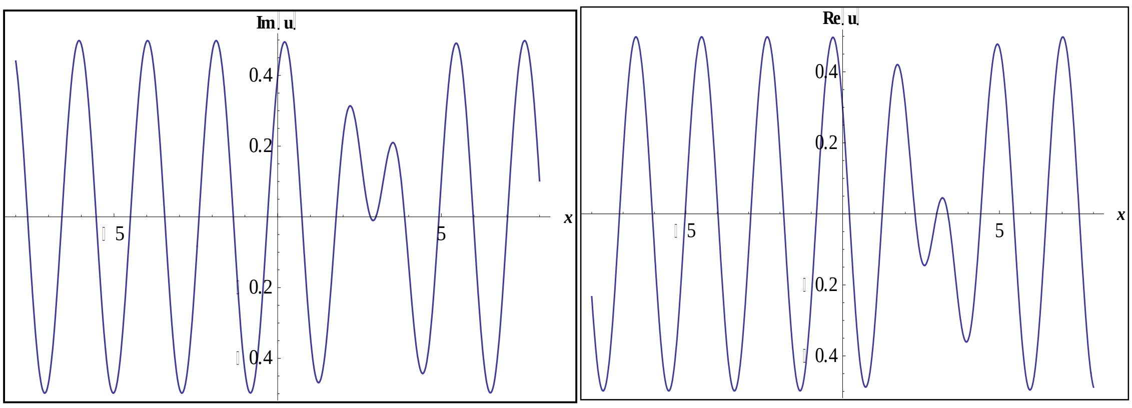

Figure 3. The 2D surfaces of u_1 of Eq.(24) under the terms of considering the values $c=5,\ a=4, \alpha=3, t=0.5, -8<x<8$.

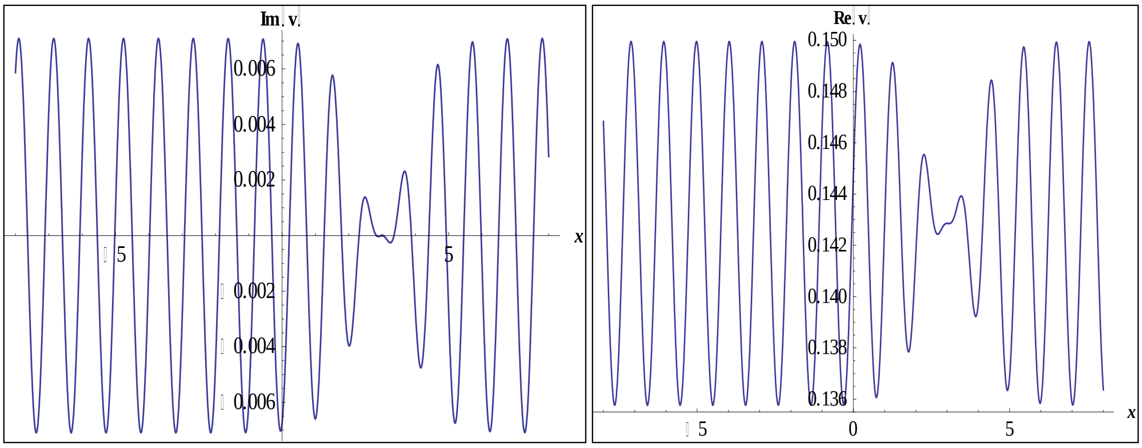

Figure 4. The 2D surfaces of v_1 of Eq.(24) under the terms of considering the values $c = 5,\ a = 4, \alpha = 3, t = 0.5, -8 < x < 8.$

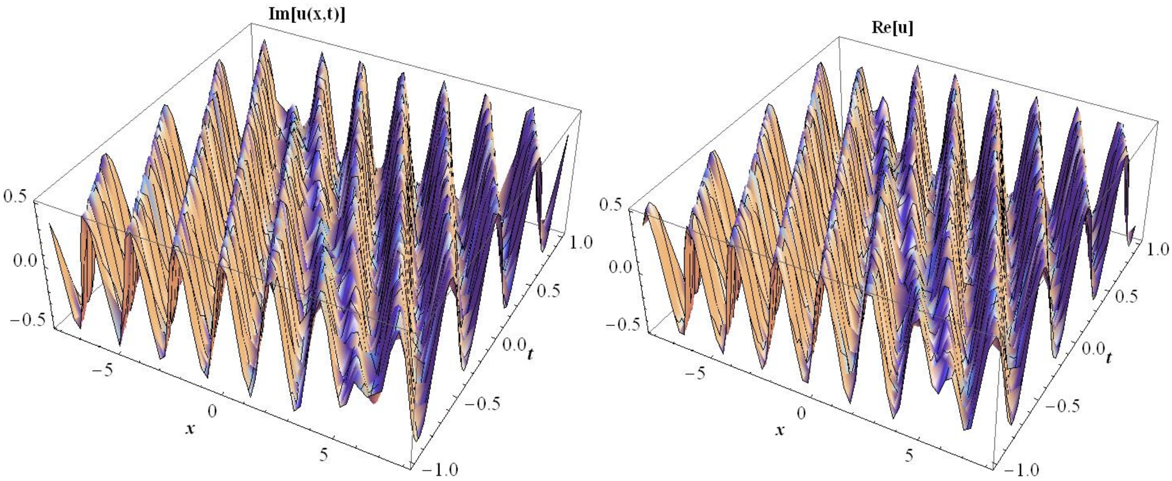

Figure 5. The 3D surfaces of u_2 of Eq.(25) under the terms of considering the values $c = -5,\ a = 4, \alpha = -3, -8 < x < 8, -1 < t < 1.$

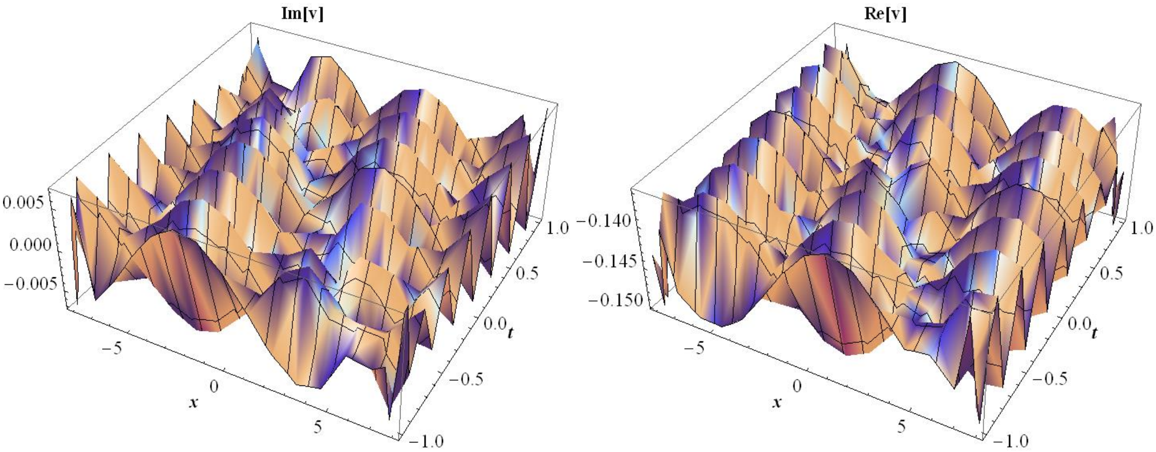

Figure 6. The 3D surfaces of v_2 of Eq.(25) under the terms of considering the values $c = -5,\ a = 4, \alpha = -3, -8 < x < 8, -1 < t < 1.$

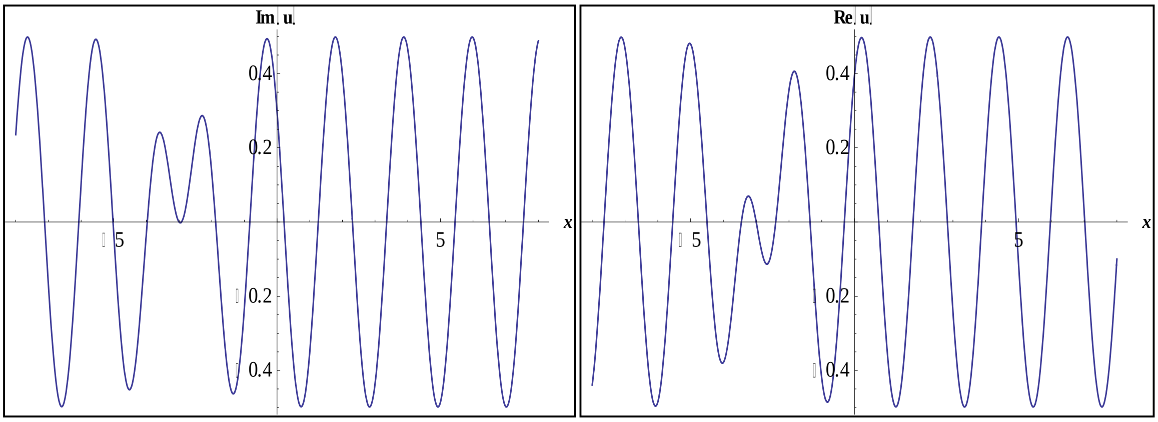

Figure 7. The 2D surfaces of u_2 of Eq.(25) under the terms of considering the values $c=-5,\ a=4, \alpha=-3, t=0.5, -8<x<8.$

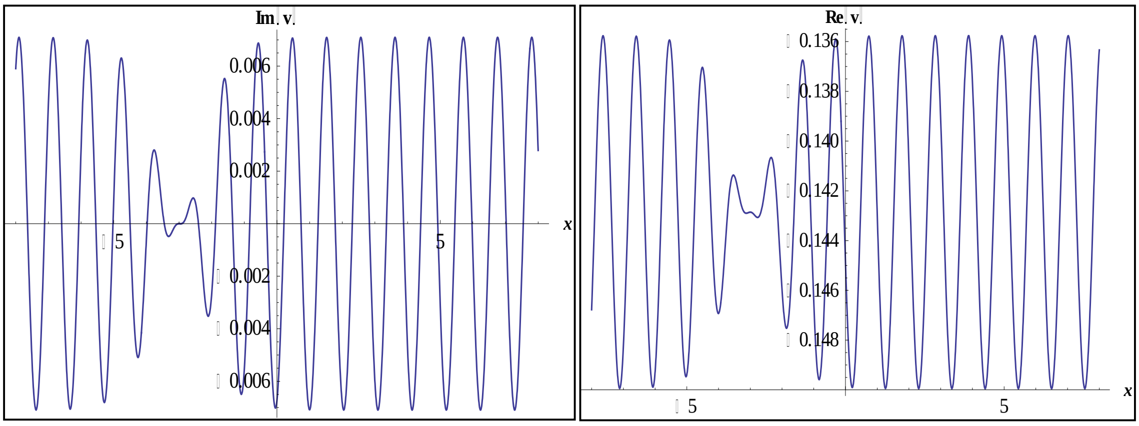

Figure 8. The 2D surfaces of v_2 of Eq.(25) under the terms of considering the values $c=-5,\ a=4, \alpha=-3, t=0.5, -8<x<8.$

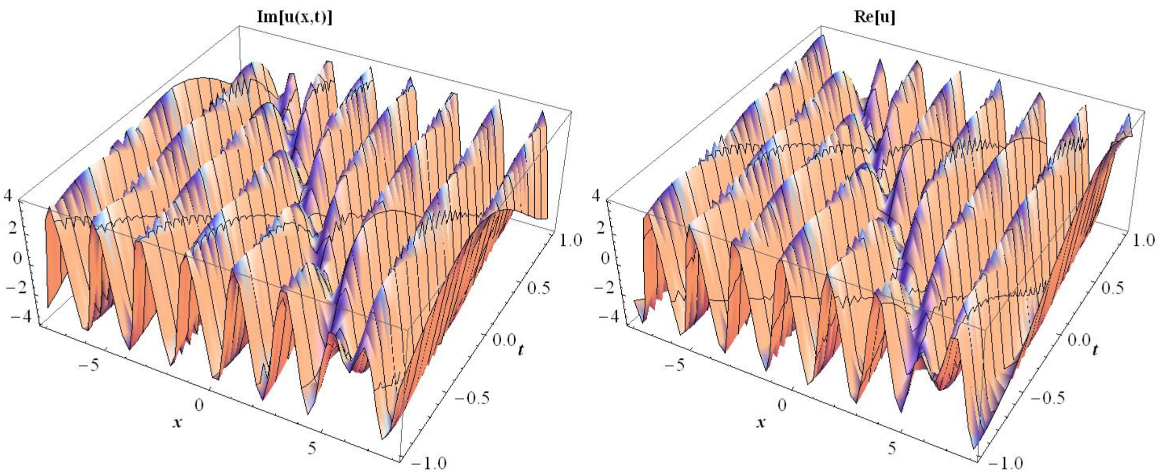

Figure 9. The 3D surfaces of u_3 of Eq.(26) under the terms of considering the values $\beta=-2,\ A_1=4, \alpha=-3, -8<x<8, -1<t<1.$

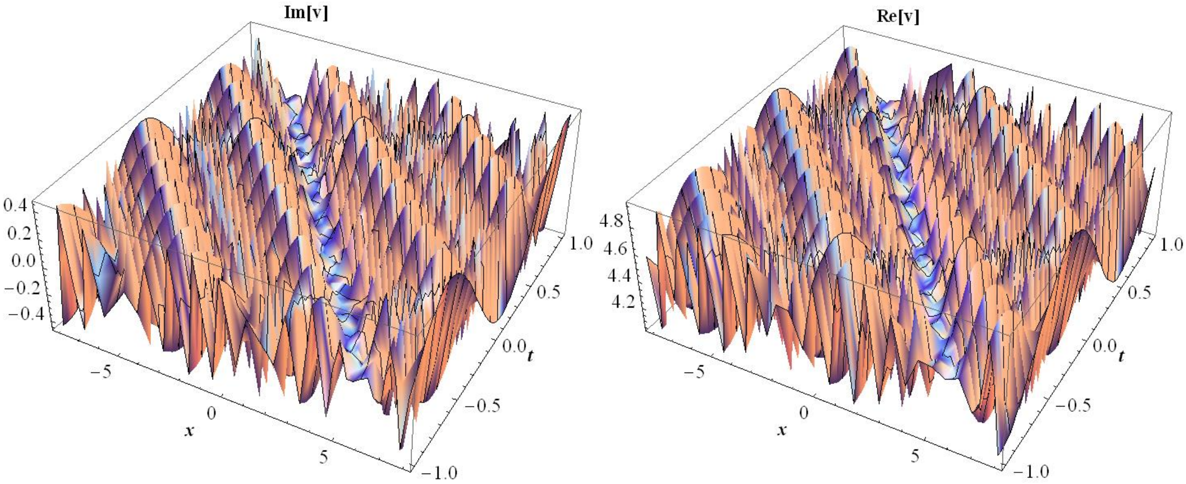

Figure 10. The 3D surfaces of v_3 of Eq.(26) under the terms of considering the values
$$\beta = -2,\ A_1 = 4, \alpha = -3, -8 < x < 8, -1 < t < 1.$$

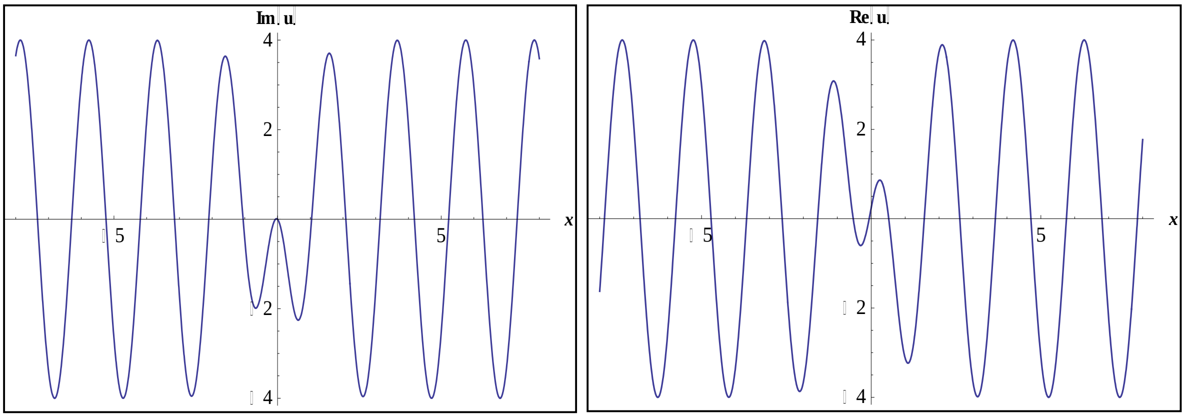

Figure 11. The 2D surfaces of u_3 of Eq.(26) under the terms of considering the values
$$\beta = -2,\ A_1 = 4, \alpha = -3, t = 0.01, -8 < x < 8.$$

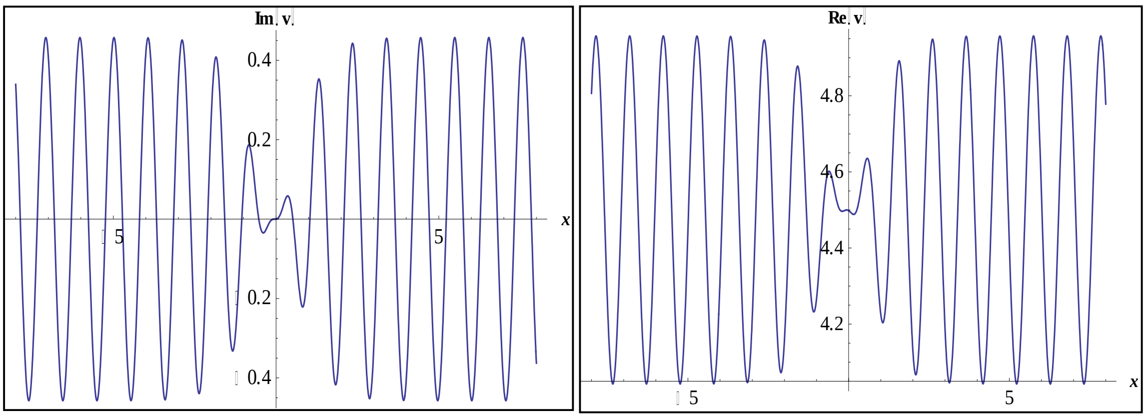

Figure 12. The 2D surfaces of v_3 of Eq.(26) under the terms of considering the values
$$\beta = -2,\ A_1 = 4, \alpha = -3, t = 0.01, -8 < x < 8.$$

5. Discussion and remark

In fact, the coefficients found in this paper such as Eqs.(21,22,23) belong to Eq.(11) defined as

$$U(w)=\sum_{i=1}^{n}\cos^{i-1}(w)\left[B_i\sin(w)+A_i\cos(w)\right]$$
$$+A_0.$$

According to fundamental properties of SGEM which includes the interesting equations such as Eq.(7) and Eq.(8), we have used the Eq.(10) because Eq.(11) equal to Eq.(10) defined by

$$U(\xi)=\sum_{i=1}^{n}\tanh^{i-1}(\xi)\left[B_i\operatorname{sec}h(\xi)+A_i\tanh(\xi)\right]$$
$$+A_0,$$

for finding the hyperbolic function solutions to the Eq.(1).

6. Conclusion

To be brief, SGEM has been successfully applied to the generalized-Zakharov equation with complex structures for obtaining the complex travelling wavesolutions. We have plotted two- and three-dimensional surfaces for the Eq.(1) under the suitable values of parameters.

When we consider all the results and Figures (1-12), we can say that this method is efficient and suitable for obtaining new travelling wave solutions to the ordinary differential equations with powerful nonlinearity. These hyperbolic function solutions have been introduced to the literature with important physical meaning about the generalized-Zakharov equation. Moreover, travelling wave solutions Eqs.(24,25,26) are dark soliton solutions to the Generalized-Zakharov equation with complex structures [19-21]. It has been observed that they are related to physical features of hyperbolic functions [22, 23]. It is estimated that they are related to the physical properties of dark soliton solutions.

We think that this method play an important role for finding travelling wave solutions to such models. To the best of our knowledge, the application of SGEM to the Eq.(1) has not been submitted to literature in advance.

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The integrated network model of pipeline, sea and road distribution of petroleum product

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Abstract. Nigeria ranks high among the community of oil producers both in the world. It is, therefore, paradoxical that Nigeria, with such profile in Organization of Petroleum Exporting Countries (OPEC) statistics finds it difficult to optimize its supply distribution while spending so much money on transportation and distribution. This paper thus reviews the petroleum product supply and distribution systems in the country. Thus, we develop a single period, single product deterministic mathematical model to effectively distribute the product to the end user through the most effective channel to the interest of the economy of the country. In our model, we first consider a perfect condition in the petroleum industry irrespective of the production crises and conflicts like pipeline vandalism, communal instability. We then consider different scnearios that presumes several breakdown cases in pipeline connection to anaylze the survivability of the network of petroleum distribution.

Keywords: Distribution logistics; network model; pipeline; petroleum product.
AMS Classification: 90C90; 90B10; 90B06

1. Introduction

All around the world, petroleum distribution enjoys transportation options which include road transportation (by trucks), rail transportation and sea transportation. Pipeline transportation has become the most viable means among these options in a long time. This could be either from exploration site to refinery or from refinery to supplying depots. This attractive means of transportation influences the oil market logistics positively: one of which includes, effective delivery of petroleum product to the required point, accidents or traffic adversity that might arise from road transportation are avoided and most importantly, transportation cost are considerably lowered when compared to other means of transportation.
The safety and reliability of petroleum is also assured via pipeline transportation as compared to road or rail transportation where product adulteration is possible on transit. Health and environmental hazards that arises from open movement of petroleum products are also avoided when transported through the pipeline. Researchers have also shown that there are lesser chances of spillage in pipeline transportation compared to other means of transporting petroleum products [1].
Although the risks of carrying petroleum products through pipelines are very rear but when they occur, they are more fatal compared to other means of transportation thus incurring great casualty. Since pipelines that transport petroleum products are submerged in the earth, there are possibilities of spillage which might arise from pipeline rapture or corrosion. These then have contact with the under-earth drinking water supply making it contaminated for consumption.

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Multiple break-downs in the pipeline network also cause product delivery delay, causing industrial down tool for the period of break down. This also affects the economy in the negative sense.

Operational and personal pressure miscalculation that might arise from pipeline transportation always has adverse effect on the environment and wildlife. Predominant failure that arises from pipeline transportation and mostly caused by corrosion, welding and material failure which might warrant evacuation is also another difficult and expensive task. This is predominant in regions within USA and Canada. Malfunction within the pipeline network are most times difficult and expensive to manage whereby the products leak unnoticed. Pipeline vandalism is another disadvantage of pipeline transportation of petroleum product; this is predominant in the sub-Sahara Africa cases. Because of this, in this paper we focus on one of the biggest petroleum produces in Africa, Nigeria.

Nigeria is a country with abundance of natural resources and most of which are of economic quantity. One of its natural resources that are readily available for trade and with high market demand is petroleum (crude oil). Exploration of crude oil have increased compared inception; production have also increased to close to a billion barrel in the year 2006. Similarly, Nigeria exported a relatively higher volume (hundreds of millions of barrels) of crude oil resulting in a corresponding increase in oil revenue making Nigeria practically abandoning other sources of revenue generation. The increasing revenue trend thus portrays Nigeria in a very good position for steady development through revenue availability for infrastructure and sustainable micro economics management. But in spite of these, the nation still wallows in decayed infrastructure and weak institutions through corrupt leadership and technological negligence.

Nigeria Oil industry has been suffering due to inadequate funding in the infrastructures, weak government policies; pipeline surveillance inefficiency and mismanagement have contributed immensely to the incessant distribution problem of the petroleum products for decades [2]. That's why, Auwal and Mamman [3] mentioned that the petroleum market in Nigeria is very sensitive; however authors claim that market players will find an opportunity to access facilities such as pipelines, depots etc. to maximize supplies to customers due to deregulation. Additionally, Ehinomen and Adeleke [4] discussed that privation of the distribution of the petroleum product in Nigeria both creates job opportunities and increase the effectiveness and the efficiency in the distribution of the product. Ehinomen and Adeleke [4] also pointed out that state-owned facilities such as pipelines, depots and refineries, storage facilities are poorly managed, hence it causes a low utilization of the facilities, inadequate distribution and increase in treasury loss. Therefore, in recent decades, there have been incessant shortages of products, long queues at gas stations due to product shortages and ineffective distribution mechanism. Hence, this paper aims to optimize the distribution of petroleum products in the downstream supply industry in Nigeria assuming that the sector is deregulated and operated by private companies.

It was evaluated that the network density and pipeline connectivity for the distribution of petroleum finished products are low thus creating room for inefficiency in the supply of these products to the eventual end users through the necessary intermediate actors [5]. Therefore, the model we develop in this study is aimed to provide answers to several questions such as "which pipeline connections are crucial for the network and what should be their capacity", "what should be the capacity of depots" and "which pipeline connections can be treated as back-up connections when a sabotage is occurred in the network."

Recently, An et al. [6] reviewed previous research about biofuel and petroleum supply chain in details and divided them into three categories: strategic, tactical and operational. This review shows that many of the research has been done in tactical level and dealt with capacity and planning of refineries and production plants, inventory, flows of product as well as scheduling multi-products. Catchpole [7] is one of the earliest study that discussed how to use a linear program to determine the optimal flow of fuel between refineries and distribution centers. However, due to computational difficulties in that time, they did not develop a model and implemented. Later on, Klingman et al. [8] developed a multi-period mathematical model to solve the planning and distribution of petroleum of product in the network of Citgo. Authors also considered the exchange contracts between nodes. Recently, Herran et al. [9] presented a non-linear model to determine the optimal sequence of products that flow through a multi-product pipeline. They solved their model for difference scenarios after they linearized their model. Additionally, Oyewale and Ozturkoglu [10] presented a deterministic linear program for the optimal distribution of petroleum products both among depots, suppliers and downstream customers in Nigeria. In this paper, we extend Oyewale and Ozturkoglu [10]'s model to evaluate the survivability of the network. Throughout the optimization, we also aim to efficiently supply the petroleum product across all nodes involved in the supply chain to affect product availability. This also serves the purpose of transportation cost reduction. After the optimal solution is obtained, different scenarios would also be analyzed to check the cost deviation in event of pipeline or flow breakage at some points in the flow network which might be as a result of pipeline vandalization, ethnic crises or other form of disruption.

2. Integrated network model

There are several assumptions that accompany the model. The model is a single-period, deterministic mathematical model. The model assumes that the transportation cost on a given route is directly proportional to the distance between two nodes.

There are assumed to be five different types of facilities in a distribution network of petroleum product. These are petroleum supply countries, import seaports, in-country refineries, and depots/pump stations and filling stations. In our model, filling stations are customers of the network. However, because there are thousands of them available in the network, we assume to that a customer point is located in the centroid of each state, and the demand of each state is calculated with respect to the accumulated demand of filling stations in that state. Depots/pump stations are points from where petroleum product is supplied to the end customer through road transportation. While depots/pump stations are connected to refineries and import seaports through a pipeline network in general, goods are transferred between import seaports and supply countries via sea transportation. Hence, considering the available route among these facilities we construct a network consisting of nodes and edges.

All facilities except for customer zones are defined as nodes in a set N where $i \in N$ represents the i^{th} facility in the network. In this set, i represents depot/pump station nodes from 1 to p, seaport nodes from $p+1$ to q, refinery nodes from $q+1$ to r, and supply country nodes from $r+1$ to u. Customer zones are also defined as nodes in a set K where $k \in K$ and there are s number of states in total. R is also a set of defined routes that vessel $e \in E$ might travel. Therefore, the parameters and decision variables are as defined in Table 1.

Table 1. Model parameters and decision variables.

Parameters	
d_{ij}	distance between node i and node j (km)
c_{ij}	unit cost of transferring of product from node i and node j (\$/km-barrel)
tc_{ik}	unit trucking cost of transporting from node i and state k (\$/km- barrel)
m_{ij}	a matrix that indicates if there is a road or pipeline connection between node i and node j ($\in \{0,1\}$)
p_{ij}	pipeline capacity between appropriate node i and node j (barrel)
D_k	total demand of the product in state k (barrel)
V_e	capacity of vessel e (barrel)
T_{eij}	cycle time of vessel e on route i and j (time/cycle)
S_i	Supply capacity of node i (barrel)
w	working period (time)
Variables	
X_{ij}	quantity of product transferred from node i and node j (barrel)
Y_{ik}	quantity of product transported from node i and state k (barrel)
N_{eij}	Number of trips that vessel e make through route nodes $i-j$
δ_{ij}	1, if connection from i to j; otherwise it is 0

$$\min \quad Z = \sum_{i \in N} \sum_{j \in N, i \neq j} X_{ij} \cdot c_{ij} + \sum_{i=1}^{p} \sum_{k \in K} Y_{ik} \cdot tc_{ik} \tag{1}$$

Subject to

$$\sum_{j=1,i\neq j}^{p} X_{ij} \cdot m_{ij} + \sum_{k\in K} Y_{ik} \leq \sum_{j\in N,i\neq j} X_{ji} \cdot m_{ji} \qquad \forall i = 1 \dots p \tag{2}$$

$$\sum_{j\in N,i\neq j} X_{ij} \cdot m_{ij} - \sum_{j\in N,i\neq j} X_{ji} \cdot m_{ji} \leq S_i \quad \forall i = (p+1) \dots m \tag{3}$$

$$X_{ij} \cdot m_{ij} \leq p_{ij} \quad \forall i = 1 \dots r, \forall j = 1 \dots p \tag{4}$$

$$\sum_{i=1}^{p} Y_{ik} \geq D_k \quad \forall k \in K \tag{5}$$

$$\sum_{i=p+1}^{q-2} \sum_{j=i+1}^{q} N_{eij} \cdot T_{eij} = w \quad \forall e \in E \tag{6}$$

$$X_{ij} \cdot m_{ij} \leq \sum_{e\in E} N_{eij} \cdot V_e \qquad \forall\, i,j = p+1 \dots q, i \neq j \tag{7}$$

$$X_{ij} \leq M \cdot \delta_{ij} \quad \forall i,j = 1,..,p; i \neq j \tag{8}$$

$$X_{ij} \geq \delta_{ij} \quad \forall i,j = 1,..,p; i \neq j \tag{9}$$

$$\delta_{ij} + \delta_{ji} \leq 1 \quad \forall i,j = 1,..,p; i \neq j \tag{10}$$

$$X_{ij}, Y_{ik}, N_{eij} \geq 0, \delta_{ij} \in \{0,1\} \; \forall i,j \in N, \forall k \in K, \forall e \in E \tag{11}$$

In formulating the model, the objective (Z) in Eq. (1) is to minimize total cost of transportation of petroleum product from supplying countries and local refineries through depots involving pipeline network to the states. The first part of the objective function presents total sea and pipeline transportation cost occurred before the products are distributed to states. Hence, the second part is related to total trucking cots to meet customer demands in states.

Because depots/pump stations are transhipment nodes, Eq. (2) maintains the equilibrium in the pipeline network such that total flow out from pump stations should be less than or equal to total flow into pump stations. Eq. (3) also enhances the equilibrium in the rest of the network in which total capacity of the refineries, supplying countries and seaport cannot be exceeded. Eq. (4) is the pipeline capacity constraint for a given period under the assumption that there is no vandalism or outage. Eq. (5) guarantees that demand of states should be satisfied by pump stations. Eq. (6) is used to calculate the number of trips that a vessel can make in a given period. We assume that a vessel visits only one location in a trip after it is loaded. As soon as it deposits all products to a location, it goes back to the beginning to be loaded. Hence no tour is allowed for vessels. Therefore, in Eq. (7), amount of products shipped among seaports cannot exceed vessel capacities. Eq. (8)-(10) provides that only one way of the pipeline flow can be allowed within the planning horizon if there is any two way flow available among the pump stations. In Eq. (8), M is a relatively big number. Eq. (11) is the non-negativity and binary constraint. Therefore, the model is a mixed-integer linear programming (MILP) model.

3. Case study: Nigerian petroleum distribution network

We implemented our model to the petroleum network in Nigeria. The data used in this case study is obtained from local sources that summarize Nigerian petroleum industry reports, analysis and expert knowledges. Most of them are not accessible in digital formats, however, some of the data are obtained from NEITI [11] by refining the appropriate tables. Even though it is not rare to face with network breakdown or pipeline vandalism in Nigeria, we first assume that the network runs properly without any shortage. In order to determine if there is any scarcity in the network in terms of depot or pipeline capacities, we assume that depots and pipelines have infinite capacity. It is a single period model; hence the working period in the model is assumed to be one week. The supply chain distribution of the petroleum industry in Nigeria is represented as a network of nodes in Figure 1. In Figure 1, the dotted lines represent sea transportation, the single line represents pipeline transportation while the thick line represents trucking transportation. Names of the locations represented by nodes are given in Table 2.

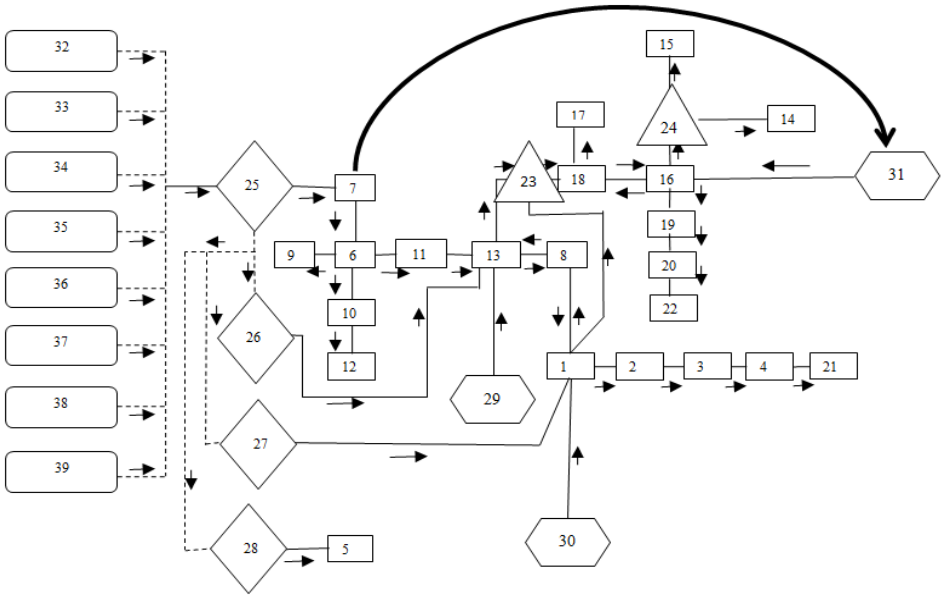

Figure 1. Representation of supply chain network of petroleum industry in Nigeria.

Table 2. Location names of nodes in Figure 1.

Node	Depot	Node	Depot	Node	Depot	Node	Suppliers
1	P.Harcourt	12	Ilorin	23	Auchi P/S	32	India
2	Aba	13	Benin	24	Suleja P/S	33	France
3	Enugu	14	Kano		**Seaport**	34	Italy
4	Makurdi	15	Gusau	25	P.Lagos	35	S/Korea
5	Calabar	16	Kaduna	26	P.Delta	36	Netherland
6	Mosimi	17	Minna	27	P.Harcourt	37	Singapore
7	Atlascove	18	Suleja	28	P.Calabar	38	Portugal
8	Warri	19	Jos		**Refinery**	39	Ivory
9	Ejigbo	20	Gombe	29	Warri		
10	Ibadan	21	Yola	30	P.Harcourt		
11	Ore	22	Maiduguri	31	Kaduna		

In Figure 1, nodes 1 through 24 are designated to represent the depot/pump stations that are connected by pipelines which are owned and operated by Nigerian government. Table A.1. in Appendix shows the distances between connected pair of nodes and the transportation cost per barrel of petroleum flowing between these pair of nodes. This data is obtained from Fantini [12]. The pipeline transportation cost is calculated by using a formula obtained from local industry experts. This formula is given in Eq. (12). The first part of the equation is related to weekly fixed maintenance and deterioration cost of pipelines per barrel under the assumption of 100% utilization of the pipelines. The second part is variable transportation cost with respect to the distance of a barrel of product flow through the pipeline. Pipeline transportation cost

$$c_{ij} = 0.5 + (1.5\ x\ \frac{d_{ij}}{1000}) \qquad (12)$$

The depots/pump stations are used to meet demand of filling stations in states via road transportation. Although there are tens of thousands of filling stations in states, we take the center of mass of each state as the demand node. Hence, we determine that 37 customer zones in the network and their total demands are obtained from NNPC [13] (see Table A.2. in Appendix for details). All of the depots/pump stations can serve any of these states by trucks. We assume that there are enough number of trucks available in the network. The connections between states and depots are not represented on the network diagram for clarification reason. However, trucking costs among the depots and state centers are given in Table A.3. in Appendix.

Nodes 25 to 28 are the local seaport nodes. These seaports are used to feed pipeline network with the imported products. As seen in Figure 1, after imported petroleum products arrive to Port Lagos, they are either transferred to the depots through pipeline network, or transferred to other local ports via sea vessels. The types of vessels that are available to use among local ports, and their capacities are given in Table 3. Additionally, average cost of shipping one barrel of product via vessels between ports are also given in Table 4, assuming that the type of vessel does not affect the transportation cost. Table 4 also shows how long a vessel travels between Port Lagos and other ports in a trip. Hence, seaport and depot/pump station nodes serve as transhipment nodes connecting the supply and demand nodes.

Table 3. Available vessels (barges) and their capacities.

Vessel Name	Capacity (barrel)
Desire I	25368
Desire II	36440
Dera I	32482
Dera II	22809
Marvel I	40483
Praise I	20745
Praise II	20813
Mnemosyne	37472
Saje 460	76139
Hera	49568
Kirikiri	56076
Demetra	18689
S215	88533
Rhea	37515
Hestia	56076
Energy 7001	27177
Energy 6503	24711

Table 4. Transportation cost and average trip cycle time from and back to Port Lagos.

	Transportation Cost				Cycle time (days)
Local seaports	**Port Lagos**	**Port Calabar**	**Port Delta**	**Port Harcourt**	**From/To Port Lagos**
Port Lagos	0,00	0,11	0,05	0,09	---
Port Calabar	0,11	0,00	0,06	0,04	4,64
Port Delta	0,05	0,06	0,00	0,04	3,155
Port Harcourt	0,09	0,04	0,04	0,00	4,145

Table 6. Capacities of suppliers and their distances to Lagos Port.

Country	Distance (Nautical mile)	Supply Capacity (barrel/week)	Transportation Cost (USD/barrel)
India	7,826.5	2,221,212	2.15
France	4,758.0	932,463	1.04
Italy	3,763.0	1,445,206	1.31
S/Korea	10,574.5	752,346	2.90
Netherland	4,260.5	2,680,545	1.17
Singapore	8,166.0	1,780,667	2.25
Portugal	3,276.0	167,727	1.90
Ivory Coast	457.0	71,001	0.13

Nodes 29 through 31 are refinery nodes where local petroleum product is produced and supplied to the depots/pump stations through pipeline network. The weekly capacities of these refineries are given in Table 5. In the case of scarcity in local supply or need of petroleum product due to excessive demand, products are imported from petroleum product supplying countries. Nodes 32 to 38 represent these suppliers. The estimated supply capacities of these countries and their distances, as well as shipping cost, to Port Lagos (node 25) are given in Table 6.

Table 5. Capacities of refineries.

Refinery	Supply Capacity (barrel/week)
Warri	131250
P.Harcourt	220500
Kaduna	115500

4. Scenario analysis and results

The mathematical model for the case study is coded using AMPL and solved by using IBM ILOG CPLEX Optimization Studio version 12.6. The optimum flows of the petroleum product that minimizes total transportation cost under ideal case are depicted in Figure 2. The ideal case refers that there is no breakage, vandalism or leakage during the transportation. The surplus, which is the difference between total inflow and total outflow, at the depots refer to the amount of products sent to the appropriate states to meet customer demands.

Therefore, the optimal solution presents what the capacity of depots and pipeline connections should be to minimize total supply chain cost of the petroleum product in Nigeria. For example, the depot at Port Harcourt (node 1) and its pipeline connection with the refinery at Port Harcourt (node 30) should be able to handle about 220,500 barrels of product. In other words, this refinery should be able to produce at least this amount of products. Additionally, it is seen that several of the depots such as depots at Ejigbo (node 9), Ore (node 11) and Maiguduri (node 22), a refinery at Kaduna (node 31), and a local Port Hartcourt are inactive in the optimal solution. However, it doesn't mean that they are useless in real life. Because of the variabilities in the production and the distribution, these nodes could be utilized as back-up or support units and play important role in smooth flow of goods to meet customer demands. Especially, in the case of vandalism or breakdown in the network which is very common in Nigeria, they are indeed actively used nodes in real life, according to our knowledge.

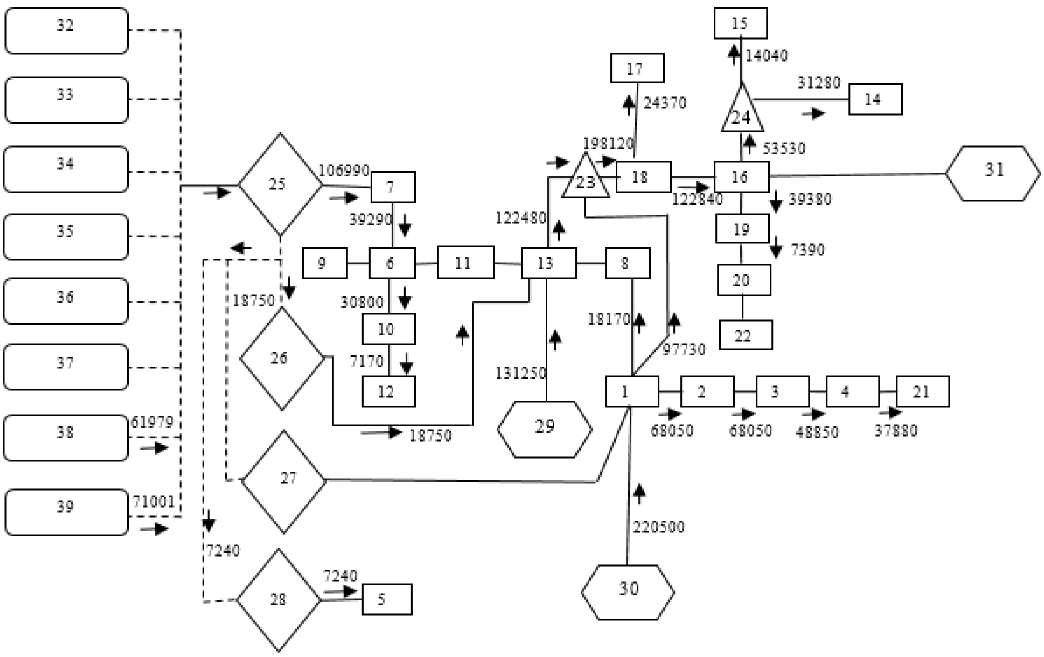

Figure 2. Representation of the optimal solution under ideal case.

The solution also shows that node 1 (depot at Port Harcourt) and node 25 (Port Lagos) have maximum flow connections (degree) in the network. In case of any problem in these connections, one can easily say that there will be a problem in flow of goods in the network. Because we run our model under ideal condition previously, we also want to show managers the survivability of their network. The term survivability refers to the ability of the network to meet customer demands in case a problem occurs in the connections. The survivability of the network is very important not only in pipeline networks but also in telecommunication networks.

Therefore, network designers always consider back-up nodes to alleviate the negative effects of breakdowns of shortages in networks.

To show the survivability of the petroleum network in Nigeria, we develop several important scenarios considering the important connections in the network and optimal flows in the ideal solution. These scenarios are described in Table 7. For example, scenario 1 represents that the pipeline connection between nodes 1 and 8 is out of work for the working period. Additionally, scenario 4 assumes that either pipeline connection between nodes 1 and 30 is broken or refinery at node 30 is out of work (no supply from this refinery).

Table 7. Several possible scenarios of breakages in the connections.

Scenarios	Breakage in connections	Reason of chosen
Scenario 1	1 – 8	Importance of node 1
Scenario 2	1 – 23	Importance of node 1 and high flow in ideal solution
Scenario 3	1 – 2	Importance of node 1
Scenario 4	30 – 1	Importance of node 1 and high flow in ideal solution
Scenario 5	25 – 7	Importance of node 25 and high flow in ideal solution
Scenario 6	18-16	High flow in ideal solution

Table 8. Scenarios and their total costs.

Cases	Total Cost (USD)	% Cost Increment
Base Scenario	1,773,019.8	---
Scenario 1	1,775,563.6	0.1
Scenario 2	1,822,212.2	2.8
Scenario 3	1,923,592.4	8.5
Scenario 4	1,959,818.5	10.5
Scenario 5	2,090,903.1	17.9
Scenario 6	1,933,835.4	9.1

Our mathematical model can easily be adjusted to reflect these scenarios by modifying the values in matrix m_{ij} which represents the availability of node connections. Currently, we assume that m_{ij} can take value of 1 or 0. Thus, we assign 0 to the value of m_{18}, $m_{1,23}$, m_{12}, $m_{30,1}$, $m_{25,7}$, and $m_{18,16}$ one at a time by recovering the value of the previously changed connection. We then evaluate both the changes in the network solution and the changes in the total cost comparing to the ideal solution. Table 8 shows the percentage increment in cost in these scenarios. The results show that the network is survivable in any of these scenarios with an additional distribution cost.

Even though the network is survivable in scenario 5, total distribution cost of the network increases about 11%, which is the second most costly scenario after scenario 4, compared to the ideal solution. We present the optimal solution for this scenario in Figure 3. The main reason of this result is that the supply capacity of the local refinery is not used due to lack connection to the distribution network. Therefore, the model buys the amount of product that the local refinery supplied (220,500 barrels) from supplying countries. Additionally, Port Hartcourt (node 27) becomes active to keep feeding node 1 and also its connected nodes to meet customer demands in states. Hence, this scenario shows that node 27 can play an important role as support units to keep the network surviving. In this scenario, it is also interesting to see that pipeline connections 1-8 and 1-23 become inactive due to lack of supply from node 31. Instead, the reduced flow to nodes 8 and 23 are met from node 13 that receives more goods from Port Delta (node 26). As a result, all of the local ports play an important role to keep product flowing in the network due to increasing amount of imported goods in this scenario.

Except for the connection between nodes 1 and 8, if any of the connections of node 1 falls down, it causes a dramatic increment in the cost of distributing goods. For example, the scenario 4 causes 10.5% increment in total distribution cost. An interesting result among the scenarios occurs in scenario 1. Interestingly, breakage in the connection 1-8 in scenario 1 almost does not change the ideal solution (see Figure 4). When we analyze the network solution for this scenario, we see that the model decides to use connection 13-8, which is not used in the ideal solution, to support node 8 to meet relevant customer demand in states. Hence, 18,170 barrels of products transferred from node 13 node 8 instead of from node 1 to 8. Additionally, to compensate the reduced flow to node 23 (from 122,480 in the ideal to 104,310), the model increases the flow from node 1 to node 23. The rest of the network is the same as the ideal solution in this scenario. As a result, this scenario shows that node 13-8 keeps the network surviving with a cheap cost. Because of the similar discussions and the length of the manuscript, we decide not to present solutions for the other scenarios. The reason why we choose to present scenarios 5 and 1 are due to inclusion of the local refinery in connection to breakage and similar solution to the ideal case, respectively.

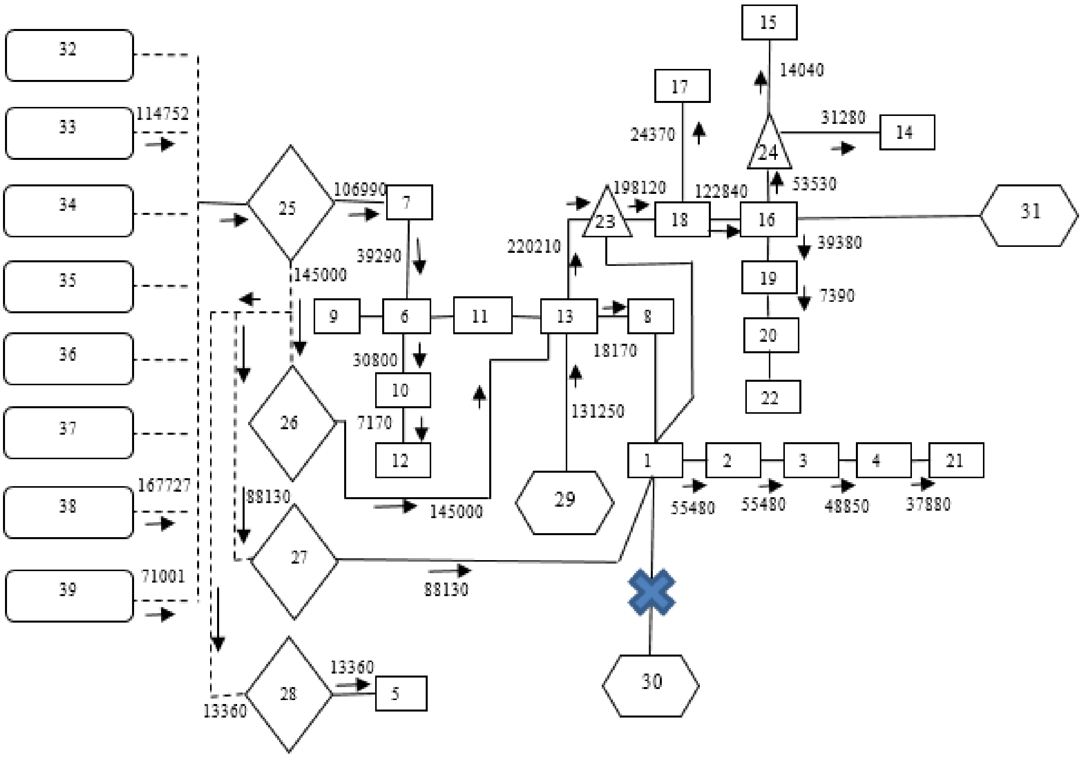

Figure 3. Representation of the solution for scenario 4.

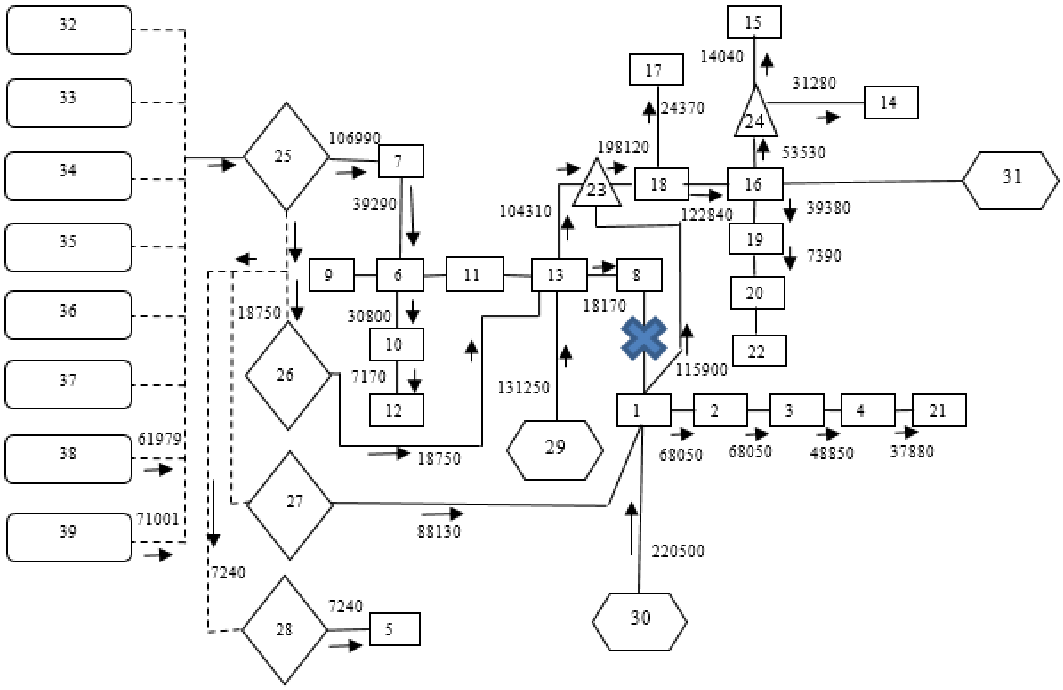

Figure 4. Representation of the solution for scenario 1.

5. Conclusion

The result of this study emphasizes on the most economical distribution of petroleum product in the downstream of the Nigerian petroleum industry to the target customers considering minimization of total transportation cost.

A resource saving is an important objective in the industry today, every progressive industry wants to procure as much saving as possible despite their interest of completing required production which the petroleum industry is not an exemption. A model of such a network would save a lot of resources, especially in an underdeveloped country such as Nigeria. Furthermore, this kind of model is liable to reasonable manipulation relative to more data availability while it still serves as a saving mechanism to transportation of petroleum product and enhancing prompt delivery as required.

From this case study, it is advisable to evacuate dormant nodes and pipeline from this network to save enough resources that would be useful in other sector of life in the Nigerian economy, thus saving transportation and distribution cost and reduces or eradicates petrol station long unnecessary queues due to product unavailability. For future direction, we are planning to develop a simulation model to investigate the potential effects of stochastic pipeline breakage on the distribution of petroleum products and on total distribution cost. Additionally, we plan to consider repair time for breakages of in the network.

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Appendix

Table A.1. Pipeline distances and transportation cost between connected nodes (OPEC, 2012).

Pipeline connections (node-node)	Distance (Km)	Pipeline transportation cost (USD/barrel)
25-7	9.7	0.51
7-6	117.5	0.67
6-9	46.7	0.57
6-10	280.0	0.91
10-12	272.0	0.90
6-11	152.9	0.72
11-13	110.0	0.66
13-8	90.1	0.63
13-23	106.2	0.66
23-18	521.4	1.27
18-17	80.5	0.62
18-16	165.0	0.74
16-18	165.0	0.74
16-24	103.0	0.65
24-14	259.1	0.88
24-15	263.9	0.89
16-19	265.5	0.89
19-20	1335.8	2.49
20-22	1335.8	2.49
8-1	218.9	0.82
1-8	218.9	0.82
1-2	156.1	0.73
2-3	54.7	0.58
3-4	268.8	0.90
4-21	756.4	1.62
26-13	4.8	0.50
27-1	33.8	0.55
28-5	16.1	0.52
29-13	8.1	0.51
30-1	23.7	0.53
31-16	17.7	0.52
1- 23	328.3	0.99
31-16	9.4	0.51

Table A.2. State demand of petroleum product (NNPC, 2012).

No	State	Demand (barrel/week)
1	Abuja	50,910
2	Abia	3,850
3	Adamawa	22,950
4	Akwa Ibom	3,900
5	Anambra	10,350
6	Bauchi	25,450
7	Bayelsa	4,240
8	Benue	5,690
9	Borno	7,080
10	Cross River	7,240
11	Delta	18,170
12	Ebonyi	2,220
13	Edo	12,230
14	Ekiti	4,310
15	Enugu	6,630
16	Gombe	7,390
17	Imo	4,720
18	Jigawa	5,260
19	Kaduna	29,930
20	Kano	26,020
21	Katsina	8,210
22	Kebbi	7,290
23	Kogi	12,360
24	Kwara	7,170
25	Lagos	52,080
26	Nasarawa	9,730
27	Niger	17,080
28	Ogun	15,620
29	Ondo	10,980
30	Osun	8,490
31	Oyo	23,630
32	Plateau	6,540
33	Rivers	19,840
34	Sokoto	6,030
35	Taraba	5,280
36	Yobe	7,850
37	Zamfara	8,010

Table A.3. Trucking cost from depots to the center of states.

Depors\States	Abuja	Abia	Adamawa	Akwa ibom	Anambra	Bauchi	Bayelsa	Benue	Borno	Cross river	Delta	Ebonyi	Edo
Aba	6,8	0,5	9,4	0,9	2,0	9,2	1,7	4,5	13,0	3,4	2,5	2,0	3,1
PH	7,2	1,2	10,0	1,5	2,3	10,4	1,4	5,1	13,6	4,0	2,2	2,6	3,4
Enugu	4,4	1,6	8,0	2,4	0,9	7,5	3,0	2,8	11,6	2,0	3,2	0,9	2,7
Makurdi	3,1	4,3	6,6	5,1	3,3	4,7	5,7	0,7	10,3	2,5	5,6	2,8	5,1
Calabar	6,9	1,7	9,4	1,2	3,4	9,3	3,4	4,5	13,0	1,6	4,2	2,0	4,7
Warri	6,1	3,0	11,5	3,5	2,5	10,4	1,6	5,9	15,1	5,4	0,3	4,2	1,5
Benin	5,1	3,1	10,9	3,9	1,9	9,4	2,5	5,3	14,5	4,8	1,2	3,6	0,5
Auchi	3,6	3,1	9,6	3,9	2,0	7,9	3,7	3,9	13,5	3,9	2,7	2,8	1,0
Mosimi	7,0	6,7	13,5	7,5	5,5	11,4	6,1	7,7	17,0	8,4	4,8	7,1	4,1
Atlascove	7,9	7,4	14,4	8,2	6,2	12,0	6,8	8,7	17,7	9,1	5,5	7,9	4,8
Satellite	7,9	7,4	14,4	8,2	6,2	12,0	6,8	8,7	17,7	9,1	5,5	7,9	4,8
Ibadan	7,2	7,6	14,6	8,4	6,4	11,0	7,0	8,9	16,6	9,4	5,7	8,1	5,0
Ore	5,5	4,3	12,1	5,1	3,1	9,8	3,7	6,5	15,7	6,0	2,4	4,8	1,7
Ilorin	5,0	6,3	12,4	7,2	5,2	9,4	6,3	6,7	14,4	7,7	5,0	6,6	6,6
Kaduna	2,0	7,8	7,9	8,6	6,3	4,3	8,8	5,3	9,3	7,1	7,8	7,2	6,1
Kano	4,6	10,4	6,8	11,2	8,9	3,6	11,4	7,3	7,4	9,1	10,4	9,9	8,7
Minna	1,7	7,1	9,7	7,9	5,5	6,2	8,0	5,0	11,1	6,8	7,0	6,5	5,3
Suleja	0,6	6,1	8,7	6,9	4,6	5,2	7,1	3,9	10,2	5,7	6,1	5,6	4,4
Zaria	2,9	8,8	7,8	9,6	7,2	4,2	9,7	6,6	8,7	8,4	8,7	8,2	7,0
Gusau	4,9	10,7	9,7	11,5	9,1	6,2	11,6	8,5	10,5	10,3	10,6	10,1	8,9
Jos	2,6	7,6	5,5	8,5	6,7	2,0	9,0	4,1	7,0	5,9	9,0	6,2	7,3
Gombe	5,6	10,2	2,5	11,0	9,7	2,1	12,3	7,2	3,9	8,1	12,1	8,9	10,4
Yola	8,2	9,0	0,2	9,8	9,0	4,9	11,1	6,0	5,0	7,0	11,3	7,7	10,8
Maiduguri	8,8	12,0	4,3	12,7	12,0	4,5	14,0	8,9	0,6	9,9	14,2	10,7	13,7

Table A.3. Trucking cost from depots to the center of states (Continued).

Depors\States	Ekiti	Enugu	Gombe	Imo	Jigawa	Kaduna	Kano	Katsina	Kebbi	Kogi	Kwara	Lagos
Aba	4,8	2,0	10,9	0,9	12,4	8,4	10,9	11,8	12,0	4,4	7,1	7,4
PH	5,3	2,8	11,5	1,2	12,8	9,1	11,3	11,7	12,6	5,1	7,7	7,2
Enugu	3,5	0,1	9,1	1,7	10,5	6,4	8,9	9,8	10,2	2,7	6,3	7,0
Makurdi	4,6	2,8	6,4	4,4	7,5	5,1	6,7	8,5	10,6	2,8	7,6	8,2
Calabar	6,4	2,9	10,7	2,5	12,1	9,2	11,7	12,6	12,9	5,5	8,7	9,0
Warri	3,2	3,4	12,1	2,4	11,8	7,7	10,2	11,1	10,5	4,1	5,6	5,4
Benin	2,1	2,8	11,1	2,5	10,8	6,7	9,2	10,1	9,4	3,1	4,5	4,3
Auchi	1,7	1,9	9,6	2,5	9,3	5,2	7,7	8,6	9,3	1,6	4,4	5,2
Mosimi	3,1	6,4	13,0	6,1	11,8	7,9	10,2	10,3	8,1	4,9	3,2	0,8
Atlascove	3,8	7,1	13,7	6,9	12,5	8,6	10,9	11,0	8,8	5,9	3,9	0,1
Satellite	3,8	7,1	13,7	6,8	12,4	8,6	10,9	11,0	8,8	5,9	3,9	0,2
Ibadan	3,5	7,3	12,6	7,0	11,4	7,5	9,8	10,0	6,5	5,7	2,9	2,6
Ore	1,7	4,0	11,4	3,7	11,1	6,9	9,6	10,5	8,4	3,4	3,5	2,9
Ilorin	1,6	5,4	10,5	5,8	9,2	6,4	7,7	7,2	5,6	3,5	0,7	3,1
Kaduna	5,7	6,0	5,4	7,7	4,4	0,3	2,8	3,3	6,2	3,7	5,0	8,4
Kano	8,4	8,6	4,3	10,3	1,6	3,8	0,4	1,6	7,0	6,3	7,3	10,7
Minna	3,6	5,3	7,2	6,9	6,3	3,3	4,8	5,2	4,8	2,9	3,3	6,7
Suleja	4,1	4,3	6,2	6,0	5,9	2,2	4,4	4,8	7,0	2,0	4,0	7,4
Zaria	6,7	7,0	5,3	8,6	3,3	2,1	1,7	2,2	5,4	4,6	5,6	9,0
Gusau	7,7	8,9	7,2	10,5	4,5	4,0	3,6	2,2	4,2	6,5	5,7	9,1
Jos	7,0	5,9	3,0	7,7	4,2	1,4	2,8	4,7	8,8	4,9	6,9	10,3
Gombe	10,1	8,9	0,1	11,0	4,4	4,4	4,0	5,8	11,5	8,0	10,0	13,4
Yola	11,0	8,4	2,7	9,8	7,0	7,0	6,6	8,5	14,1	9,5	12,6	14,3
Maiduguri	13,3	11,3	3,4	12,8	4,8	7,6	6,2	8,0	13,3	11,2	13,2	16,6

Table A.3. Trucking cost from depots to the center of states (Continued).

Depors\States	Nasarawa	Niger	Ogun	Ondo	Osun	Oyo	Plateau	Rivers	Sokoto	Taraba	Yobe	Zamfara
Aba	5,3	9,1	7,3	4,7	5,5	6,4	7,6	0,7	13,4	6,9	11,9	11,8
PH	6,5	9,7	7,4	4,6	5,6	7,3	8,2	0,4	13,8	7,5	12,5	12,2
Enugu	3,5	7,2	6,9	4,3	5,1	6,0	5,8	2,7	11,5	5,5	10,5	9,8
Makurdi	2,9	7,7	8,0	6,7	5,8	6,8	3,1	5,4	10,1	4,1	8,1	8,5
Calabar	6,0	10,0	8,9	6,3	7,1	8,0	7,3	2,4	14,2	6,9	11,8	12,6
Warri	5,8	7,6	5,3	2,7	3,6	4,4	8,9	2,0	12,3	8,9	13,8	11,1
Benin	4,9	6,5	4,2	1,6	2,5	3,3	8,3	3,0	11,2	8,4	12,8	10,1
Auchi	3,4	6,4	5,1	2,4	2,8	3,9	6,4	3,6	10,3	7,1	11,3	8,6
Mosimi	6,7	5,2	0,8	2,0	1,7	1,4	11,3	6,5	9,9	11,0	14,8	8,5
Atlascove	7,6	5,9	0,8	2,7	2,4	2,1	12,0	7,3	10,6	11,9	15,5	9,2
Satellite	7,6	5,9	0,6	2,7	2,4	2,1	12,0	7,2	10,6	11,9	15,5	9,2
Ibadan	7,5	4,8	2,0	2,8	2,2	0,4	10,9	7,4	9,6	12,1	14,4	8,2
Ore	7,9	5,4	3,2	0,3	1,4	3,0	9,5	4,2	10,6	9,6	13,2	8,8
Ilorin	7,2	2,7	3,0	2,7	1,6	2,0	8,8	6,7	7,8	9,9	12,2	6,0
Kaduna	4,2	3,8	8,4	6,8	7,0	7,3	4,4	8,7	5,4	7,6	7,3	3,7
Kano	5,8	5,9	10,6	9,4	9,2	9,6	5,0	11,3	5,2	8,7	5,0	3,5
Minna	4,0	2,3	6,7	4,9	4,4	5,6	5,5	7,9	7,0	8,1	8,9	4,3
Suleja	2,8	4,0	7,4	5,1	5,3	6,3	4,5	7,0	6,9	7,0	8,0	5,3
Zaria	5,0	4,2	9,0	7,8	7,5	7,9	4,5	9,6	4,3	7,7	6,2	2,6
Gusau	7,0	3,4	9,1	8,7	7,6	8,0	6,4	11,5	2,4	9,7	8,0	0,8
Jos	2,5	6,3	10,2	8,0	8,2	9,2	0,5	8,7	6,8	5,2	4,8	5,2
Gombe	5,5	7,9	13,3	11,2	11,3	12,3	3,2	12,0	9,6	4,4	2,8	7,9
Yola	6,8	10,5	14,4	12,2	12,1	14,0	4,3	10,8	12,2	3,2	3,8	10,5
Maiduguri	8,7	11,0	16,6	15,1	14,5	15,5	6,8	13,8	11,6	6,2	1,5	10,0

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Sales plan generation problem on TV broadcasting

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Abstract. Major advertisers and/or advertisement agencies purchase hundreds of slots during a given broadcast period. Deterministic optimization approaches have been well developed for the problem of meeting client requests. The challenging task for the academic research currently is to address optimization problem under uncertainty. This paper is concerned with the sales plan generation problem when the audience levels of advertisement slots are random variables with known probability distributions. There are several constraints the TV networks must meet including client budget, product category and demographic information, plan weighting by week, program mix requirements, and the lengths of advertisement slots desired by the client. We formulate the problem as a chance constrained goal program and we demonstrate that it provides a robust solution with a user specified level of reliability.

Keywords: Chance constrained goal programming; media planning; scheduling.

AMS Classification: 90C29, 90C90, 90B36

1. Introduction

The Broadcasting companies make most of their revenues from selling impressions through advertisement space during various programs or shows. U.S. ad spending was USD 168.8 billion in 2008, where it is approximately USD 500 billion worldwide. Of this, television accounts for approximately 50%, and TV advertising is still the biggest player followed by the internet which has a 30% share of total spend. Advertisers are ready to pay up to approximately half a million dollars for a 20 or 30 second advertisement in a popular show [1].

In North America and several European countries, most of the advertising space is sold before the broadcast season which is also called "upfront market" --occurring for a couple of weeks in May--, following the announcement of program schedules and prices for the following year. During upfront market period, major advertisers and companies request from the TV networks to purchase time for the entire season. A typical request consists of the dollar amount, the demographic in which the client is interested, the program mix, weekly weighting, unit-length distribution, and a negotiated cost per 1,000 viewers. Broadcast Companies must develop a detailed sales plan consisting of the schedule of advertisements to be aired to meet the requirements and they have to pay a penalty when it is unable to meet its commitment for client requests. This happens when a broadcast company sells a large amount of rating points during the upfront market and its shows turn out to be misses or when the broadcast company cannot meet the weekly demand or demand per show. In addition, the plan should also meet the objectives of the company, whose goal is to minimize the penalties or to maximize the revenues for the available fixed amount of inventory.

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The objective of this paper is to prepare a sales plan that meets all the client requirements after clients' sales requests have been received. This plan is modeled to include a complete schedule of advertisements that will be aired for each client and target audience levels requested by these clients. Audience uncertainty has not been taken into account in this problem of media planning, to the best of our knowledge. Due to penalty rates that Broadcast Companies accept to pay to the clients in case of not meeting their specific constraints, we try to model sales plan generation problem using chance constrained programming to minimize the penalty costs under audience uncertainty.

This paper is organized as follows. The next section provides a review of the related literature. We give a brief description of goal programming, chance constrained programming and represent our model in Section 3. In Section 4, we present computational studies that complement our analytical findings. We then describe some managerial implications and give some future directions in Section 5.

2. Literature review

There are several studies dealing with scheduling programs for the television networks to optimize some specified criteria which are usually audience ratings. The placement strategy called "lead in" is widely used to schedule programs which use the strength of the preceding program to boost the ratings of a newly introduced program. A comprehensive review of these models was provided by Rust [2]. Models and heuristic methods for scheduling programs were developed by Horen [3], Rust and Echambadi [4], and Reddy et al. [5]. Simon [6], Mahajan and Muller [7] have studied on strategies for scheduling advertisements. These studies are concerned with whether advertising programs should be steady or turned on and off. A review of these models is collected by Lilien et al. [8].

There are also several studies about advertising allocation problem concerning the distribution of available budget to different media channels. Mihiotis and Tsakiris [9] considered advertisement allocation problem by using mathematical programming. In their problem the best possible combination of placements of an advertisement is asked including the channel, time, and frequency with the objective of the highest rating. The clients have a budget limitation for advertising. They used integer programming to solve the model. Cetin and Esen [10] have studied media allocation problem giving a good example of military operations research models that can be adapted to contemporary business world applications. They modelled the problem as a weapon-target model and solved it using integer nonlinear programming. Saha et al. [11] have also studied media allocation problem. They applied a linear time algorithm that finds a solution to the 'maximum weight 1 colouring' problem for an interval graph with interval weight. To solve the problem that involves selecting different program slots they telecast on different television channels in a day so as to reach the maximum number of viewers.

Despite its richness and complexity, the problem of scheduling advertisements on broadcast television or sales plan generation problem has received very little attention in the literature. A math-programming-based algorithm to rapidly generate near-optimal sales plans that meet advertiser requirements have been developed by Bollapragada et al. [12] where a sales plan consists of a complete schedule of advertisements to be aired for an advertiser during a broadcast year to meet its requirements. Bollapragada et al. [13] have also developed an algorithm to schedule client videotapes in the advertisement slots they purchased to meet certain client specific objectives. Araman and Popescu [14] developed stylized stochastic optimization models of airtime inventory planning and allocation across multiple clients under audience uncertainty. They devised a simple procedure for accepting upfront client contracts and estimating their overall inventory requirements.

In this paper, we formulate the problem as a chance constrained goal program to satisfy the client needs by a specified service level and we demonstrate that it provides a strong solution with a user specified level of reliability. The problem that is discussed in this paper has not been addressed in the literature to the best of our knowledge.

3. Problem definition

The problem of Broadcasting Companies is to prepare sales plans to meet clients' requirements. During upfront market, clients declare their special requests by using standard forms supplied by Broadcasting Companies (Figure 1). There are three most important requests of clients to be satisfied. The first one is the target audience level. Clients generally appraise the audience level in terms of number of people that they want to reach. But audience levels per show per week are unknown. Broadcasting Companies have to use estimated data derived from past data by the media rating agencies to prepare a future plan for client specific requests. Second one is the weekly distribution. Because of seasonal factors, special days of the year such as Mother's Day or St. Valentine's day or the special weeks, clients want their advertisements to be aired in these specific weeks because of the demographic features of their target customers. The request form includes the distribution of the advertisements over various weeks. These distribution criteria may be specified as a fraction of the total number of equivalent length advertisements or sometimes as a percentage of the total amount of expenditure or number of people achieved in the plan. The third important request is to satisfy the distribution of advertisements by shows. Clients generally specify the shows in the program schedule that they want their advertisements to be aired in. This is why their target customers are known to be potential viewers of these shows. Clients normally specify these requirements as fractions of the total number of equivalent length advertisements.

Prime Time Plan Request

Client		Date	
Agency		Contact	
Product		Phone	
Demographic		Email	
Date Range			

Week	%	Week	%	Week	%	Week	%
1		14	3.75	27	3.75	40	
2		15		28	1.88	41	
3		16	3.75	29	1.88	42	1.88
4		17	0.94	30	3.75	43	1.88
5	3.75	18	0.94	31	3.75	44	3.75
6	1.88	19	1.88	32	1.88	45	3.75
7	1.88	20		33	1.88	46	
8	3.75	21		34	3.75	47	3.75
9	0.94	22	1.88	35	2.5	48	
10	0.94	23	3.75	36		49	
11	1.88	24	1.88	37	1.88	50	1.88
12	3.75	25	1.88	38	3.75	51	3.75
13	1.88	26		39	3.75	52	3.75

Show	Number	Show	Number	Show	Number	Show	Number
1	3	5		7		11	4
2	7	6		8	5	12	
3		7	10	9	5	13	12
4		8		10		14	10

Comments:

Figure 1. Sample client request form

For instance, a client prepares a request form as in Figure 1. They request that 3.75% of all their advertisements to be aired in week 5 and 1.88% of all advertisements in week 6, etc.

In addition to this, 3 of their advertisements should be aired in show 1 and 7 of them should be aired in show 2, etc. They also define their target audiences in demographic part of the form. Then the broadcasting company tries to optimize the clients' requests.

To use minimum advertisement inventory to meet the client requests is also important for Broadcasting Companies. The inventory that is not used during upfront market period will be sold during the following broadcast period which is also called scatter market. The scatter market sales prices per second of advertisement inventory are generally higher than the upfront market sale prices. Broadcasting Companies have also an offset option where they may use scatter market inventory rather than to pay penalty in terms of dollars.

The problem of sales departments is to prepare a sales plan that meets all the client requirements after clients' sales requests are received. Broadcasting Companies legally accept to pay penalties in case of not meeting the client requests before a client request an advertisement slot. A sales plan generally includes a complete schedule of advertisements to be aired, target audience levels, and legal terms and options to cut back or expand the plan. We try to model a sales plan to satisfy client requirements, to minimize the penalty costs incurred not meeting the client requests, and to minimize the amount of premium inventory assigned to a plan.

4. Model

We use chance constrained goal programming in our model. The goal programming (GP) model is one of the well-known multi-objective mathematical programming models. This model allows to take into account simultaneously several objectives in a problem for choosing the most satisfactory solution within a set of feasible solutions. More precisely, the GP designed to find a solution that minimizes the deviations between the achievement level of the objectives and the goals set for them. In the case where the goal is surpassed, the deviation will be positive and in the case of the underachievement of the goal, the deviation will be negative. First developed by Charnes and Cooper [17] and Charnes et al. [18] then applied by Lee [19] and Lee and Clayton [20], the GP model gained a great deal of popularity and its use has spread in diversified field such as management of water basins, management of solid waste, accounting and financial aspect of stock management, marketing, quality control, human resources, production, transportation and site selection, space studies, telecommunications, agriculture and forestry and aviation. The goal or aspiration levels assigned to the various objectives can be probabilistic where the decision maker does not know its value with complete certainty. Several techniques have been proposed to solve the Stochastic GP model. But the most popular technique is a chance constrained programming developed by Charnes and Cooper [15, 16, 22, 23]. Belaid et al. [21] have exploited the concept of the satisfaction function to explicitly integrate the decision maker's preferences in the stochastic goal programming model.
In our study, we have also many objectives to be satisfied and one of the constraints related to meeting target audience level is defined as probabilistically. That is, a client may say that their target audience level is satisfied by 95% for example. Therefore, we use chance constrained goal programming that the model is given in the next section.

4.1. Model formulation

Our problem under audience uncertainty is described as a chance constrained goal programming model. We assume that the lengths of all advertisements for all clients are same.

The parameters used in the model are as follows:
A_{swk} : number of audiences of break *k* in show *s* that is aired in week *w*; which has a probability distribution function.
σ_{swk} : standard deviation of audiences of break k in show s that is aired in week w.
β_{si} : rate of ads demanded per show per week for client i.
α_{wi} : rate of ads demanded per week for client i.
S : set of slots in week w.
Z : set of slots in show s.
C : set of competitor companies conflicted.
g_i : target number of audience to be reached for client i.
n_i : total slots purchased by client i.
U_k : number of available slots in break *k*.
γ_i : service level of meeting target audience demand of client i.
π_1^+, π_1^-: penalty rate of not meeting and exceeding target audience demand, respectively.
π_2^+, π_2^-: penalty rate of not meeting and exceeding audience per week demand.
π_3^+, π_3^-: penalty rate of not meeting and exceeding audience per show demand.

Decision variables of the model are:
Y_{ijswk} : Binary variable (1, if *j*th advertisement of client *i* is aired in the *k*th break of show *s* in week *w*; 0, otherwise).
NG_i, PG_i : negative and positive deviation from target number of audience of client *i*.
NW_i, PW_i : negative and positive deviation from target number of weekly aired ads of client *i.*
NS_i, PS_i: negative and positive deviation from target number of ads per show of client *i*.
The mathematical formulation of the problem is as follows:

min $$\pi_1^+ \sum_i PG_i + \pi_1^- \sum_i NG_i + \pi_2^+ \sum_i PW_i + \pi_2^- \sum_i NW_i + \pi_3^+ \sum_i PS_i + \pi_3^- \sum_i NS_i \qquad (13)$$

subject to $$P(g_i \geq \sum_j \sum_s \sum_w \sum_k Y_{ijswk}.A_{swk}) \geq \gamma_i \qquad \forall i \qquad (14)^1$$

$$\alpha_{wi}.n_i - \sum_j \sum_s \sum_k Y_{ijswk} = PW_i - NW_i \qquad \forall i, w \qquad (15)$$

$$\beta_{si}.n_i - \sum_j \sum_w \sum_k Y_{ijswk} = PS_i - NS_i \qquad \forall i, s \qquad (16)$$

$$\sum_j Y_{ijswk} \leq 1 \qquad \forall i, s, w, k \qquad (17)$$

$$\sum_i \sum_j Y_{ijswk} \leq U_k \qquad \forall s, w, k \qquad (18)$$

$$\sum_j \sum_s \sum_w \sum_k Y_{ijswk} = n_i \qquad \forall i \qquad (19)$$

$$\sum_j \sum_i Y_{ijswk} \leq 1 \qquad \forall s, w, k \qquad (20)$$

$$Y_{ijswk} = 0,1 \qquad \forall i, j, s, w, k \qquad (21)$$

$$NG_i, PG_i, NW_i, PW_i, NS_i, PS_i \geq 0 \qquad \forall i \qquad (22)$$

The objective function of our model (13) is to minimize the total penalty incurred in meeting requirements. The first two terms in the objective function is the penalty incurred in not meeting and exceeding the total audience demand respectively. Third and fourth terms are the penalty incurred in not meeting and exceeding the number of ads per week demand, respectively, and the last two terms represent the penalty arising from not meeting and exceeding the number of ads per show demand. Since these objectives are wanted to be satisfied definitely, the negative and positive deviations corresponding to each constraint are available in the objective function.

Constraints (14), (15) and (16) are the goal constraints in the model. Moreover constraint (14) is a chance goal constraint and ensures that the target number of audience should be satisfied by a pre-specified service level and therefore the probability that the difference between the target number of audience to be reached for company *i* (g_i) and the total expected number of audience reached by scheduling the advertisements $(Y(.) \times A(.))$ will be greater than the probability level γ_1 specified by the client. To define constraint (14) as a goal constraint, it should be written in equality form. An important pre-requisite for this model is to understand the structure of past audience metrics. Audience means the gross sum of the number of impressions watching a given show [13]. Media rating agencies have realized data to be used by Broadcasting Companies. In our model, A_{swk}, the number of audience in break *k* of show *s* that is aired in week *w*, is a random variable and normally distributed. Constraint (14) was given as below:

[1] This inequality will be replaced by (25)

$$P(g_i \geq \sum\sum\sum\sum Y_{ijswk} A_{swk}) \geq \gamma_i \qquad (14)$$

When we arrange (14), we get the following inequalities. $\Phi(.)$ denotes the cumulative distribution function of a standard normal random variable.

$$\Phi\left(\frac{g_i - \sum_j\sum_s\sum_w\sum_k Y_{ijswk} A_{swk}}{\sum_j\sum_s\sum_w\sum_k Y_{ijswk}\sigma_{swk}}\right) \leq 1-\gamma_i \qquad (23)$$

$$\frac{g_i - \sum_j\sum_s\sum_w\sum_k Y_{ijswk} A_{swk}}{\sum_j\sum_s\sum_w\sum_k Y_{ijswk}\sigma_{swk}} \leq \Phi^{-1}(1-\gamma_i) \qquad (24)$$

Then (24) can be converted to the goal constraint easily and (25) is obtained. $\Phi^{-1}(1-\gamma_i)$ denotes the standard value of $(1-\gamma_i)$. Therefore, (25) should be replaced with (14) to solve the model. Furthermore, deviation terms PG_i and NG_i were added to the objective function (13) because of this reason.

$$\begin{aligned} & g_i - \sum_j\sum_s\sum_w\sum_k Y_{ijswk} A_{swk} - \\ & \Phi^{-1}(1-\gamma_i)\sum_j\sum_s\sum_w\sum_k Y_{ijswk}\sigma_{swk} \qquad (25) \\ & = PG_i - NG_i \qquad \forall i, \end{aligned}$$

Constraint (15) ensures that the number of advertisements of a company that are planned to be aired in a specific week should be equal to the total scheduled number of advertisements that will be aired in aforementioned week for each client *i*. Constraint (16) ensures that there is no difference between the number of advertisements of a company that are planned to be aired in a specific show and the total scheduled number of advertisements that will be aired in aforementioned show for each client *i*.

The remaining constraints are the functional constraints in the model. Constraint (17) ensures that an advertisement can be placed only one which means that it can only be assigned to one position in one break. Constraint (18) ensures that the number of advertisements scheduled cannot exceed the available number of slots in a break. Constraint (19) ensures that the total number of advertisements assigned to a break must be equal to the total number of advertisements bought by all clients. Constraint (20) ensures that the number of advertisements assigned to a break from clients that has conflicts must be less than or equal to 1, so conflicting companies' advertisements will not be aired in the same break. Decision variable Y is a binary variable and all other variables are greater than or equal to zero.

5. Computational study

We define two hypothetical examples in which we try to observe if the model is relevant in meeting client requirements. We assume that penalties for all clients are constant. Penalty cost of not meeting the audience demand is 2.50 TRY, and it is 2000 TRY per show. Moreover penalty cost of not meeting weekly demand is 1500 TRY, penalty of exceeding the audience demand is 2.45 TRY, it is 1900 TRY per show and the penalty of exceeding weekly demand is 1400 TRY. We used GAMS high-level modeling system for mathematical programming and optimization problems to solve these hypothetical examples.

5.1. 2 clients, 2 weeks, 7 shows, 3 breaks case

In this case we suppose there are 2 clients. The broadcast period is 2 weeks and there are 7 shows to be aired in each week. Only one show is aired in a day and there are 3 advertisement breaks in each show. We observe two subcases in this case: one of them is that the clients are non-conflicting and the other is they are conflicting.

2 non-conflicting clients, 2 weeks, 7 shows, breaks case

We assume that clients are non-conflicting with each other. Therefore, constraint (20) is removed from the model. Client 1 requests 15,000 audiences by using 15 advertisements, and wants 8 of its ads to be aired in week 1 and 4 of its ads to be aired in show 1. Client 2 request 13,500 audiences by using 10 advertisements, and 8 of its ads to be aired in week 2 and 7 of its ads to be aired in show 6. These 2 clients are not conflicting so any 2 advertisements of these clients can be aired in same break.

We suppose that Broadcast Company have 2000 potential viewers, and we know that the number of these potential viewers who see the advertisements on each break is normally distributed. The expected numbers of viewers which watch the advertisements in break *k* of show *s* in week *w* and variances of them are known.

The clients wanted their target audience level is satisfied by 99%. According to the GAMS results, the advertisements of each company were placed in ad slots given in Table 1.

Table 1. Advertisement plan for each company

Company	Ad	Show	Week	Break	Company	Ad	Show	Week	Break
1	1	6	2	1	**2**	1	1	2	3
1	2	2	2	3	**2**	2	6	1	3
1	3	3	2	2	**2**	3	6	2	3
1	4	3	2	1	**2**	4	6	2	1
1	5	3	1	2	**2**	5	5	2	3
1	6	7	1	2	**2**	6	3	2	3
1	7	2	1	1	**2**	7	6	1	2
1	8	6	1	1	**2**	8	4	2	3
1	9	1	2	3	**2**	9	6	2	2
1	10	5	2	3	**2**	10	7	2	3
1	11	7	1	3					
1	12	4	1	1					
1	13	1	1	3					
1	14	1	1	2					
1	15	1	2	1					

Due to the restrictions the number of viewers that watch all advertisements of client 1 is 14,790 although 15,000 viewers were contracted, and the number of viewers that watch all advertisements of client 2 is 14,661 although 13,500 viewers were contracted.

On the other hand, the week and show constraints are met for client 1. For client 1, 8 advertisements are aired in week 1 where 8 of 15 advertisements were contracted and 4 advertisements are aired in show 1 where 4 of 15 advertisements were contracted. Week constraint is also met for client 2. 8 advertisements are aired in week 1 where 8 of 10 advertisements were contracted for client 2. Show constraint is not exactly met for second client because 5 advertisements are aired in show 6 where 7 of 15 advertisements were contracted.

Because the 2 clients are non-conflicting for this example, the advertisements of client 1 and client 2 are both aired in the slots (6, 2, 1), (1, 2, 3), (5, 2, 3) corresponding to the (show, week, break). Then the minimized cost of the deviations from goals is calculated as 9323.384 TRY.

2 conflicting clients, 2 weeks, 7 shows, 3 breaks case

Suppose the clients in this example are conflicting clients. They don't want that any of their advertisements to be aired in the same break of any show in any week. All other constraints are same with the first case. According to the GAMS results, the advertisements of each company were placed in ad slots given in Table

Table 2. Placement of the advertisements for each company

Company	Ad	Show	Week	Break	Company	Ad	Show	Week	Break
1	1	1	1	2	2	1	5	2	3
1	2	4	1	3	2	2	4	2	3
1	3	7	2	2	2	3	7	2	3
1	4	5	1	3	2	4	6	1	3
1	5	2	2	2	2	5	6	2	3
1	6	1	1	3	2	6	6	2	2
1	7	4	2	2	2	7	1	2	3
1	8	7	1	3	2	8	6	2	1
1	9	2	2	3	2	9	6	1	2
1	10	2	1	3	2	10	3	2	3
1	11	1	2	2					
1	12	1	1	1					
1	13	2	2	1					
1	14	3	1	3					
1	15	5	2	2					

Due to the restrictions the number of viewers that watch all advertisements of client 1 is 15,080 although 15,000 viewers were contracted, and the number of viewers that watch all advertisements of client 2 is 14,661 although 13,500 viewers were contracted.

The week and show constraints are satisfied for client 1. 8 advertisements are aired in week 1. Week constraint is met for client 2 although show constraint is not exactly met for second client because 5 advertisements are aired in show 6 where 7 of 15 advertisements were contracted. There are also no conflicting advertisements according to the results so the conflicting constraint is also satisfied in this example. Then the minimized cost of the deviations from goals is calculated as 9102.972 TRY.

5.2. 10 clients, 13 weeks, 10 shows, 7 breaks case

In this case, we try to model a problem much closer to the real life. In real life, there are also many conflicting companies and to satisfy their constraints together is a very hard and complex issue. Suppose we have 10 clients with some conflict constraints. Client 1 and Client 2 do not want their advertisements to be aired in the same break. Client 5 and Client 6 are also conflicting and want their advertisements not to be aired in the same breaks. Client 7 is conflicting with Client 1 and Client 2, and Client 5 and Client 6, respectively.

Client 1 requests 10 advertisements to be aired in week 1 while Client requests 12 advertisements. Both Client 5 and Client 6 want 12 of their advertisements to be aired separately in week 7 and week 10. The demand of Client 7 for week 1, week 7 and week 10 are 8, 12 and 12, respectively. Client 3 also requests 6 advertisements to be aired in week 4.

Furthermore, Client 1 requests 8 of its advertisements to be aired in show 1 while Client 2 wants 10 of its advertisements. In addition to this, Client 5 and Client 6 informs that the number of advertisements that will be aired in show 5 as 10 and 12 respectively. The demand of Client 7 is 12 for show 1 and 12 for show 5. Client 7 also restricted its plan by informing that none of its advertisements to be aired in show 6. Finally, Client 10 demands for 10 advertisements to be aired in show 2.

The target and realized data for number of advertisements and audience levels per client are given in Table 3.

Table 3. Results for number of advertisements and audience level

Client	Target		Realized	
	No.of Ads	Audience Level	No.of Ads	Audience Level
1	20	20,000	20	19,901
2	25	25,000	25	25,005
3	6	8000	6	7284
4	10	13,000	10	14,303
5	35	40,000	35	39,296
6	35	40,000	35	38,773
7	40	50,000	40	49,388
8	15	15,000	15	21,591
9	15	15,000	15	21,280
10	10	8000	10	9715

The total number of viewers who watch the Client 1 advertisements are 19,901 although 20,000 viewers were contracted, 25,005 viewers for Client 2 although 25,000 viewers were contracted, and 49,388 viewers for Client 7 although 50,000 viewers were contracted. They are not much different from the target values as seen in Table 3. All conflict constraints are met, so that there are no breaks that are used for 2 different clients. The week and show constraints related with Client 1 and Client 2 are exactly satisfied. Only two show constraints of Client 7 are met. In show 5, 12 advertisements are aired and in show 6 no ads are aired. The other show and week constraints related to Client 7are not met. For example, 5 advertisements are aired in show 1 but it should have been 12. Moreover, only one advertisement is aired in week 1 but it should have been 8.

The parameters in parenthesis shown in Table 4 are related to company, advertisement, show, week and break indexes, respectively. For example, the first row for Client 1 shows that advertisement 1 of Client 1 was planned to be aired in break 5 of show 3 in week 1.

Table 4. Advertisement plans of the Client 1, Client 2, and Client 7

Client 1	Client 2	Client 7
(1 , 1 , 3 , 1 , 5)	(2 , 1 , 1 , 1 , 6)	(7 , 1 , 8 , 7 , 5)
(1 , 2 , 1 , 1 , 2)	(2 , 2 , 1 , 3 , 4)	(7 , 2 , 1 , 12 , 5)
(1 , 3 , 1 , 1 , 1)	(2 , 3 , 6 , 1 , 5)	(7 , 3 , 4 , 7 , 5)
(1 , 4 , 1 , 13 , 6)	(2 , 4 , 8 , 1 , 5)	(7 , 4 , 5 , 12 , 5)
(1 , 5 , 5 , 1 , 2)	(2 , 5 , 7 , 1 , 2)	(7 , 5 , 7 , 7 , 5)
(1 , 6 , 9 , 10 , 2)	(2 , 6 , 3 , 1 , 2)	(7 , 6 , 7 , 13 , 5)
(1 , 7 , 5 , 1 , 5)	(2 , 7 , 1 , 13 , 5)	(7 , 7 , 10 , 1 , 5)
(1 , 8 , 10 , 3 , 5)	(2 , 8 , 3 , 13 , 5)	(7 , 8 , 8 , 4 , 5)
(1 , 9 , 3 , 1 , 3)	(2 , 9 , 6 , 1 , 2)	(7 , 9 , 10 , 7 , 5)
(1 , 10 , 4 , 1 , 2)	(2 , 10 , 1 , 1 , 5)	(7 , 10 , 5 , 10 , 3)
(1 , 11 , 1 , 10 , 6)	(2 , 11 , 4 , 9 , 5)	(7 , 11 , 7 , 2 , 5)
(1 , 12 , 1 , 1 , 4)	(2 , 12 , 7 , 3 , 2)	(7 , 12 , 1 , 9 , 5)
(1 , 13 , 1 , 5 , 4)	(2 , 13 , 3 , 7 , 5)	(7 , 13 , 2 , 2 , 5)
(1 , 14 , 1 , 1 , 3)	(2 , 14 , 10 , 1 , 2)	(7 , 14 , 3 , 8 , 5)
(1 , 15 , 5 , 7 , 5)	(2 , 15 , 1 , 13 , 4)	(7 , 15 , 5 , 13 , 5)
(1 , 16 , 6 , 9 , 5)	(2 , 16 , 1 , 9 , 2)	(7 , 16 , 5 , 3 , 6)
(1 , 17 , 8 , 1 , 2)	(2 , 17 , 4 , 1 , 5)	(7 , 17 , 1 , 6 , 5)
(1 , 18 , 1 , 7 , 3)	(2 , 18 , 1 , 1 , 7)	(7 , 18 , 8 , 13 , 5)
(1 , 19 , 4 , 13 , 5)	(2 , 19 , 1 , 3 , 2)	(7 , 19 , 3 , 3 , 5)
(1 , 20 , 2 , 8 , 5)	(2 , 20 , 10 , 1 , 6)	(7 , 20 , 8 , 2 , 5)
	(2 , 21 , 8 , 6 , 5)	(7 , 21 , 9 , 5 , 5)
	(2 , 22 , 1 , 2 , 6)	(7 , 22 , 1 , 5 , 5)
	(2 , 23 , 9 , 1 , 2)	(7 , 23 , 10 , 8 , 5)
	(2 , 24 , 1 , 12 , 6)	(7 , 24 , 4 , 4 , 5)
	(2 , 25 , 5 , 2 , 5)	(7 , 25 , 5 , 10 , 6)
		(7 , 26 , 5 , 5 , 5)
		(7 , 27 , 5 , 4 , 2)
		(7 , 28 , 1 , 10 , 2)
		(7 , 29 , 7 , 3 , 5)
		(7 , 30 , 5 , 11 , 5)
		(7 , 31 , 1 , 3 , 5)
		(7 , 32 , 5 , 3 , 5)
		(7 , 33 , 4 , 10 , 5)
		(7 , 34 , 9 , 11 , 5)
		(7 , 35 , 7 , 11 , 5)
		(7 , 36 , 5 , 5 , 2)
		(7 , 37 , 5 , 6 , 5)
		(7 , 38 , 5 , 4 , 5)
		(7 , 39 , 10 , 10 , 5)
		(7 , 40 , 2 , 7 , 4)

Table 3 and Table 5 give the realized data and scheduled slots for Client 5, Client 6, and Client 7. The total number of viewers related to Client 5, Client 6, and Client 7 are 39,296, 38,773 and 49,388 respectively. No single breaks are used for 2 different clients, so all conflict constraints are met. Client 5 have 8 ads and 8 of its advertisements been aired in week 7 and week 10 respectively when 12 of their advertisements to be separately aired. 8 advertisements in week 7 and 8 advertisements in week 10 of Client 6 are aired, when Client 6 requests 12 of their advertisements to be separately aired. The demands of Client 5 and Client 6 for show 5 are 10 and 12, respectively. Show constraints are met for both clients.

In Table 6, the scheduled slots for Client 3, Client 4, Client 8, Client 9, and Client 10 are given. For Client 3, all the advertisements are contracted to be aired in week 4, therefore week request is met. Client 10 has a show constraint. They request that all of their advertisements to be aired in show 2 and this constraint is also met. Remaining constraints are not related to show or week constraints. The realized number of advertisements aired and the audience level reached are given in Table 3.

Table 5. Advertisement plans of the Client 5, Client 6, and Client 7

Client 5	Client 6	Client 7
(5 , 1 , 5 , 1 , 5)	(6 , 1 , 5 , 1 , 2)	(7 , 1 , 8 , 7 , 5)
(5 , 2 , 10 , 12 , 5)	(6 , 2 , 6 , 3 , 5)	(7 , 2 , 1 , 12 , 5)
(5 , 3 , 5 , 8 , 5)	(6 , 3 , 4 , 7 , 2)	(7 , 3 , 4 , 7 , 5)
(5 , 4 , 5 , 2 , 2)	(6 , 4 , 10 , 7 , 2)	(7 , 4 , 5 , 12 , 5)
(5 , 5 , 5 , 7 , 4)	(6 , 5 , 10 , 9 , 5)	(7 , 5 , 7 , 7 , 5)
(5 , 6 , 5 , 10 , 1)	(6 , 6 , 5 , 7 , 5)	(7 , 6 , 7 , 13 , 5)
(5 , 7 , 4 , 9 , 5)	(6 , 7 , 2 , 7 , 5)	(7 , 7 , 10 , 1 , 5)
(5 , 8 , 6 , 9 , 5)	(6 , 8 , 6 , 10 , 5)	(7 , 8 , 8 , 4 , 5)
(5 , 9 , 6 , 10 , 2)	(6 , 9 , 6 , 7 , 5)	(7 , 9 , 10 , 7 , 5)
(5 , 10 , 9 , 10 , 5)	(6 , 10 , 8 , 11 , 5)	(7 , 10 , 5 , 10 , 3)
(5 , 11 , 2 , 5 , 5)	(6 , 11 , 1 , 13 , 5)	(7 , 11 , 7 , 2 , 5)
(5 , 12 , 8 , 3 , 5)	(6 , 12 , 6 , 8 , 5)	(7 , 12 , 1 , 9 , 5)
(5 , 13 , 6 , 12 , 5)	(6 , 13 , 6 , 11 , 5)	(7 , 13 , 2 , 2 , 5)
(5 , 14 , 10 , 13 , 5)	(6 , 14 , 6 , 2 , 5)	(7 , 14 , 3 , 8 , 5)
(5 , 15 , 7 , 8 , 5)	(6 , 15 , 5 , 13 , 2)	(7 , 15 , 5 , 13 , 5)
(5 , 16 , 2 , 10 , 5)	(6 , 16 , 5 , 9 , 4)	(7 , 16 , 5 , 3 , 6)
(5 , 17 , 9 , 7 , 2)	(6 , 17 , 4 , 3 , 5)	(7 , 17 , 1 , 6 , 5)
(5 , 18 , 7 , 9 , 5)	(6 , 18 , 4 , 13 , 5)	(7 , 18 , 8 , 13 , 5)
(5 , 19 , 4 , 11 , 5)	(6 , 19 , 8 , 6 , 5)	(7 , 19 , 3 , 3 , 5)
(5 , 20 , 6 , 7 , 2)	(6 , 20 , 6 , 6 , 5)	(7 , 20 , 8 , 2 , 5)
(5 , 21 , 5 , 7 , 6)	(6 , 21 , 7 , 7 , 6)	(7 , 21 , 9 , 5 , 5)
(5 , 22 , 3 , 7 , 5)	(6 , 22 , 4 , 8 , 5)	(7 , 22 , 1 , 5 , 5)
(5 , 23 , 6 , 5 , 5)	(6 , 23 , 5 , 7 , 2)	(7 , 23 , 10 , 8 , 5)
(5 , 24 , 10 , 3 , 5)	(6 , 24 , 8 , 10 , 5)	(7 , 24 , 4 , 4 , 5)
(5 , 25 , 2 , 8 , 5)	(6 , 25 , 5 , 5 , 6)	(7 , 25 , 5 , 10 , 6)
(5 , 26 , 7 , 10 , 2)	(6 , 26 , 9 , 7 , 5)	(7 , 26 , 5 , 5 , 5)
(5 , 27 , 8 , 12 , 5)	(6 , 27 , 5 , 2 , 5)	(7 , 27 , 5 , 4 , 2)
(5 , 28 , 3 , 10 , 5)	(6 , 28 , 3 , 10 , 2)	(7 , 28 , 1 , 10 , 2)
(5 , 29 , 5 , 10 , 2)	(6 , 29 , 5 , 10 , 4)	(7 , 29 , 7 , 3 , 5)
(5 , 30 , 3 , 13 , 5)	(6 , 30 , 4 , 10 , 2)	(7 , 30 , 5 , 11 , 5)
(5 , 31 , 1 , 7 , 5)	(6 , 31 , 5 , 10 , 5)	(7 , 31 , 1 , 3 , 5)
(5 , 32 , 5 , 7 , 3)	(6 , 32 , 5 , 9 , 5)	(7 , 32 , 5 , 3 , 5)
(5 , 33 , 5 , 7 , 1)	(6 , 33 , 7 , 10 , 5)	(7 , 33 , 4 , 10 , 5)
(5 , 34 , 5 , 12 , 2)	(6 , 34 , 5 , 8 , 6)	(7 , 34 , 9 , 11 , 5)
(5 , 35 , 1 , 10 , 5)	(6 , 35 , 5 , 10 , 7)	(7 , 35 , 7 , 11 , 5)
		(7 , 36 , 5 , 5 , 2)
		(7 , 37 , 5 , 6 , 5)
		(7 , 38 , 5 , 4 , 5)
		(7 , 39 , 10 , 10 , 5)
		(7 , 40 , 2 , 7 , 4)

Table 6. Advertisement plan of Client 3, Client 4, Client 8, Client 9, and Client 10

Client 3	Client 4	Client 8	Client 9	Client 10
(3 , 1 , 8 , 4 , 5)	(4 , 1 , 10 , 1 , 5)	(8 , 1 , 8 , 6 , 5)	(9 , 1 , 10 , 1 , 5)	(10 , 1 , 2 , 13 , 3)
(3 , 2 , 4 , 4 , 5)	(4 , 2 , 5 , 7 , 5)	(8 , 2 , 3 , 7 , 5)	(9 , 2 , 8 , 2 , 5)	(10 , 2 , 2 , 12 , 6)
(3 , 3 , 9 , 4 , 5)	(4 , 3 , 5 , 11 , 5)	(8 , 3 , 4 , 9 , 5)	(9 , 3 , 8 , 6 , 5)	(10 , 3 , 2 , 7 , 1)
(3 , 4 , 5 , 4 , 5)	(4 , 4 , 6 , 9 , 5)	(8 , 4 , 10 , 8 , 5)	(9 , 4 , 5 , 11 , 5)	(10 , 4 , 2 , 8 , 2)
(3 , 5 , 10 , 4 , 5)	(4 , 5 , 7 , 8 , 5)	(8 , 5 , 2 , 8 , 5)	(9 , 5 , 3 , 8 , 5)	(10 , 5 , 2 , 4 , 4)
(3 , 6 , 3 , 4 , 5)	(4 , 6 , 4 , 10 , 5)	(8 , 6 , 5 , 11 , 5)	(9 , 6 , 3 , 3 , 5)	(10 , 6 , 2 , 11 , 1)
	(4 , 7 , 5 , 12 , 5)	(8 , 7 , 3 , 3 , 5)	(9 , 7 , 7 , 13 , 5)	(10 , 7 , 2 , 1 , 5)
	(4 , 8 , 8 , 6 , 5)	(8 , 8 , 3 , 13 , 5)	(9 , 8 , 5 , 12 , 5)	(10 , 8 , 2 , 9 , 5)
	(4 , 9 , 3 , 3 , 5)	(8 , 9 , 5 , 7 , 5)	(9 , 9 , 7 , 8 , 5)	(10 , 9 , 2 , 3 , 5)
	(4 , 10 , 3 , 8 , 5)	(8 , 10 , 5 , 3 , 5)	(9 , 10 , 2 , 1 , 5)	(10 , 10 , 2 , 8 , 5)
		(8 , 11 , 8 , 2 , 5)	(9 , 11 , 8 , 13 , 5)	
		(8 , 12 , 10 , 1 , 5)	(9 , 12 , 6 , 9 , 5)	
		(8 , 13 , 2 , 1 , 5)	(9 , 13 , 5 , 13 , 5)	
		(8 , 14 , 10 , 3 , 5)	(9 , 14 , 5 , 7 , 5)	
		(8 , 15 , 8 , 7 , 5)	(9 , 15 , 8 , 7 , 5)	

6. Conclusion and future directions

Media planning is a complex problem and sales plan generation is a subproblem of this area. We studied the sales plan generation problem as a stochastic decision problem because the number of audience that is wanted to be reached is uncertain for any show in any week.

In our examples, we give a relatively high penalty rate of not meeting and/or exceeding total audience demand than the penalty rates of not meeting and/or exceeding audience per week and show demands. Therefore, some of the week and show constraints are not met for all companies. These results and unsatisfied constraints will be changed if the penalty rates are changed. By using results derived from the model, company should increase its revenue from advertisement sales, so these results can be used in stochastic revenue management models.

In this study, we plan an optimal schedule for advertisement slots bought at upfront market. By generating these kinds of optimal plans, broadcast companies can save millions of dollars. These plans also increase revenues because of a better use of advertisement inventory. The time needed to produce a sales plan and the time needed to rework on created plans will decrease by using optimization models. Broadcasting companies will also be able to respond more quickly to their clients, hence they will have a stronger position against their competitors.

We formulate the problem from the broadcast companies' point of view. The penalties, weekly rates and show rates are assumed to be constant for all clients. In future, same problem can be reformulated from the clients' point of view or an integrated approach can be formulated. In addition to this, the problem modeled in this paper is a minimization problem where it can be reformulated as a profit maximization problem with available budget data of the clients. Some other constraints from real world can be added to these models such that different penalty rates for more influential clients which will be more realistic especially for countries in which the business environment is more of a relationship-based type.

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The prediction of the wind speed at different heights by machine learning methods

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Abstract. In Turkey, many enterprisers started to make investment on renewable energy systems after new legal regulations and stimulus packages about production of renewable energy were introduced. Out of many alternatives, production of electricity via wind farms is one of the leading systems. For these systems, the wind speed values measured prior to the establishment of the farms are extremely important in both decision making and in the projection of the investment. However, the measurement of the wind speed at different heights is a time consuming and expensive process. For this reason, the success of the techniques predicting the wind speeds is fairly important in fast and reliable decision-making for investment in wind farms. In this study, the annual wind speed values of Kutahya, one of the regions in Turkey that has potential for wind energy at two different heights, were used and with the help of speed values at 10 m, wind speed values at 30 m of height were predicted by seven different machine learning methods. The results of the analysis were compared with each other. The results show that support vector machines is a successful technique in the prediction of the wind speed for different heights.

Keywords: Wind speed prediction; support vector machines; wind farm investment.
AMS Classification: 68Q32, 68T05, 62M20, 68Q32.

1. Introduction

Renewable energy technologies such as solar, wind, biomass, geothermal, etc., become more important for the future of countries, since there are local resources and indefinite sources of energy [1]. Similar to other countries, Turkey is also making progress in the use of renewable energies. Within this scope, according to the 2015-2019 Strategic Plan of the Ministry of Energy and Natural Sources, in 2014, it is intended to increase the share of renewable energy resources in primary energy supply and electricity generation. In the plan, it is aimed to increase the established wind power capacity from 5600MW in 2015 to 10000MW in 2019 [2]. In order to achieve these targets, the government introduced new stimulus packages and provided some convenience for renewable energy investments. This is quite encouraging for enterprisers to make investments in this field.

Wind is available virtually everywhere on earth, although there are wide variations in wind strengths. Wind energy is being developed in the industrialized world for environmental reasons and it has attractions in the developing world as it can be installed quickly in areas where electricity is urgently needed [3]. In Turkey, the production of electricity through wind energy connected to the grid started in 1998 and increased one fold in each year after 2006, see Figure 1. As seen from the Figure 2, in 2015, the overall energy produced via wind energy reached to 4718,3 MW [4].

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The wind speed is one of the most important parameters in determination of the wind energy potential of a region. For this reason, in a potential region, wind speed data are measured hourly and saved for one year and these data are used in measurement of the wind potential of that region. For this purpose, the measurement station is placed at a point of the region which is representative of that field. In the farm field, the height of the measurement station, which is located perpendicular to the direction of the dominating wind, is commonly two-third of the height of the wind turbine. The measurements could be performed at different heights, e.g. 10m, 30m and 50m height of observation pole. These measurements are necessary to make a decision for investment. However, as they are long-term and expensive, they bring about extra cost and also prolonged the duration to the investment. For this reason, the success of the wind speed prediction methods for different heights could offer fast, reliable and cost-effective way by which the investment could be planned well-in advance.

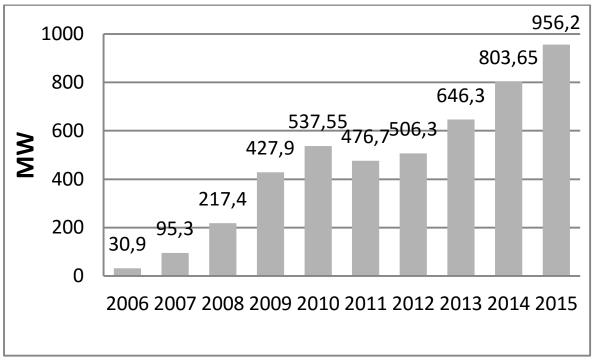

Figure 1. Annual installations for wind power plants in Turkey (MW) [4].

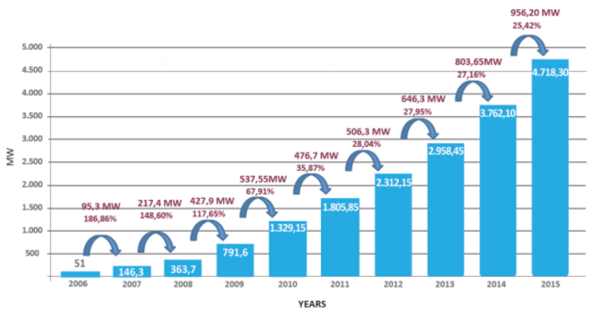

Figure 2. Cumulative installations for wind power plants in Turkey (MW) [4].

Wind energy industry depends on wind speed forecasts to help determine facility location, facility layout, as well as the optimal use of turbines in day today operations [5]. There are physical, statistical, artificial neural and hybrid methods on the prediction of wind speed. Especially, in recent years, artificial intelligence techniques, like artificial neural networks (ANN), fuzzy logic and support vector machines (SVM), and hybrids of these methods are widely used in the prediction of the wind speeds. In a review study, presenting the previous studies on the prediction of the wind speed and the energy produced, Lei et al. state that artificial techniques are more successful than the traditional techniques and hybrid models, which come out nowadays, of cause are advanced ones and have less error than others [6]. A research was developed by Mohandes et al. based on ANN and autoregressive model (AR), the results indicated that the ANN was superior to the AR model [7]. Mohandes et al. also introduced SVM to wind speed prediction and compared the performance with the multilayer perceptron (MLP) neural networks. Results indicate that SVM compare favorably with the MLP model [8]. A fuzzy model based on spatial correlation method was proposed by Damousis et al. to predict wind speed and power generation [9]. Kurban et al. analyzed the wind energy potential by using Weibull and Rayleigh statistical distribution functions. Results indicate that Weibull modeled wind speed better than Rayleigh [10]. Shi et al. investigated the applicability of hybrid models based on two case studies on wind speed and wind power generation. Two hybrid models, namely, ARIMA–ANN and ARIMA–SVM, are selected to compare with the single ARIMA, ANN, and SVM forecasting models [11].

To the best our knowledge, this is the first study that handles the wind speed prediction problem via seven different algorithms. The remainder of this paper is organized as follows. In section 2, machine learning regression models used in wind speed forecasting are introduced. In Section 3, data set is given and in section 4 forecasting models are compared and evaluated. Finally, Section 5 presents the main conclusion of the paper.

2. Machine learning regression methods

Machine learning (ML) regression methods predict an unknown dependency between the inputs and output from a dataset [12]. Table 1 demonstrates a list of the ML regression methods, which are utilized in this paper. Most of these regression algorithms have been widely used for modeling many real-life regression problems. These methods are stated by The Waikato Environment for Knowledge Analysis (WEKA, http://www.cs.waikato.ac.nz/~ml/weka/), the leading open-source platform in machine learning. WEKA is a comprehensive collection of machine-learning algorithms for data mining tasks written in Java, containing tools for data pre-processing, classification, regression, clustering, association rules, and visualization [13]. In this study the algorithms are directly applied with WEKA platform and we utilized three categories of WEKA 3.6.13 platform as Functions, Lazy-learning, and Tree-based learning algorithms. Functions incorporate algorithms, which are based on the mathematical models. Lazy-learning algorithms handle with training data until a query is answered. The lazy-learning algorithms aggregate the training data in memory and find out associated data in the database to satisfy a specific query. Tree-based learning algorithms are proper for making predictions via a tree structure. Leaves of the trees exemplify classifications and branches of the trees indicate conjunctions of features. The brief summary of the methods, used in this paper, are presented in Table 1.

Table 1. Regression methods used in this paper.

Categories	Method	Abbreviation
Functions	Multilayer Perceptron	MLP
	Radial Basis Function Neural Networks	RBFNetwork
	Support Vector Regression	SVMr
Lazy-learning algorithms	KStar	K*
	Locally Weighted Learning	LWL
Tree-based learning algorithms	DecisionStump	DecisionStump
	RandomTree	RandomTree

2.1. Support vector regression (SVMr)

The foundations of support vector machines (SVMs) have been developed by Vapnik [14] and have been increasingly used in different forecasting problems. Successful forecasting studies were performed with support vector regression (SVMr) in different fields such as production forecasting [15], speed of traffic flow forecasting [16] and financial time series forecasting [17, 18]. Also SVMr is used as a predictor to determine wind speed [8, 19].

SVMr formulation is given below [20, 21, 22];

The simplest classification problem is two-class linear separable case. Assume that there is a training set which has "l" number points.

$$(x_1, y_1),....,(x_n, y_n),\ x_i \in R^d,\ y_i \in \{-1,+1\} \tag{1}$$

Suppose that there are some hyper planes that separates two classes can shown as

$$w.x + b = 0 \tag{2}$$

where w is weight vector which is normal to hyperplane, and b is the threshold value. In the simplest linearly separable case, we seek for "largest margin". Margin borders can be formulated as

$$w.x_i + b \geq 1 \quad y_i = 1,\ w.x_i + b \leq -1 \quad y_i = -1 \tag{3}$$

Eq. (3) can be generalizable as

$$y_i(w.x_i + b) \geq 1, \quad i = 1,....,l \tag{4}$$

The distance between margin borders is

$$d = \frac{2}{\|w\|} \tag{5}$$

Here $\|w\|$ is the Euclidean norm of w. According to theory, to determine unique solution with finding optimal hyperplane "d" must be maximized. To calculate optimal hyperplane we have to minimize

$$1/2\|w\|\ 2 \tag{6}$$

subject to Eq. (3). This quadratic optimization problem can be solved with Lagrange Multipliers.

$$L(w,b,\alpha) = \frac{1}{2}\|w\|^2 - \sum_{i=1}^{l} \alpha_i [y_i(w.x + b) - 1] \tag{7}$$

Eq. (7) is a Lagrangian where w and b are primal variables and α_i is dual variable. To find the optimal solution of the primal optimization problem (Eq. 7) we have to minimize primal variables w and b.

$$\frac{\partial L(w,b,\alpha)}{\partial w} = 0 \tag{8}$$

$$\frac{\partial L(w,b,\alpha)}{\partial b} = 0 \tag{9}$$

After calculating above differential operations, Eqs. (10, 11) are found.

$$\sum_{i=1}^{l} \alpha_i y_i = 0 \qquad \alpha_i \geq 0, \quad i = 1,...,l \tag{10}$$

$$w = \sum_{i=1}^{l} \alpha_i y_i x_i \quad \alpha_i \geq 0, \quad i = 1,...,l \tag{11}$$

By using a generalized method of Lagrange multipliers called Karush–Kuhn–Tucker conditions we can provide below equation where $\alpha \neq 0$ points from the Eq. (4). Those points are subset of training data with the non-zero Lagrangian multipliers called Support Vectors.

$$\alpha_i[y_i(w.x_i + b) - 1] = 0, \qquad i = 1,...,l \tag{12}$$

We can transform Eq. (12) into Eq. (13) subject to Eqs. (10, 11) . In our Lagrangian equation, there are only dual variables after substitution primal variables w and b . Now, our problem is a dual optimization problem, it can be solved as shown below,

Maximize

$$L(\alpha) = \sum_{i=1}^{l} \alpha_i - \frac{1}{2} \sum_{i=1}^{l} \sum_{j=1}^{l} \alpha_i \alpha_j y_i y_j (x_i x_j) \tag{13}$$

subject to Eq. (10).

2.2. Multi layer perceptron (MLP)

The MLP is a feed forward artificial neural network (ANN) trained with the back propagation algorithm that consists of neurons with substantially weighted interconnections, where signals always travel to the direction of the output layer. These neurons are mapped as sets of input data onto a set of proper outputs with hidden layers. The input signals are sent by the input layer to the hidden layer without execution of any operations. Then the hidden and output layers multiply the input signals by a set of weights, and either linearly/non-linearly transforms results into output values. The connection between units in following layers has an associated weight. These weights are optimized to compute reasonable accuracy of prediction [23, 24]. A typical MLP with one hidden layer can be mathematically describe in Eqs. (14, 18) as below [25, 26, 27]:

$$u_j = \sum_{i=1}^{N_{input}} X_i a_{ij} + a_{0j} \tag{14}$$

Eq. (14) defines summing products of the inputs (X_i) and weight vectors (a_{ij}) and a bias term of hidden layer (a_{0j}). In Eq. (15), the outputs of hidden layer (Z_j) are obtained as transforming this sum, which is defined in Eq. (14), by using activation function g .

$$Z_j = g(u_j) \tag{15}$$

The most widely used activation function is sigmoid function [28], which is defined in Eq. (16) for input x. The hidden and output layers are based on this sigmoid function.

$$g(x) = sigmoid(x) = \frac{1}{(1 + e^{-x})} \tag{16}$$

Eq. (17) defines summing products of hidden layer's outputs (Z_j) and weight vectors (b_{jk}) and bias term of output layer (b_{0k}).

$$v_k = \sum_{j=1}^{N_{hidden}} Z_j b_{jk} + b_{0k} \tag{17}$$

In Eq. (18), the outputs of the output layer (Y_k) are obtained by transforming this sum, that is calculated in Eq. (17), using sigmoid function g, which is defined in Eq. (16).

$$Y_k = g(v_k) \tag{18}$$

2.3. Radial basis function neural networks (RBFNetwork)

An RBFNetwork is a type of a feed-forward neural network comprised of three layers: input, hidden and output layers. Even though the computations between input and hidden layers are nonlinear, they are linear between hidden and output layers. An RBFNetwork can build both regression and classification models [29]. It differs from an MLP in the way the hidden layer units perform calculations. In an RBFNetwork, inputs from the input layer are mapped to each of the hidden units. The hidden units use radial functions such as the bell-shaped Gaussian function for activation. The activation h(x) of the Gaussian function for a given input x decreases monotonically as the distance between x and the center c of the Gaussian function increases. The most general RBFNetwork can be mathematically defined as below [30]:

$$h(x) = \phi((x - c)^T R^{-1}(x - c)) \tag{19}$$

where c is the center, R is the metric and ϕ is the function The metric is often Euclidean so that $R = r^2 I$ for some scalar radius r and the Eq.(20) simplifies to

$$h(x) = \phi\left(\frac{(x - c)^T (x - c)}{r^2}\right) \tag{20}$$

The simplification is a one-dimensional input space in which case

$$h(x) = \phi\left(\frac{-(x - c)^2}{r^2}\right) \tag{21}$$

The Gaussian function $\phi(z) = e^{-z}$ is used.

Therefore a typical radial function is the Gaussian which, in the case of a scalar input, is

$$h(x) = \exp\left(\frac{-(x-c)^2}{r^2}\right) \quad (22)$$

2.4. KStar (K*)

KStar is an instance-based classifier used for regression problems [31]. It uses entropic measure, based on probability of transforming instance into another by randomly selecting between all possible transformations. Using entropy as appraise of distance has numerous utility. Tackling with the missing values by classifiers pose a problem. Usually missing values treated as a separate value, thought as maximally different, substitute for average value, which otherwise would simply be ignored. Entropy based classifier is a solution for such issues [32].

2.5. Locally weighted learning (LWL)

The LWL uses an instance-based algorithm, assigns instance weights. This algorithm can perform both classification and regression [33]. The basic idea of the LWL is that any non-linearity can be approximated by a linear model, if the output surface is smooth. Therefore, instead of looking for a complex global model, it is simple to approximate non-linear functions by using individual local models [34].

2.6. DecisionStump

DecisionStump, constructs one-level binary decision trees for datasets with a categorical or numeric class, handling with missing values by treating them as a separate value and extending a third branch from the stump [35]. It makes (i) regression based on mean-squared errors or (ii) classification based on entropy depending on the data type to be predicted [36]. It also finds a single attribute that provides the best discrimination between the classes and then bases future predictions on this attribute [37].

2.7. RandomTree

RandomTree is also a regression-based decision tree algorithm. Trees built by RandomTree consider randomly selected attributes at each node. It performs no pruning. Also has an option to allow prediction of class probabilities based on a hold-out set (backfitting) [35].

3. The data set

In this study, twelve months data used which consists of 10 m and 30 m heights. Measurements were generally taken at 10–30 m heights from the ground [1]. An annual set of data, collected for a wind farm which is planned to be established in Kutahya was investigated. Kutahya is a region of Turkey has potential of wind power. By using wind speed values obtained for 10 m of height, wind speed values for 30 m of height were predicted by SVMr, MLP (the most commonly used technique in prediction of wind speed), RBFNetwork, K*, LWL, DecisionStump, RandomTree techniques. The results of prediction were compared by each other and actual wind speed.

From whole set of data, the first eleven months of the year data is used for training stage and one month (December) data is used for validating the results obtained. Daily averaged datas are used and the data collected in the first five days of the June were ignored and were not considered in calculations due to maintenance of the station that was performed in that period. The data are summarized in Table 2 monthly used in this paper.

Table 2. Summary of used datas.

	10 meters			30 meters		
	Min	Max	Average	Min	Max	Average
Jan.	1,36	8,56	4,11	1,86	9,33	4,62
Feb.	1,77	7,79	3,92	1,89	8,64	4,28
March	1,98	9,33	4,68	1,78	9,30	4,94
April	2,77	7,32	4,30	3,00	7,70	4,58
May	2,59	6,25	4,00	2,56	6,68	4,23
June	3,02	7,92	4,48	3,24	8,41	4,81
July	3,09	5,59	4,33	3,30	5,98	4,64
Aug.	2,78	6,71	4,10	2,86	7,20	4,41
Sept.	2,74	7,02	4,01	2,86	7,50	4,28
Oct.	1,99	7,30	3,98	1,90	8,00	4,31
Nov.	2,52	9,18	4,73	2,72	9,83	5,09

4. Findings

In this paper, a comparative assessment on wind speed prediction has been performed via seven machine learning regression methods. Forecasting results obtained were compared to each other and actual data sets. For wind prediction, data of 331 days wind speed measured at 10 and 30 meters were used for training, while data of 31 days in December were used for testing. After many different trials for each model, polynomial kernel was selected for SVMr; where p and C (complexity coefficient) were taken as 1. In MLP method, learning coefficient was L=0.3, moment was

M=0.2, training number was N=500 and hidden layer number was H=2. Minimum standard deviation was 0.1, random seed was 1 and number of clusters was 2 for RBFnetwork. In random tree the number of randomly chosen attributes K and maximum depth for unlimited was taken as 0, the minimum total weight was 1, one random seed and no allow of unclassified instances. The parameter for global blending was determined as 20 for K* and weighting function was determined as linear for LWL.

Wind speed forecasting comparison results are presented in Figure 3 and Figure 4 via Bar Chart with lower and upper bounds and Cumulative Line Chart, also the numerical descriptors are depicted with Boxplots presented in Figure 5. The boxes indicate the interquartile ranges, the whiskers show the 5th and 95th percentile of observed and predicted data and the horizontal line within each boxes indicate the median values. Skewness, a description of wind speed distribution asymmetry, is shown in the Figure 5. Typically, wind speed data are positively skewed, placing the mean in the upper half of the data, i.e., they have a long right tail. This means that large positive deviations from the mean wind speed are more frequent than negative deviations of the same magnitude. This is, because wind speed values are one-sidedly bounded. The degree of positive skewness illustrates that wind speed typically occurs as many small events with a few large events that elevate the mean. The predictions fit the observed data well. The MLP and RandomTree did a fairly good job at capturing the observed data, however the overall performance of SVMr are the best when compared to the patterns of the observed wind speed data. Overall, results suggest that SVMr forecasting results are more realistic than other six methods' forecasting results.

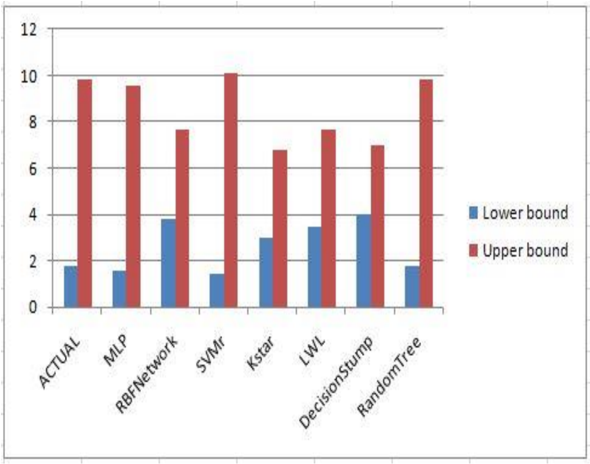

Figure 3. Bar chart comparison of actual wind speed with lower and upper bounds.

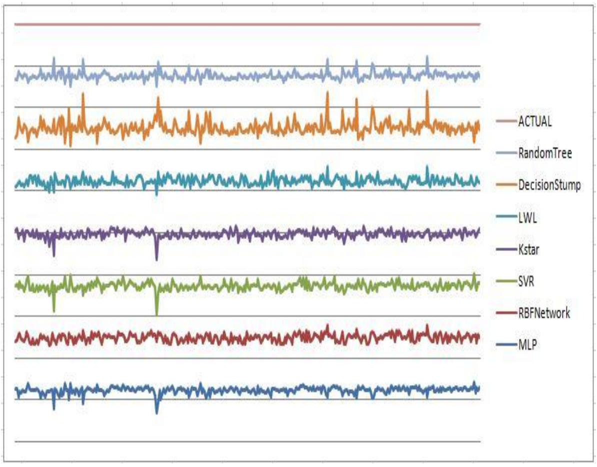

Figure 4. Cumulative line chart comparison with machine learning regression methods.

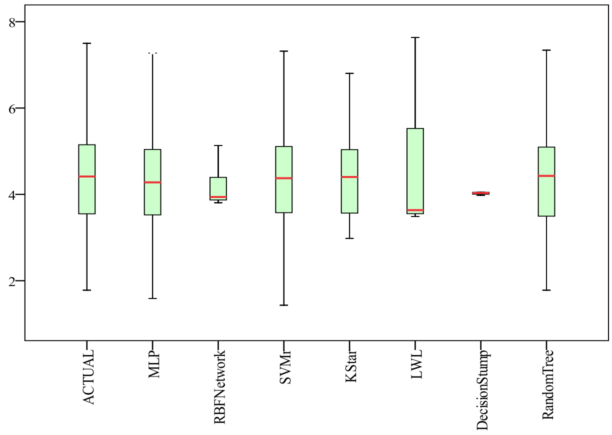

Figure 5. Boxplot comparison of actual wind speed with machine learning regression methods.

Performances of the methods employed were compared using different statistical measures. Mean absolute error (MAE), root mean square error (RMSE) and coefficient of determination (R^2) are among the widely used measures that are based on the notion of "mean error". Relative absolute error (RAE) and root relative squared error (RRSE) are based on the notion of "relative error". Successes of SVMr, MLP, RBFNetwork, K*, LWL, DecisionStump, RandomTree methods were compared using the measures of R^2, MAE, RMSE, RAE and RRSE. Calculated values related to statistical measures are given in Table 3.

Similarly when we test the data with traditional regression method, we obtained relatively worse prediction performance (MAE: 0.727, RMSE: 0.8902, R^2: 0.5975, RAE: %69.02 and RRSE: %63.26).

Table 3. Comparison of statistical measures.

Machine learning	Model	R^2	MAE	RMSE	RAE	RRSE
Functions	MLP	0,98	0,15	0,21	14,13	14,98
	RBF Network	0,75	0,57	0,70	54,28	49,64
	SVMr	**0,98**	**0,11**	**0,18**	**10,35**	**12,75**
Lazy-learning algorithms	KStar	0,94	0,27	0,51	25,41	36,36
	LWL	0,80	0,50	0,63	47,79	44,92
Tree-based learning algorithms	Decision Stump	0,60	0,73	0,89	68,91	63,41
	Random Tree	0,97	0,15	0,22	14,58	15,90

5. Result

Within the scope of the study, wind speed predictions were performed by seven machine learning regression methods. To the best our knowledge, it is the first so comprehensive comparative study that handles the wind speed prediction problem via seven different algorithms. All seven methods are compared and it is shown that each of the three methods, SVMr, MLP and RandomTree, are highly successful in wind speed forecasting than the other four methods. When the methods are compared, the correlation between wind speed at 30 m and prediction result are very close to each other for these three techniques. As seen from Figure 5, the wind speed predictions of the SVMr, MLP and RandomTree methods were fairly accurate. If statistical analysis criteria were applied, however, it can be seen that MAE, RMSE, RAE and RRSE values are much smaller for SVM technique. Thus, it can be stated that, in this sample study, SVMr shows a better performance compared to others.

The study shows that three methods are quite successful in the prediction of the wind speeds and the predicted values are very close to the real measurements. For this reason, it can be stated that wind speed predictions for different heights made by SVMr, MLP and RandomTree may help in decision making for establishment of wind farms and in wind farm planning activities.

Finding suitable and accurate wind speed predictor is crucial in wind energy applications. it's obtained that prediction success of SVM has been found more satisfactory than the other's. It is concluded that the SVM's can be used effectively as an alternative method by researchers and the investors for predicting the wind speed.

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On semi-G-V-type I concepts for directionally differentiable multiobjective programming problems

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Abstract. In this paper, a new class of nonconvex nonsmooth multiobjective programming problems with directionally differentiable functions is considered. The so-called G-V-type I objective and constraint functions and their generalizations are introduced for such nonsmooth vector optimization problems. Based upon these generalized invex functions, necessary and sufficient optimality conditions are established for directionally differentiable multiobjective programming problems. Thus, new Fritz John type and Karush-Kuhn-Tucker type necessary optimality conditions are proved for the considered directionally differentiable multiobjective programming problem. Further, weak, strong and converse duality theorems are also derived for Mond-Weir type vector dual programs.

Keywords: multiobjective programming; (weak) Pareto optimal solution; G-V-invex function; G-Fritz John necessary optimality conditions; G-Karush-Kuhn-Tucker necessary optimality conditions; duality.
AMS Classification: 90C70,90C29.

1. Introduction

It is well known that the convexity notion plays a vital role in many aspects of mathematical programming including sufficient optimality conditions and duality theorems. However, it is not possible to prove under convexity fundamental results in optimization theory for a lot of optimization problems. During the past decades, therefore, generalized convex functions received much attention. Various generalizations of convex functions have appeared in literature, not only for scalar optimization problems, but also for multiobjective programming problems. This is simply a consequence of the fact that, in recent years, the analysis of optimization problems with several objectives conflicting with one another has been a focal issue. Such multiobjective optimization problems are useful mathematical models for the investigation of real-world problems, for example, in engineering, economics, and human decision making.

One of a generalization of convexity is invexity defined by Hanson [11]. Hanson showed that the Kuhn-Tucker necessary conditions are sufficient for a minimum in differentiable scalar optimization problems involving invex functions with respect to the same function η. Craven [10] has shown that f has the previous property when $f = h \circ \phi$, with h convex, ϕ differentiable, and ϕ' having full rank. Thus some invex functions, at least, may be obtained from convex functions by a suitable transformation of the domain spaces. Such transformation destroy convexity, but not

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the invex property; the term invex, from invariant convex, was introduced by Craven in [10] to express this fact. Further, Ben-Israel and Mond [9] considered a class of functions called pre-invex and also showed that the class of invex functions is equivalent to the class of functions whose stationary points are global minima.

Later, Hanson and Mond [12] defined two new classes of functions called type-I and type-II functions, and they established sufficient optimality conditions and duality results for differentiable scalar optimization problems by using these concepts. Mukherjee and Singh [21] derived a set of sufficient conditions for a solution to be efficient for a multiobjective programming problem where the objective as well as constraint functions are semi-differentiable and η-convex. In [16], Kaul et al. proved Kuhn-Tucker type necessary and sufficient conditions for a feasible point to be an efficient or properly efficient solution in the considered multiobjective programming problems with (generalized) type I functions. Following Jeyakumar and Mond [15], Hanson et al. [13] introduced the V-type I vector optimization problem, including positive real-valued functions α_i and β_j in their definition, and they obtained optimality conditions and duality results under various types of generalized V-type I requirements. They showed that V-type I property can replace invex, in proving sufficient KKT conditions. Further, Aghezzaf and Hachimi [2] [14] introduced classes of generalized type I functions for a differentiable multiobjective programming problem and derived some Mond-Weir type duality results under the generalized type I assumptions. In [17], Kuk and Tanino derived optimality conditions and duality theorems for non-smooth multiobjective programming problems involving generalized Type I vector valued functions. Suneja and Srivastava [24] introduced generalized d-type I functions in terms of directional derivative for a multiobjective programming problem and discussed Wolfe type and Mond-Weir type duality results. Suneja and Gupta [25] established necessary optimality conditions for a multiobjective nonlinear programming involving semilocally convex functions and Wolfe type and Mond-Weir type duals are formulated.

In [4], Antczak studied d-invexity as one of the nondifferentiable generalization of an invex function. He established, under weaker assumptions than Ye [27], the Fritz John type and Karush-Kuhn-Tucker type necessary optimality conditions for weak Pareto optimality and duality results which have been stated in terms of the right differentials of functions involved in the considered multiobjective programming problem. Some authors proved that the Karush-Kuhn-Tucker type necessary conditions [4] are sufficient under various generalized d-invex functions (see, for instance, [1], [3], [18], [19], [20]). In [5], Antczak defined the classes of d-r-type I objective and constraint functions and, moreover, the various classes of generalized d-r-type I objective and constraint functions for multiobjective programming problems with directionally differentiable functions. He corrected the Karush-Kuhn-Tucker necessary conditions proved in [4] and established sufficient optimality conditions and various Mond Weir duality results for nondifferentiable multiobjective programming problems in which the functions involved belong to the introduced classes of directionally differentiable generalized invex functions. Finally, he showed that the introduced d-r-type I notion with $r \neq 0$ is not a sufficient condition for Wolfe weak duality to hold. Slimani and Radjef [23] introduced new concepts of d_I-invexity and generalized d_I-invexity in which each component of the objective and constraint functions is directionally differentiable in its own direction d_i. Further, they proved new Fritz–John type necessary and Karush–Kuhn–Tucker type necessary and sufficient optimality conditions for a feasible point to be weakly efficient, efficient or properly efficient and, moreover, weak, strong, converse and strict duality results for a Mond–Weir type dual under various types of generalized d_I-invexity assumptions. Ahmad [3] introduced a new class of generalized d_I-univexity in which each component of the objective and constraint functions is directionally differentiable in its own direction d_i for a nondifferentiable multiobjective programming problem. Based upon these generalized functions, he proved sufficient optimality conditions for a feasible point to be efficient and properly efficient and duality results under the generalized d_I-univexity requirements.

This paper represents the study concerning nonconvex nonsmooth multiobjective programming with a new class of directionally differentiable functions. Thus, for the considered nonsmooth multiobjective programming problem with inequality constraints, we define a new class of directionally differentiable nonconvex vector-valued functions, namely directionally differentiable G-V-type I objective and constraint functions and various classes of its generalizations. The class of directionally differentiable G-V-type

I objective and constraint functions is a generalization of the class of d-invex functions introduced by Ye [27], the class of G-invex functions introduced by Antczak [6] and [7] for differentiable multiobjective programming problems and also the class of V-invex functions defined by Jeyakumar and Mond [15] for differentiable vector optimization problems to the directionally differentiable vectorial case.

The class of G-invex functions extends the notion of invexity since it contains many various invexity concepts. A characteristic global optimality property of various classes of invex functions is also proved in the case of a G-invex functions. It turns out that every stationary point of G-invex function is its global minimum point. The importance of the G-invex functions is because, similarly to Craven's work [10], the transformations of functions do not destroy properties of invex functions.

The main purpose of this article is, however, to apply the concept of directionally differentiable G-V-type I objective and constraint functions to develop optimality conditions for a new class of nonconvex directionally differentiable multiobjective programming problems. Considering the concept of a (weak) Pareto solution, we establish both the so-called G-Fritz John necessary optimality conditions and the so-called G-Karush-Kuhn-Tucker necessary optimality conditions for directionally differentiable vector optimization problems under the constraint qualification introduced in this work. The G-Fritz John necessary optimality conditions and the G-Karush-Kuhn-Tucker necessary optimality conditions proved in this paper are more general than the classical Fritz John necessary optimality conditions and the classical Karush-Kuhn-Tucker necessary optimality conditions well-known in the literature. Furthermore, based on the introduced G-Karush-Kuhn-Tucker necessary optimality conditions, we prove sufficient optimality for both weak Pareto and Pareto optimality in multiobjective programming problems involving directionally differentiable G-V-type I objective and constraint functions. In particular, the sufficient optimality conditions established under semi-G-V-type I assumptions are more useful for some class of nonconvex directionally differentiable vector optimization problems than the sufficient optimality conditions proved for vector optimization problems with directionally differentiable vector-valued invex functions. Furthermore, a so-called G-Mond–Weir type dual is formulated for the considered directionally differentiable vector optimization problem and weak, strong, converse and strict duality results are proved under semi-G-V-type I assumptions.

2. Directionally differentiable G-type I functions and generalized G-type I functions

In this section, we provide some definitions and some results that we shall use in the sequel. The following convention for equalities and inequalities will be used throughout the paper.

For any $x = (x_1, x_2, ..., x_n)^T$, $y = (y_1, y_2, ..., y_n)^T$, we define:

(i) $x = y$ if and only if $x_i = y_i$ for all $i = 1, 2, ..., n$;
(ii) $x < y$ if and only if $x_i < y_i$ for all $i = 1, 2, ..., n$;
(iii) $x \leqq y$ if and only if $x_i \leqq y_i$ for all $i = 1, 2, ..., n$;
(iv) $x \leq y$ if and only if $x \leqq y$ and $x \neq y$.

Throughout the paper, we will use the same notation for row and column vectors when the interpretation is obvious.

Definition 1. *A function $\theta : R \to R$ is said to be strictly increasing if and only if*

$$\forall x, y \in R \quad x < y \quad \Rightarrow \quad \theta(x) < \theta(y).$$

Definition 2. [9] *Let $\Phi : X \to R$ be defined on a nonempty invex set $X \subset R^n$ and $u \in X$. If there exists a vector-valued function $\eta : X \times X \to R^n$ such that the inequality*

$$\Phi\left(u + \lambda\eta(x, u)\right) \leqq \lambda\Phi(x) + (1 - \lambda)\,\Phi(u) \quad (1)$$

holds for all $x \in X$ and any $\lambda \in [0, 1]$, then Φ is said to be a pre-invex function (with respect to η) at u on X. If inequality (1) holds for each $u \in X$, then Φ is said to be a pre-invex function (with respect to η) on X.

Definition 3. *We say that a mapping $f : X \to R^k$ defined on a nonempty set $X \subseteq R^n$ is directionally differentiable at $u \in X$ into a direction v if, for every $i = 1, ..., k$, the limit*

$$f_i^+(u; v) = \lim_{\lambda \to 0^+} \frac{f_i\left(u + \lambda v\right) - f_i\left(u\right)}{\lambda}$$

exists finite. We say that f is directionally differentiable or semi-differentiable at u, if its directional derivative $f_i^+(u; v)$ exists finite for all $v \in R^n$.

In the paper, we consider the following constrained multiobjective programming problem (VP):

$$\begin{aligned} V\text{-minimize } & f(x) := (f_1(x), ..., f_k(x)) \\ & g(x) \leqq 0, \\ & x \in X, \end{aligned} \quad \text{(VP)}$$

where $f_i : X \to R$, $i \in I = \{1, ..., k\}$, $g_j : X \to R$, $j \in J = \{1, ..., m\}$, are directionally differentiable functions on a nonempty set $X \subset R^n$.

Let $D = \{x \in X : g_j(x) \leqq 0,\ j \in J\}$ be the set of all feasible solutions for problem (VP). Further, we denote by $J(\overline{x}) := \{j \in J : g_j(\overline{x}) = 0\}$ the set of constraint indices active at $\overline{x} \in D$ and by $\widetilde{J}(x) = \{j \in \{1, ..., m\} : g_j(x) < 0\}$ the set of constraint indices inactive at $\overline{x} \in D$. Then $J(x) \cup \widetilde{J}(x) = \{1, ..., m\}$. Furthermore, let $g_{J(\overline{x})}$ denote the vector of active constraints at $\overline{x}$.

Before studying optimality in multiobjective programming, one has to define clearly the well-known concepts of optimality and solutions in multiobjective programming problem. The (weak) Pareto optimality in multiobjective programming associates the concept of a solution with some property that seems intuitively natural.

Definition 4. *A feasible point $\overline{x}$ is said to be a Pareto solution (an efficient solution) for the multiobjective programming problem (VP) if and only if there exists no $x \in D$ such that*

$$f(x) \leq f(\overline{x}).$$

Definition 5. *A feasible point $\overline{x}$ is said to be a weak Pareto solution (a weakly efficient solution, a weak minimum) for the multiobjective programming problem (VP) if and only if there exists no $x \in D$ such that*

$$f(x) < f(\overline{x}).$$

As it follows from the definition of (weak) Pareto optimality, $\overline{x}$ is nonimprovable with respect to the vector cost function f. The quality of nonimprovability provides a complete solution if $\overline{x}$ is unique. However, usually this is not the case, and then one has to find the entire exact set of all Pareto optimality solutions in a multiobjective programming problem.

Let $f : X \to R^k$ and $g : X \to R^m$ be vector-valued directionally differentiable functions defined on a nonempty open set $X \subset R^n$ at $u \in X$. Further, let $I_{f_i}(X)$, $i = 1, ..., k$, be the range of f_i, that is, the image of X under f_i and $I_{g_j}(X)$, $j = 1, ..., m$, be the range of g_j, that is, the image of X under g_j.

Definition 6. *If there exist a differentiable vector-valued function $G_f = (G_{f_1}, ..., G_{f_k}) : R \to R^k$ such that any its component $G_{f_i} : I_{f_i}(X) \to R$, $i = 1, ..., k$, is a strictly increasing function on its domain, a differentiable vector-valued function $G_g = (G_{g_1}, ..., G_{g_m}) : R \to R^m$ such that any its component $G_{g_j} : I_{g_j}(X) \to R$, $j = 1, ..., m$, is a strictly increasing function on its domain, functions $\alpha_i, \beta_j : X \times X \to R_+ \setminus \{0\}$, $i = 1, ..., k$, $j = 1, ..., m$, and a vector-valued function $\eta : X \times X \to R^n$ such that the following inequalities*

$$G_{f_i}(f_i(x)) - G_{f_i}(f_i(u)) \\ -\alpha_i(x, u)\, G'_{f_i}(f_i(u))\, f_i^+(u; \eta(x, u)) \geqq 0, \\ \imath = 1, ..., k, \tag{2}$$

$$-G_{g_j}(g_j(u)) \\ \geqq \beta_j(x, u)\, G'_{g_j}(g_j(u))\, g_j^+(u; \eta(x, u)), \\ j = 1, ..., m \tag{3}$$

hold for all $x \in X$, then (f, g) is said to be semi-G-V-type I objective and constraint functions at u on X with respect to η (and with respect to G_f and G_g). If (2) and (3) are satisfied for each $u \in X$, then (f, g) is said to be semi-G-V-type I objective and constraint functions on X with respect to η (and with respect to G_f and G_g).
If (2) and (3) are satisfied for all $x \in X$, but (2) is strict for all $x \in X$, $(x \neq u)$, then (f, g) is said to be semi-strictly-G-V-type I objective and constraint functions at u on X with respect to η (and with respect to G_f and G_g).

Remark 1. *In the case when $G_{f_i}(a) \equiv a$, $i \in I$, for any $a \in I_{f_i}(X)$, $G_{g_j}(a) \equiv a$, $j \in J$, for any $a \in I_{g_j}(X)$, we obtain the definition of directionally differentiable V-type I objective and constraint functions. In the case when the functions are differentiable, it reduces to the definition of V-type I objective and constraint functions (see Hanson et al.* [13] *for differentiable multiobjective programming problems).*

Now, we introduce the concepts of generalized semi-G-V-type I functions for the considered multiobjective programming problem (VP).

Definition 7. *If there exist a differentiable vector-valued function $G_f = (G_{f_1}, ..., G_{f_k}) : R \to R^k$ such that any its component $G_{f_i} : I_{f_i}(X) \to R$ is a strictly increasing function on its domain, a differentiable vector-valued function $G_g = (G_{g_1}, ..., G_{g_m}) : R \to R^m$ such that any its component $G_{g_j} : I_{g_j}(X) \to R$ is a strictly increasing function on its domain, functions $\alpha_i, \beta_j : X \times X \to R_+ \setminus \{0\}$, $i = 1, ..., k$, $j = 1, ..., m$, and a vector-valued function $\eta : X \times X \to R^n$ such that, for all $x \in X$, the following relations*

$$\sum_{i=1}^{k} G'_{f_i}(f_i(u))\, f_i^+(u; \eta(x, u)) \geqq 0 \Longrightarrow \\ \sum_{i=1}^{k} \alpha_i(x, u) \left[G_{f_i}(f_i(x)) - G_{f_i}(f_i(u))\right] \geqq 0 \tag{4}$$

$$\sum_{j=1}^{m} G'_{g_j}(g_j(u))\, g_j^+(u; \eta(x, u)) \geqq 0 \Longrightarrow \\ -\sum_{j=1}^{m} \beta_j(x, u)\, G_{g_j}(g_j(u)) \geqq 0 \tag{5}$$

hold, then (f,g) is said to be semi-pseudo-G-V-type I objective and constraint functions at u on X with respect to η (and with respect to G_f and G_g). If (4) and (5) are satisfied for each $u \in X$, then (f,g) is said to be semi-pseudo-G-V-type I on X with respect to η (and with respect to G_f and G_g).
If the second (implied) inequalities in (4) and (5) are strict for all $x \in X$, $x \neq u$, then (f,g) is said to be semi-strictly-pseudo-G-V-type I objective and constraint functions at u on X with respect to η (and with respect to G_f and G_g).

Now, we give an example of semi-pseudo-G-V-type I objective and constraint functions with respect to η not being semi-G-V-type I objective and constraint functions with respect to the same function η.

Example 1. *Let $f : R \to R^2$ and $g : R \to R$ be defined as follows: $f(x) = (f_1(x), f_2(x)) = \left(\arctan\left(e^{-x}|x|\right), e^{-x^2}\right)$ and $g(x) = g_1(x) = e^{x^2+2|x|} - 1$. We show that (f,g) is semi-pseudo-G-V-type I objective and constraint functions with respect to η at $u = 0$ on $X = R$. In order to do this, we set*

$$\eta(x,u) = x - u,$$

$$G_{f_1}(t) = \tan(t), \quad G_{f_2}(t) = \ln(t), \quad G_{g_1}(t) = \ln(t+1),$$

$$\alpha_1(x,u) = e^x\left(x^2+1\right), \quad \alpha_2(x,u) = \frac{1}{2(x^2+1)}, \quad \beta_1(x,u) = 1.$$

Then, by Definition 7, it follows that (f,g) is semi-pseudo-G-V-type I objective and constraint functions with respect to η (and with respect to G_f and G_g) at $u = 0$ on R. Further, by Definition 6, it follows that (f,g) is not semi-G-V-type I objective and constraint functions with respect to the same function η at $u = 0$ on R.

Definition 8. *If there exist a differentiable vector-valued function $G_f = (G_{f_1}, ..., G_{f_k}) : R \to R^k$ such that any its component $G_{f_i} : I_{f_i}(X) \to R$ is a strictly increasing function on its domain, a differentiable vector-valued function $G_g = (G_{g_1}, ..., G_{g_m}) : R \to R^m$ such that any its component $G_{g_j} : I_{g_j}(X) \to R$ is a strictly increasing function on its domain, functions $\alpha_i, \beta_j : X \times X \to R_+ \setminus \{0\}$, $i = 1, ..., k$, $j = 1, ..., m$, and a vector-valued function $\eta : X \times X \to R^n$ such that, for all $x \in X$, the following relations*

$$\sum_{i=1}^k \alpha_i(x,u)\left[G_{f_i}(f_i(x)) - G_{f_i}(f_i(u))\right] \leqq 0 \Longrightarrow \sum_{i=1}^k G'_{f_i}(f_i(u))\, f_i^+(u;\eta(x,u)) \leqq 0, \quad (6)$$

$$-\sum_{j=1}^m \beta_j(x,u)\, G_{g_j}(g_j(u)) \leqq 0 \Longrightarrow \sum_{j=1}^m G'_{g_j}(g_j(u))\, g_j^+(u;\eta(x,u)) \leqq 0 \quad (7)$$

hold, then (f,g) is said to be semi-quasi-G-V-type I objective and constraint functions at u on X with respect to η (and with respect to G_f and G_g). If (6) and (7) are satisfied for each $u \in X$, then (f,g) is said to be semi-quasi G-V-type I objective and constraint functions on X with respect to η (and with respect to G_f and G_g).

Example 2. *Let $f : R \to R^2$ and $g : R \to R$ be defined as follows: $f(x) = (f_1(x), f_2(x)) = \left(e^{-|x|}, e^{x^3}\right)$ and $g(x) = g_1(x) = e^{\frac{1}{2}(|x|+x)} - 1$. We prove that (f,g) is semi-quasi-G-V-type I objective and constraint functions with respect to η at $u = 0$ on $X = R$. In order to do this, we set*

$$\eta(x,u) = -|x-u|,$$

$$G_{f_1}(t) = \ln(t), \quad G_{f_2}(t) = \ln(t), \quad G_{g_1}(t) = \ln(t+1),$$

$$\alpha_1(x,u) = 1, \ \alpha_2(x,u) = \frac{1}{3(x^4+1)}, \ \beta_1(x,u) = 1.$$

Then, by Definition 8, (f,g) is semi-quasi-G-V-type I objective and constraint functions at $u = 0$ on R with respect to η given above (and with respect to G_f and G_g also given above).

Definition 9. *If there exist a differentiable vector-valued function $G_f = (G_{f_1}, ..., G_{f_k}) : R \to R^k$ such that any its component $G_{f_i} : I_{f_i}(X) \to R$ is a strictly increasing function on its domain, a differentiable vector-valued function $G_g = (G_{g_1}, ..., G_{g_m}) : R \to R^m$ such that any its component $G_{g_j} : I_{g_j}(X) \to R$ is a strictly increasing function on its domain, functions $\alpha_i, \beta_j : X \times X \to R_+ \setminus \{0\}$, $i = 1, ..., k$, $j = 1, ..., m$, and a vector-valued function $\eta : X \times X \to R^n$ such that, for all $x \in X$, the following relations*

$$\sum_{i=1}^k G'_{f_i}(f_i(u))\, f_i^+(u;\eta(x,u)) \geqq 0 \Longrightarrow \sum_{i=1}^k \alpha_i(x,u)\left[G_{f_i}(f_i(x)) - G_{f_i}(f_i(u))\right] \geqq 0, \quad (8)$$

$$-\sum_{j=1}^m \beta_j(x,u)\, G_{g_j}(g_j(u)) \leqq 0 \Longrightarrow \sum_{j=1}^m G'_{g_j}(g_j(u))\, g_j^+(u;\eta(x,u)) \leqq 0 \quad (9)$$

hold, then (f,g) is said to be semi-pseudo-quasi-G-V-type I objective and constraint functions at u on X with respect to η (and with respect to G_f and G_g). If (8) and (9) are satisfied for each

$u \in X$, then (f,g) is said to be semi-pseudo-quasi-G-V-type I objective and constraint functions on X with respect to η (and with respect to G_f and G_g).
If the second (implied) inequality in (8) is strict for all $x \in X$, $x \neq u$, then (f,g) is said to be semi-strictly-pseudo-quasi-G-V-type I at u on X with respect to η (and with respect to G_f and G_g).

Example 3. *Let $f : R \to R^2$ and $g : R \to R$ be defined as follows: $f(x) = (f_1(x), f_2(x)) = \left(\arctan\left(e^{-x}|x|\right), e^{-x^2}\right)$ and $g(x) = g_1(x) = e^{\frac{1}{2}(|x|+x)} - 1$. We prove that (f,g) is semi-pseudo-quasi-G-V-type I objective and constraint functions with respect to η at $u = 0$ on $X = R$. In order to do this, we set*

$$\eta(x,u) = -|x-u|,$$

$$G_{f_1}(t) = \tan(t),\ G_{f_2}(t) = \ln(t),$$
$$G_{g_1}(t) = \ln(t+1),$$

$$\alpha_1(x,u) = e^x\left(x^2+1\right),$$
$$\alpha_2(x,u) = \frac{1}{2\left(x^4+1\right)},$$
$$\beta_1(x,u) = 1.$$

Then, by Definition 9, (f,g) is semi-pseudo-quasi-G-V-type I objective and constraint functions at $u = 0$ on R with respect to η given above. Note that (f,g) is not semi-G-V-type I objective and constraint functions at $u = 0$ on R with respect to η given above (and with respect to G_f and G_g also given above).

Definition 10. *If there exist a differentiable vector-valued function $G_f = (G_{f_1},...,G_{f_k}) : R \to R^k$ such that any its component $G_{f_i} : I_{f_i}(X) \to R$ is a strictly increasing function on its domain, a differentiable vector-valued function $G_g = (G_{g_1},...,G_{g_m}) : R \to R^m$ such that any its component $G_{g_j} : I_{g_j}(X) \to R$ is a strictly increasing function on its domain, functions $\alpha_i, \beta_j : X \times X \to R_+ \setminus \{0\}$, $i = 1,...,k$, $j = 1,...,m$, and a vector-valued function $\eta : X \times X \to R^n$ such that, for all $x \in X$, the following relations*

$$\sum_{i=1}^k \alpha_i(x,u)\left[G_{f_i}(f_i(x)) - G_{f_i}(f_i(u))\right] \leqq 0 \Longrightarrow \sum_{i=1}^k G'_{f_i}(f_i(u))\, f_i^+(u;\eta(x,u)) \leqq 0, \quad (10)$$

$$\sum_{j=1}^m G'_{g_j}(g_j(u))\, g_j^+(u;\eta(x,u)) \geqq 0 \Longrightarrow -\sum_{j=1}^m \beta_j(x,u)\, G_{g_j}(g_j(u)) \geqq 0 \quad (11)$$

hold, then (f,g) is said to be semi-quasi-pseudo-G-V-type I objective and constraint functions at u on X with respect to η (and with respect to G_f and G_g). If (10) and (11) are satisfied for each $u \in X$, then (f,g) is said to be semi-quasi-pseudo-G-V-type I objective and constraint functions on X with respect to η (and with respect to G_f and G_g). If the second (implied) inequality in (11) is strict for all $x \in X$, $x \neq u$, then (f,g) is said to be semi-quasi-strictly-pseudo-G-V-type I at u on X with respect to η (and with respect to G_f and G_g).

Now, we give an example of semi-quasi-pseudo-G-V-type I objective and constraint functions with respect to η not being semi-G-V-type I objective and constraint functions with respect to the same function η.

Example 4. *Let $f : R \to R^2$ and $g : R \to R$ be defined as follows: $f(x) = (f_1(x), f_2(x)) = \left(e^{-|x|}, e^{x^3}\right)$ and $g(x) = g_1(x) = \arctan(e^{-|x|}x)$. We show that (f,g) is semi-quasi-pseudo-G-V-type I objective and constraint functions with respect to η at $u = 0$ on $X = R$. In order to do this, we set*

$$\eta(x,u) = |x-u|,$$

$$G_{f_1}(t) = \ln(t),\ G_{f_2}(t) = \ln(t),\ G_{g_1}(t) = \tan(t),$$

$$\alpha_1(x,u) = 1,\ \alpha_2(x,u) = \frac{1}{2\left(x^4+1\right)},\ \beta_1(x,u) = 1.$$

Then, by Definition 10, it follows that (f,g) is semi-quasi-pseudo-G-V-type I objective and constraint functions with respect to η (and with respect to G_f and G_g) at $u = 0$ on R. Further, by Definition 6, it follows that (f,g) is not semi-G-V-type I objective and constraint functions at $u = 0$ on R with respect to the same functions η, G_f and G_g.

3. Optimality conditions for directionally differentiable multiobjective programming

In this section, we prove necessary and sufficient optimality conditions for the considered directionally differentiable multiobjective programming problem (VP). Before we prove necessary optimality conditions of Fritz John type and Karush-Kuhn-Tucker type for problem (VP), we establish the following useful lemma:

Lemma 1. *If $\overline{x}$ is a weak Pareto solution for (VP) and g_j, $j \in \widetilde{J}(\overline{x})$, is continuous at $\overline{x}$, then the system*

$$G'_{f_i}(f_i(\overline{x}))\, f_i^+(\overline{x};\eta(x,\overline{x})) < 0,\ i = 1,...,k, \quad (12)$$

$$G'_{g_j}(g_j(\overline{x}))\, g_j^+(\overline{x};\eta(x,\overline{x})) < 0,\ j \in J(\overline{x}) \quad (13)$$

has no solution $x \in X$, where G_{f_i}, $i \in I$, is a differentiable real-valued strictly increasing function *defined on $I_{f_i}(D)$ and G_{g_j}, $j \in J$, is a differentiable real-valued strictly increasing function defined on $I_{g_j}(D)$, such that $G_{g_j}(0) = 0$, $j \in J(\overline{x})$.*

Proof. Let $\overline{x}$ be a weak Pareto solution in problem (VP) and suppose, contrary to the result, that there exists $\widetilde{x} \in X$ such that the inequalities (12)-(13) are fulfilled. Let $\varphi_{f_i}(\overline{x}, \widetilde{x}, \lambda) = G_{f_i}(f_i(\overline{x} + \lambda\eta(\widetilde{x}, \overline{x}))) - G_{f_i}(f_i(\overline{x}))$, $i = 1, ..., k$. We observe that this function vanishes at $\lambda = 0$. Therefore, the right differential of $\varphi_{f_i}(\overline{x}, \widetilde{x}, \lambda)$ with respect to λ at $\lambda = 0$ satisfies the following relations

$$\begin{aligned} &\lim_{\lambda \to 0^+} \frac{\varphi_{f_i}(\overline{x}, \widetilde{x}, \lambda) - \varphi_{f_i}(\overline{x}, \widetilde{x}, 0)}{\lambda} \\ = &\lim_{\lambda \to 0^+} \frac{G_{f_i}(f_i(\overline{x} + \lambda\eta(\widetilde{x}, \overline{x}))) - G_{f_i}(f_i(\overline{x}))}{\lambda} \\ = &\ G'_{f_i}(f_i(\overline{x}))\, f_i^+(\overline{x}; \eta(x, \overline{x})) < 0, \quad i = 1, ..., k, \end{aligned}$$

where the last inequality follows from (12). Therefore $\varphi_{f_i}(\overline{x}, \widetilde{x}, \lambda) < 0$ if λ is in some open interval $(0, \delta_{f_i})$. Hence, it follows that

$$G_{f_i}(f_i(\overline{x} + \lambda\eta(\widetilde{x}, \overline{x}))) - G_{f_i}(f_i(\overline{x})) < 0, \quad \lambda \in (0, \delta_{f_i}), \ i = 1, ..., k.$$

Since $G_{f_i} : I_{f_i}(X) \to R$, $i = 1, ..., k$, is a strictly increasing function on its domain, the above inequality yields

$$f_i(\overline{x} + \lambda\eta(\widetilde{x}, \overline{x})) < f_i(\overline{x}), \quad \lambda \in (0, \delta_{f_i}), \ i = 1, ..., k.$$

Similarly, we define $\varphi_{g_j}(\overline{x}, \widetilde{x}, \lambda) = G_{g_j}(g_j(\overline{x} + \lambda\eta(\widetilde{x}, \overline{x}))) - G_{g_j}(g_j(\overline{x}))$, $j \in J(\overline{x})$. Hence, by (13), it follows

$$G_{g_j}(g_j(\overline{x} + \lambda\eta(\widetilde{x}, \overline{x}))) < G_{g_j}(g_j(\overline{x})), \quad \lambda \in (0, \delta_{g_j}), \ j \in J(\overline{x}). \tag{14}$$

Since each $G_{g_j} : I_{g_j}(X) \to R$, $j \in J$, is a strictly increasing function on its domain, (14) yields

$$g_j(\overline{x} + \lambda\eta(\widetilde{x}, \overline{x})) < g_j(\overline{x}), \quad \lambda \in (0, \delta_{g_j}), \ j \in J(\overline{x}).$$

By definition of $J(\overline{x})$, it follows that

$$g_j(\overline{x} + \lambda\eta(\widetilde{x}, \overline{x})) < 0, \quad \lambda \in (0, \delta_{g_j}), \ j \in J(\overline{x}).$$

Since g_j, $j \in \widetilde{J}(\overline{x})$, is continuous at $\overline{x}$, therefore, there exists δ_j such that

$$g_j(\overline{x} + \lambda\eta(\widetilde{x}, \overline{x})) < 0, \quad \lambda \in (0, \delta_j), \ j \in \widetilde{J}(\overline{x}).$$

Let $\overline{\delta} = min\left\{\delta_{f_i}, \ i = 1, ..., k, \ \delta_{g_j}, \ j \in J(\overline{x}), \ \delta_j, \ j \in \widetilde{J}(\overline{x})\right\}$, then

$$(\overline{x} + \lambda\eta(\widetilde{x}, \overline{x})) \in N_{\overline{\delta}}(\overline{x}), \qquad \lambda \in (0, \overline{\delta}), \tag{15}$$

where $N_{\overline{\delta}}(\overline{x})$ is a neighbourhood of $\overline{x}$.
Hence, we have that

$$f(\overline{x} + \lambda\eta(\widetilde{x}, \overline{x})) < f(\overline{x}), \tag{16}$$

$$g_j(\overline{x} + \lambda\eta(\widetilde{x}, \overline{x})) < 0, \ j \in J(\overline{x}), \tag{17}$$

$$g_j(\overline{x} + \lambda\eta(\widetilde{x}, \overline{x})) < 0, \ j \in \widetilde{J}(\overline{x}). \tag{18}$$

By (17) and, (18), it follows that

$$(\overline{x} + \lambda\eta(\widetilde{x}, \overline{x})) \in N_{\overline{\delta}}(\overline{x}) \cap D, \qquad \lambda \in (0, \overline{\delta}),$$

and this means that $\overline{x} + \lambda\eta(\widetilde{x}, \overline{x})$ is a feasible solution in problem (VP). Hence, (16) is a contradiction to the assumption that $\overline{x}$ is a weak Pareto solution in problem (VP). Thus, there exists no $x \in X$ satisfying the system (12)-(13). The proof of this lemma is completed. □

In order to prove the next result, we need the following lemma:

Lemma 2. [26] *Let S be a nonempty set in R^n and $\psi : S \to R^m$ be a pre-invex function on S. Then either*

$$\psi(x) < 0 \text{ has a solution } x \in S$$

or

$$\lambda^T \psi(x) \geqq 0 \text{ for all } x \in S, \text{ for some } \lambda \in R^m_+ \backslash \{0\},$$

but both alternatives are never true.

Now, we give the necessary optimality criteria of Fritz John type for $\overline{x} \in D$ to be a weak Pareto optimal solution in the considered directionally differentiable multiobjective programming problem in which the right differentials of f and $g_{J(\overline{x})}$ at $\overline{x}$ are pre-invex functions.

Theorem 1. *(G-Fritz John Type Necessary Optimality Conditions). Let $\overline{x} \in D$ be a weak Pareto optimal solution for problem (VP). Further, assume that g_j, $j \in \widetilde{J}(\overline{x})$, is continuous, f and g are directionally differentiable at $\overline{x}$ with $f'(\overline{x}, \eta(x, \overline{x}))$, $g'_{J(\overline{x})}(\overline{x}, \eta(x, \overline{x}))$ are pre-invex functions of x on D. Then, there exist $\overline{\lambda} \in R^k$, $\overline{\xi} \in R^m$ such that the following conditions*

$$\begin{gathered} \textstyle\sum_{i=1}^k \overline{\lambda}_i G'_{f_i}(f_i(\overline{x}))\, f_i^+(\overline{x}; \eta(x, \overline{x})) \\ + \textstyle\sum_{j=1}^m \overline{\xi}_j G'_{g_j}(g_j(\overline{x}))\, g_j^+(\overline{x}; \eta(x, \overline{x})) \geqq 0, \\ \forall\, x \in D, \end{gathered} \tag{19}$$

$$\overline{\xi}_j G_{g_j}(g_j(\overline{x})) = 0, \quad j \in J, \tag{20}$$

$$(\overline{\lambda}, \overline{\xi}) \geq 0 \tag{21}$$

hold, where G_{f_i}, $i \in I$, is a differentiable real-valued strictly increasing function defined on $I_{f_i}(D)$ and G_{g_j}, $j \in J$, is a differentiable real-valued strictly increasing function defined on $I_{g_j}(D)$, such that $G_{g_j}(0) = 0$, $j \in J(\overline{x})$.

Proof. If $\overline{x}$ is a weak Pareto solution for problem (VP) then, by Lemma 1, the system

$$G'_{f_i}\left(f_i\left(\overline{x}\right)\right) f_i^+\left(\overline{x};\eta(x,\overline{x})\right) < 0,\ i = 1, ..., k,$$

$$G'_{g_j}\left(g_j\left(\overline{x}\right)\right) g_j^+\left(\overline{x};\eta(x,\overline{x})\right) < 0,\ j \in J(\overline{x})$$

has no solution $x \in D$. Since the right differential of f, $g_{J(\overline{x})}$ are pre-invex functions on D, therefore, by Lemma 2, there exist $\overline{\lambda} \in R^k$, $\overline{\lambda} \geqq 0$, $\overline{\mu}_j \geqq 0$ for $j \in J(\overline{x})$, $\left(\overline{\lambda}, \overline{\mu}\right) \neq 0$, such that the inequality

$$\sum_{i=1}^k \overline{\lambda}_i G'_{f_i}\left(f_i\left(\overline{x}\right)\right) f_i^+\left(\overline{x};\eta(x,\overline{x})\right) + \sum_{j\in J(\overline{x})} \overline{\mu}_j G'_{g_j}\left(g_j\left(\overline{x}\right)\right) g_j^+\left(\overline{x};\eta(x,\overline{x})\right) \geqq 0 \quad (22)$$

hold for all $x \in D$. We put $\overline{\xi}_j = \overline{\mu}_j$ for $j \in J(\overline{x})$ and $\overline{\xi}_j = 0$ for $j \in \widetilde{J}(\overline{x})$. Hence, (22) implies that (19) is satisfied. Using the feasibility of $\overline{x}$ together with the assumption $G_{g_j}(0) = 0$, $j \in J(\overline{x})$, we obtain that the relation (20) is satisfied. This completes the proof of this theorem. □

In order to prove the next result, we need the following G-constraint qualification (G-CQ).

Definition 11. *It is said that the directionally differentiable multiobjective programming problem (VP) satisfies the G-constraint qualification (G-CQ) at $\overline{x} \in D$ if there exists $\widetilde{x} \in D$ such that $G'_{g_j}\left(g_j\left(\overline{x}\right)\right) g_j^+\left(\overline{x};\eta(\widetilde{x},\overline{x})\right) < 0$, $j \in J(\overline{x})$.*

Now, the following Karush-Kuhn-Tucker type necessary optimality conditions for the considered vector optimization problem (VP) are satisfied:

Theorem 2. *(G-Karush-Kuhn-Tucker Type Necessary Optimality Conditions). Let $\overline{x}$ be a weak Pareto solution for the directionally differentiable multiobjective programming problem (VP) and g_j be continuous for $j \in \widetilde{J}(\overline{x})$. Further, assume that f, g are directionally differentiable at $\overline{x}$ with $f'\left(\overline{x}, \eta(x,\overline{x})\right)$, $g'_{J(\overline{x})}\left(\overline{x}, \eta(x,\overline{x})\right)$ being pre-invex functions of x on D, G_{f_i}, $i \in I$, is a differentiable real-valued strictly increasing function defined on $I_{f_i}(D)$ and G_{g_j}, $j \in J$, is a differentiable real-valued strictly increasing function defined on $I_{g_j}(D)$, such that $G_{g_j}(0) = 0$, $j \in J(\overline{x})$. If the G-constraint qualification (G-CQ) is satisfied at $\overline{x}$ for problem (VP) (with G_g), then there exist $\overline{\lambda} \in R^k$, $\overline{\xi} \in R^m$ such that the following conditions*

$$\sum_{i=1}^k \overline{\lambda}_i G'_{f_i}\left(f_i\left(\overline{x}\right)\right) f_i^+\left(\overline{x};\eta(x,\overline{x})\right) + \sum_{j=1}^m \overline{\xi}_j G'_{g_j}\left(g_j\left(\overline{x}\right)\right) g_j^+\left(\overline{x};\eta(x,\overline{x})\right) \geqq 0, \quad \forall\, x \in D, \quad (23)$$

$$\overline{\xi}_j G_{g_j}\left(g_j\left(\overline{x}\right)\right) = 0, \quad j \in J, \quad (24)$$

$$\overline{\lambda} \geq 0, \ \ \overline{\xi} \geqq 0 \quad (25)$$

hold.

Proof. Let the constraint qualification (G-CQ) be satisfied at $\overline{x}$ for the considered directionally differentiable multiobjective programming problem (VP). Suppose, contrary to the result, that $\overline{\lambda} = 0$. Hence, by the G-Fritz John type necessary optimality condition (19) (Theorem 1), it follows that

$$\sum_{j\in J(\overline{x})} \overline{\xi}_j G'_{g_j}\left(g_j\left(\overline{x}\right)\right) g_j^+\left(\overline{x};\eta(x,\overline{x})\right) \geqq 0,\ \forall\, x \in D. \quad (26)$$

By assumption, the constraint qualification (G-CQ) is satisfied at $\overline{x}$ for problem (VP). Hence, there exist $\widetilde{x} \in D$ such that $G'_{g_j}\left(g_j\left(\overline{x}\right)\right) g_j^+\left(\overline{x}, \eta(\widetilde{x},\overline{x})\right) < 0$, $j \in J(\overline{x})$. Thus, the following inequality

$$\sum_{j\in J(\overline{x})} \overline{\xi}_j G'_{g_j}\left(g_j\left(\overline{x}\right)\right) g_j^+\left(\overline{x};\eta(\widetilde{x},\overline{x})\right) < 0 \quad (27)$$

holds. Then $\widetilde{x} \in D$ implies that the inequality (27) contradicts (26). Hence, $\overline{\lambda} \neq 0$ and, therefore, the proof of this theorem is completed. □

We now prove the sufficiency of the G-Karush-Kuhn-Tucker type necessary optimality conditions.

Theorem 3. *(Sufficient optimality conditions). Let $\overline{x} \in D$, $G_f = (G_{f_1}, ...G_{f_k})$ be a differentiable vector-valued function such that each its G_{f_i}, $i \in I$, is a strictly increasing function defined on $I_{f_i}(D)$, $G_g = (G_{g_1}, ...G_{g_m})$ be a differentiable vector-valued function such that each its component G_{g_j}, $j \in J$, is a strictly increasing function defined on $I_{g_j}(D)$ with $G_{g_j}(0) = 0$, $j \in J$. Assume that there exist vectors $\overline{\lambda} \in R^k$ and $\overline{\xi} \in R^m$ such that the G-Karush-Kuhn-Tucker type necessary optimality conditions (23)-(25) are satisfied at $\overline{x}$ with functions G_f and G_g. Furthermore, assume that (f, g) is semi-G-V-type I objective and constraint functions at $\overline{x}$ on D with respect to η and with respect to G_f and G_g. If the Lagrange multiplier $\overline{\lambda}$ is assumed to satisfy $\overline{\lambda} > 0$, then $\overline{x}$ is an efficient solution in problem (VP).*

Proof. Let $\overline{x} \in D$. Further, assume that there exist a differentiable vector-valued function $G_f = (G_{f_1}, ...G_{f_k})$ such that each its G_{f_i}, $i \in I$, is a strictly increasing function defined on $I_{f_i}(D)$, and a differentiable vector-valued function $G_g = (G_{g_1}, ...G_{g_m})$ such that each its component G_{g_j}, $j \in J$, is a strictly increasing function defined on $I_{g_j}(D)$ with $G_{g_j}(0) = 0$, $j \in J$. Furthermore, assume that there exist vectors $\overline{\lambda} \in R^k$ and $\overline{\xi} \in R^m$

such that the G-Karush-Kuhn-Tucker type necessary optimality conditions (23)-(25) are fulfilled at $\overline{x}$ with functions G_f and G_g.

We proceed by contradiction. Suppose, contrary to the result, that $\overline{x}$ is not an efficient solution in problem (VP). Thus, by Definition 4, it follows that there exists $\widetilde{x} \in D$ such that

$$f(\widetilde{x}) \leq f(\overline{x}). \tag{28}$$

By assumption, (f, g) is vector semi-G-V-type I at $\overline{x}$ on D (with respect to η and with respect to G_f and G_g). Then, by Definition 6, the following inequalities

$$\begin{gathered} G_{f_i}\left(f_i(x)\right) - G_{f_i}\left(f_i(\overline{x})\right) \\ -\alpha_i\left(x, \overline{x}\right) G'_{f_i}\left(f_i(\overline{x})\right) f_i^+\left(\overline{x}; \eta(x, \overline{x})\right) \geqq 0, \\ i = 1, ..., k, \\ -G_{g_j}\left(g_j(\overline{x})\right) \geqq \beta_j\left(x, \overline{x}\right) G'_{g_j}\left(g_j(\overline{x})\right) g_j^+\left(\overline{x}; \eta(x, \overline{x})\right), \\ j = 1, ..., m \end{gathered}$$

are satisfied for all $x \in D$. Since the inequalities above are fulfilled for all $x \in D$, therefore, they are also satisfied for $x = \widetilde{x}$. Thus,

$$\begin{aligned} & G_{f_i}\left(f_i(\widetilde{x})\right) - G_{f_i}\left(f_i(\overline{x})\right) \\ & -\alpha_i\left(\widetilde{x}, \overline{x}\right) G'_{f_i}\left(f_i(\overline{x})\right) f_i^+\left(\overline{x}; \eta(\widetilde{x}, \overline{x})\right) \geqq 0, \\ & i = 1, ..., k, \end{aligned} \tag{29}$$

$$\begin{aligned} & -G_{g_j}\left(g_j(\overline{x})\right) \geqq \beta_j\left(\widetilde{x}, \overline{x}\right) G'_{g_j}\left(g_j(\overline{x})\right) g_j^+\left(\overline{x}; \eta(\widetilde{x}, \overline{x})\right), \\ & j = 1, ..., m. \end{aligned} \tag{30}$$

By Definition 6, the functions $G_{f_i} : I_{f_i}(D) \to R$, $i \in I$, are strictly increasing on their domains. Hence, (28) yields

$$G_{f_i}\left(f_i\left(\widetilde{x}\right)\right) \leqq G_{f_i}\left(f_i\left(\overline{x}\right)\right), \; i = 1, ..., k, \tag{31}$$

but for at least $i^* \in I$,

$$G_{f_{i^*}}\left(f_{i^*}\left(\widetilde{x}\right)\right) < G_{f_{i^*}}\left(f_{i^*}\left(\overline{x}\right)\right). \tag{32}$$

Combining (29), (31) and (32), we get

$$\begin{gathered} \alpha_i\left(\widetilde{x}, \overline{x}\right) G'_{f_i}\left(f_i(\overline{x})\right) f_i^+\left(\overline{x}; \eta(\widetilde{x}, \overline{x})\right) \leqq 0, \\ i = 1, ..., k, \end{gathered} \tag{33}$$

but for at least $i^* \in I$,

$$\alpha_{i^*}\left(\widetilde{x}, \overline{x}\right) G'_{f_{i^*}}\left(f_{i^*}(\overline{x})\right) f_{i^*}^+\left(\overline{x}; \eta(\widetilde{x}, \overline{x})\right) < 0. \tag{34}$$

By Definition 6, it follows that $\alpha_i\left(\widetilde{x}, \overline{x}\right) > 0$, $i = 1, ..., k$. Therefore, (33) and (34) imply, respectively,

$$G'_{f_i}\left(f_i(\overline{x})\right) f_i^+\left(\overline{x}; \eta(\widetilde{x}, \overline{x})\right) \leqq 0, \; i = 1, ..., k, \tag{35}$$

$$G'_{f_{i^*}}\left(f_{i^*}(\overline{x})\right) f_{i^*}^+\left(\overline{x}; \eta(\widetilde{x}, \overline{x})\right) < 0 \text{ for some } i^* \in I. \tag{36}$$

Since the vector of the Lagrange multipliers $\overline{\lambda}$ associated to the objective function is assumed to satisfy $\overline{\lambda} > 0$, (35) and (36) yield, respectively,

$$\overline{\lambda}_i G'_{f_i}\left(f_i(\overline{x})\right) f_i^+\left(\overline{x}; \eta(\widetilde{x}, \overline{x})\right) \leqq 0, \; i = 1, ..., k,$$

$$\overline{\lambda}_i G'_{f_{i^*}}\left(f_{i^*}(\overline{x})\right) f_{i^*}^+\left(\overline{x}; \eta(\widetilde{x}, \overline{x})\right) < 0 \text{ for some } i^* \in I.$$

Adding both sides of the above inequalities, we get

$$\sum_{i=1}^{k} \overline{\lambda}_i G'_{f_i}\left(f_i(\overline{x})\right) f_i^+\left(\overline{x}; \eta(\widetilde{x}, \overline{x})\right) < 0. \tag{37}$$

Using the G-Karush-Kuhn-Tucker type necessary optimality condition (25) together with (30), we obtain

$$\begin{aligned} & -\overline{\xi}_j G_{g_j}\left(g_j(\overline{x})\right) \\ & \geqq \overline{\xi}_j \beta_j\left(\widetilde{x}, \overline{x}\right) G'_{g_j}\left(g_j(\overline{x})\right) g_j^+\left(\overline{x}; \eta(\widetilde{x}, \overline{x})\right), \\ & j = 1, ..., m. \end{aligned} \tag{38}$$

By the G-Karush-Kuhn-Tucker type necessary optimality condition (24), it follows that

$$\begin{aligned} & \overline{\xi}_j \beta_j\left(\widetilde{x}, \overline{x}\right) G'_{g_j}\left(g_j(\overline{x})\right) g_j^+\left(\overline{x}; \eta(\widetilde{x}, \overline{x})\right) \leqq 0, \\ & j = 1, ..., m. \end{aligned} \tag{39}$$

By Definition 6, it follows that $\beta_j\left(\widetilde{x}, \overline{x}\right) > 0$, $j = 1, ..., m$. Hence, (39) yields

$$\overline{\xi}_j G'_{g_j}\left(g_j(\overline{x})\right) g_j^+\left(\overline{x}; \eta(\widetilde{x}, \overline{x})\right) \leqq 0, \; j = 1, ..., m.$$

Adding both sides of the above inequalities, we get

$$\sum_{j=1}^{m} \overline{\xi}_j G'_{g_j}\left(g_j(\overline{x})\right) g_j^+\left(\overline{x}; \eta(\widetilde{x}, \overline{x})\right) \leqq 0. \tag{40}$$

Then, adding both sides of (37) and (40), we obtain that the following inequality

$$\begin{aligned} & \sum_{i=1}^{k} \overline{\lambda}_i G'_{f_i}\left(f_i(\overline{x})\right) f_i^+\left(\overline{x}; \eta(\widetilde{x}, \overline{x})\right) \\ + & \sum_{j=1}^{m} \overline{\xi}_j G'_{g_j}\left(g_j(\overline{x})\right) g_j^+\left(\overline{x}; \eta(\widetilde{x}, \overline{x})\right) < 0 \end{aligned}$$

holds, which contradicts the G-Karush-Kuhn-Tucker type necessary optimality condition (23). This means that $\overline{x}$ is an efficient solution in problem (VP) and completes the proof of this theorem. □

Remark 2. *In order to prove the analogous result for $\overline{x} \in D$ to be a weak efficient solution in problem (VP), it is sufficient that the vector of the Lagrange multipliers $\overline{\lambda}$ associated to the objective function is assumed to satisfy $\overline{\lambda} \geq 0$.*

Now, we illustrate the sufficient optimality conditions established in Theorem 3 by an example of a nonconvex directionally differentiable vector optimization problem.

Example 5. *Consider the following directionally differentiable vector optimization problem*

$$\begin{array}{c} V\text{-minimize } f(x) = \left(\ln(|x|+1), \arctan\left(e^{-x}|x|\right)\right) \\ g_1(x) = \arctan\left(x^2 - |x|\right) \leqq 0. \end{array} \quad (VP1)$$

Note that $D = \{x \in R : -1 \leqq x \leqq 1\}$ and $\overline{x} = 0$ is a feasible solution in the considered directionally differentiable vector optimization problem (VP1). It can be proved, by Definition 6, that the functions constituting problem (VP1) are semi-G-V-type I objective and constraint functions at $\overline{x}$ on D with respect to the same function η, where

$$\eta(x,u) = x - u,$$

$$G_{f_1}(t) = e^t,\ G_{f_2}(t) = \tan(t),\ G_{g_1}(t) = \tan(t),$$

$$\alpha_1(x,u) = 1,\ \alpha_2(x,u) = e^{u-x},\ \beta_1(x,u) = 1.$$

Thus, the G-Karush-Kuhn-Tucker type necessary optimality conditions (23)-(25) are satisfied at $\overline{x}$ with the functions G_f and G_g defined above and with the Lagrange multiplier $\overline{\lambda} > 0$. Since all hypotheses of Theorem 3 are fulfilled, by Theorem 3, $\overline{x} = 0$ is an efficient solution in the considered directionally differentiable vector optimization problem (VP1).

Before we prove the sufficient optimality conditions for $\overline{x} \in D$ to be (weakly) efficient in the considered directionally differentiable vector optimization problem (VP) under assumptions that the functions constituting the considered vector optimization problem are generalized semi-G-type I, we introduce some useful denotations. Let $\overline{x}$ be such a feasible solution in problem (VP) at which the G-Karush-Kuhn-Tucker type necessary optimality conditions (23)-(25) are fulfilled with the Lagrange multipliers $\overline{\lambda} \in R^k$ and $\overline{\xi} \in R^m$. Further, let us denote $I(\overline{x}) = \left\{i \in I : \overline{\lambda}_i > 0\right\}$. Then, $\left(f_{I(\overline{x})}, g_{J(\overline{x})}\right)$ denotes vectors of objective function components f_i, $i \in I(\overline{x})$ and constraint function components g_j, $j \in J(\overline{x})$, respectively. In other words, $\left(f_{I(\overline{x})}, g_{J(\overline{x})}\right)$ denotes vectors of such objective function components and such constraint function components for which the associated Lagrange multiplier is positive.

Theorem 4. *(Sufficient optimality conditions). Let $\overline{x} \in D$, $G_f = (G_{f_1}, ..., G_{f_k})$ be a differentiable vector-valued function such that each its G_{f_i}, $i \in I$, is a strictly increasing function defined on $I_{f_i}(D)$, $G_g = (G_{g_1}, ..., G_{g_m})$ be a differentiable vector-valued function such that each its component G_{g_j}, $j \in J$, is a strictly increasing function defined on $I_{g_j}(D)$ with $G_{g_j}(0) = 0$, $j \in J$. Assume that there exist vectors $\overline{\lambda} \in R^k$ and $\overline{\xi} \in R^m$ such that the G-Karush-Kuhn-Tucker type necessary optimality conditions (23)-(25) are satisfied at $\overline{x}$ with functions G_f and G_g and with the Lagrange multipliers $\overline{\lambda} \in R^k$ and $\overline{\xi} \in R^m$. Further, assume that one of the following conditions is satisfied:*

a) *$\left(f_{I(\overline{x})}, g_{J(\overline{x})}\right)$ is semi-strictly-pseudo $\widetilde{G}$-V-type I objective and constraint functions at $\overline{x}$ on D with respect to η, $\widetilde{G}_f = \left(\widetilde{G}_{f_1}, ..., \widetilde{G}_{f_k}\right)$ and $\widetilde{G}_g = \left(\widetilde{G}_{g_1}, ..., \widetilde{G}_{g_m}\right)$, where $\widetilde{G}_{f_i} = \overline{\lambda}_i G_{f_i}$, $i \in I(\overline{x})$, $\widetilde{G}_{g_j} = \overline{\xi}_j G_{g_j}$, $j \in J(\overline{x})$,*
b) *$\left(f_{I(\overline{x})}, g_{J(\overline{x})}\right)$ is semi-strictly-pseudo-quasi $\widetilde{G}$-V-type I objective and constraint functions at $\overline{x}$ on D with respect to η, $\widetilde{G}_f = \left(\widetilde{G}_{f_1}, ..., \widetilde{G}_{f_k}\right)$ and $\widetilde{G}_g = \left(\widetilde{G}_{g_1}, ..., \widetilde{G}_{g_m}\right)$, where $\widetilde{G}_{f_i} = \overline{\lambda}_i G_{f_i}$, $i \in I(\overline{x})$, $\widetilde{G}_{g_j} = \overline{\xi}_j G_{g_j}$, $j \in J(\overline{x})$,*
c) *$\left(f_{I(\overline{x})}, g_{J(\overline{x})}\right)$ is semi-quasi-strictly-pseudo $\widetilde{G}$-V-type I at u on X with respect to η, $\widetilde{G}_f = \left(\widetilde{G}_{f_1}, ..., \widetilde{G}_{f_k}\right)$ and $\widetilde{G}_g = \left(\widetilde{G}_{g_1}, ..., \widetilde{G}_{g_m}\right)$, where $\widetilde{G}_{f_i} = \overline{\lambda}_i G_{f_i}$, $i \in I(\overline{x})$, $\widetilde{G}_{g_j} = \overline{\xi}_j G_{g_j}$, $j \in J(\overline{x})$.*

Then $\overline{x}$ is an efficient solution in problem (VP).

Proof. We now prove the theorem under hypothesis a). Suppose, contrary to the result, that $\overline{x}$ is not an efficient solution of problem (VP). Then, there exists $\widetilde{x} \in D$ such that

$$f_i(\widetilde{x}) \leqq f_i(\overline{x}),\ i \in I, \quad (41)$$

$$f_{i^*}(\widetilde{x}) < f_{i^*}(\overline{x}),\ \text{for at least one } i^* \in I. \quad (42)$$

By assumption, (f, g) is semi-strictly-pseudo-$\widetilde{G}$-V-type I objective and constraint functions at $\overline{x}$ on D with respect to η, $\widetilde{G}_f$ and $\widetilde{G}_g$. Note that, if G_{f_i}, $i \in I(\overline{x})$, and G_{g_j}, $j \in J(\overline{x})$, are strictly increasing functions on their domains, then $\widetilde{G}_{f_i} = \overline{\lambda}_i G_{f_i}$, $i \in I(\overline{x})$, and $\widetilde{G}_{g_j} = \overline{\xi}_j G_{g_j}$, $j \in J(\overline{x})$, are also strictly increasing functions on these sets. Further, by the definition of semi-strictly-pseudo-$\widetilde{G}$-V-type I objective and constraint functions (see Definition 7), it follows that there exist functions $\alpha_i, \beta_j : X \times X \to R_+ \setminus \{0\}$, $i = 1, ..., k$, $j = 1, ..., m$. We multiply each inequality (41) and (42) by the associated $\alpha_i(\widetilde{x}, \overline{x}) > 0$, $i \in I$. Adding both sides of (41) and (42), and then adding the obtained inequalities, we get

$$\sum_{i \in I(\overline{x})} \alpha_i(\widetilde{x}, \overline{x}) \left[\overline{\lambda}_i G_{f_i}(f_i(\widetilde{x})) - \overline{\lambda}_i G_{f_i}(f_i(\overline{x}))\right] \leqq 0. \quad (43)$$

By the G-Karush-Kuhn-Tucker necessary optimality condition (24), it follows that

$$\beta_j(\widetilde{x}, \overline{x})\, \overline{\xi}_j G_{g_j}(g_j(\overline{x})) = 0,\ \ j \in J. \quad (44)$$

Adding both sides of the inequalities (44), we get

$$\sum_{j\in J(\overline{x})} \beta_j\left(\widetilde{x},\overline{x}\right)\overline{\xi}_j G_{g_j}\left(g_j(\overline{x})\right)=0. \quad (45)$$

Since (f,g) is semi-strictly-pseudo-$\widetilde{G}$-V-type I objective and constraint functions at $\overline{x}$ on D (with respect to η), (43) and (45) imply, respectively,

$$\sum_{i\in I(\overline{x})} \overline{\lambda}_i G'_{f_i}\left(f_i(\overline{x})\right) f_i^+\left(\overline{x};\eta(\widetilde{x},\overline{x})\right)<0, \quad (46)$$

$$\sum_{j\in J(\overline{x})} \overline{\xi}_j G'_{g_j}\left(g_j(\overline{x})\right) g_j^+\left(\overline{x};\eta(\widetilde{x},\overline{x})\right)<0. \quad (47)$$

Taking into account the Lagrange multipliers equal to 0 and then adding both sides of the inequalities above, we get that the following inequality

$$\sum_{i=1}^k \overline{\lambda}_i G'_{f_i}\left(f_i(\overline{x})\right) f_i^+\left(\overline{x};\eta(\widetilde{x},\overline{x})\right) + \sum_{j=1}^m \overline{\xi}_j G'_{g_j}\left(g_j(\overline{x})\right) g_j^+\left(\overline{x};\eta(\widetilde{x},\overline{x})\right)<0 \quad (48)$$

holds, contradicting the G-Karush-Kuhn-Tucker necessary optimality condition (23). Hence, the proof of this theorem under hypothesis a) is completed.

The proofs of this theorem under hypotheses b) and c) are similar and, therefore, they are omitted in the paper. □

In order to prove that $\overline{x}$ is a weakly efficient solution in problem (VP), the hypotheses of Theorem 4 can be weakened.

Theorem 5. *(Sufficiency). Let $\overline{x}\in D$. Assume that there exist a differentiable real-valued strictly increasing function G_{f_i}, $i\in I$, defined on $I_{f_i}(D)$, a differentiable real-valued strictly increasing function G_{g_j}, $j\in J$, defined on $I_{g_j}(D)$ with $G_{g_j}(0)=0$, $j\in J$, and vectors $\overline{\lambda}\in R^k$ and $\overline{\xi}\in R^m$ such that the G-Karush-Kuhn-Tucker type necessary optimality conditions (23)-(25) are satisfied at $\overline{x}$.*

Further, assume that one of the following conditions is satisfied:

a) $\left(f_{I(\overline{x})},g_{J(\overline{x})}\right)$ *is semi-pseudo-$\widetilde{G}$-V-type I objective and constraint functions at $\overline{x}$ on D (with respect to η), where $\widetilde{G}_{f_i}=\overline{\lambda}_i G_{f_i}$, $i\in I(\overline{x})$, $\widetilde{G}_{g_j}=\overline{\xi}_j G_{g_j}$, $j\in J(\overline{x})$,*

b) $\left(f_{I(\overline{x})},g_{J(\overline{x})}\right)$ *is semi-pseudo-quasi-$\widetilde{G}$-V-type I objective and constraint functions at $\overline{x}$ on D (with respect to η), where $\widetilde{G}_{f_i}=\overline{\lambda}_i G_{f_i}$, $i\in I(\overline{x})$, $\widetilde{G}_{g_j}=\overline{\xi}_j G_{g_j}$, $j\in J(\overline{x})$,*

c) $\left(f_{I(\overline{x})},g_{J(\overline{x})}\right)$ *is semi-quasi-pseudo-$\widetilde{G}$-V-type I at u on X (with respect to η), where $\widetilde{G}_{f_i}=\overline{\lambda}_i G_{f_i}$, $i\in I(\overline{x})$, $\widetilde{G}_{g_j}=\overline{\xi}_j G_{g_j}$, $j\in J(\overline{x})$.*

Then $\overline{x}$ is a weakly efficient solution in problem (VP).

Proof. Proof of this theorem is similar to that one for Theorem 4 and, therefore, it is omitted in the paper. □

4. G-Mond-Weir type duality

In this section, for the considered directionally differentiable multiobjective programming problem (VP), we define the following vector dual problem in the sense of Mond-Weir:

$$\begin{aligned}
& f(y)=(f_1(y),f_2(y),...,f_k(y))\to \max \\
& \sum_{i=1}^k \lambda_i G'_{f_i}\left(f_i(y)\right) f_i^+\left(x;\eta(x,y)\right) \\
& +\sum_{j=1}^m \xi_j G'_{g_j}\left(g_j(y)\right) g_j^+\left(x;\eta(x,y)\right) \geqq 0,\ \forall x\in D, \\
& \xi_j G_{g_j}\left(g_j\left(y\right)\right)\geqq 0,\ j=1,...,m, \\
& y\in X,\ \lambda\in R^k,\ \lambda\geq 0,\ \lambda\mathbf{e}=1, \\
& \xi\in R^m,\ \xi\geqq 0, \qquad (G\text{-VMWD})
\end{aligned}$$

where $e=(1,...,1)\in R^k$, $G_f=(G_{f_1},...,G_{f_k})$, where each G_{f_i}, $i\in I$, is a differentiable real-valued strictly increasing function defined on $I_{f_i}(X)$, and $G_g=(G_{g_1},...,G_{g_m})$, where each G_{g_j}, $j\in J$, is a differentiable real-valued strictly increasing function defined on $I_{g_j}(X)$. We call (G-VMWD) the G-Mond-Weir vector dual problem (with respect to η, G_f and G_g) for the considered directionally differentiable multiobjective programming problem (VP).

Let W denote the set of all feasible points of (G-VMWD) and $pr_X W$ be the projection of the set W on X, that is, $pr_X W := \{y\in X:(y,\lambda,\xi)\in W\}$. Moreover, for a given $(y,\lambda,\xi)\in W$, we denote by $I(y) := \{i\in I:\lambda_i>0\}$ and, moreover, by $J(y) := \{j\in J:\xi_j>0\}$. Then, $\left(f_{I(y)},g_{J(y)}\right)$ denotes vectors of f_i, $i\in I(y)$ and g_j, $j\in J(y)$, respectively.

Now, we prove duality results between the primal multiobjective programming problem (VP) and its vector dual problem in the sense of Mond-Weir under assumption that the functions constituting these problems are semi-G-V-type I objective and constraint functions.

Theorem 6. *(G-weak duality): Let x and (y,λ,ξ) be any arbitrary feasible solutions in the considered multiobjective programming problem (VP) and its G-Mond-Weir vector dual problem*

(G-VMWD) with respect to η, G_f *and* G_g, *respectively. Further, we assume that* (f, g) *is semi-G-V-type I objective and constraint functions at* y *on* $D \cup pr_X W$ *with respect to* η, G_f *and* G_g. *Then*

$$f(x) \not< f(y).$$

Proof. Let x and (y, λ, ξ) be any arbitrary feasible solutions in problems (VP) and (G-VMWD) (with respect to η, G_f and G_g), respectively. By assumption, (f, g) is vector semi-G-V-type I at y on $D \cup pr_X W$ with respect to η (and with respect to η, G_f and G_g). Then, by Definition 6, the following inequalities

$$G_{f_i}(f_i(z)) - G_{f_i}(f_i(y)) \\ -\alpha_i(z, y)\, G'_{f_i}(f_i(y))\, f_i^+(y; \eta(z, y)) \geqq 0, \\ i = 1, ..., k,$$

$$-G_{g_j}(g_j(y)) \geqq \beta_j(z, y)\, G'_{g_j}(g_j(y))\, g_j^+(y; \eta(z, y)), \\ j = 1, ..., m$$

are satisfied for all $z \in D \cup pr_X W$. Therefore, they are also satisfied for $z = x \in D$. Thus, the above inequalities yield

$$G_{f_i}(f_i(x)) - G_{f_i}(f_i(y)) \\ -\alpha_i(x, y)\, G'_{f_i}(f_i(y))\, f_i^+(y; \eta(x, y)) \geqq 0, \\ i = 1, ..., k, \tag{49}$$

$$-G_{g_j}(g_j(y)) \\ \geqq \beta_j(x, y)\, G'_{g_j}(g_j(y))\, g_j^+(y; \eta(x, y)), \\ j = 1, ..., m. \tag{50}$$

We proceed by contradiction. Suppose, contrary to the result, that

$$f(x) < f(y).$$

Thus,

$$f_i(x) < f_i(y),\ i = 1, ..., k.$$

Taking into account the increasing property of each function G_{f_i}, $i = 1, ..., k$, the inequalities above imply

$$G_{f_i}(f_i(x)) < G_{f_i}(f_i(y)),\ i = 1, ..., k. \tag{51}$$

Combining (49) and (51), we get

$$\alpha_i(x, y)\, G'_{f_i}(f_i(y))\, f_i^+(y; \eta(x, y)) < 0,\ i = 1, ..., k.$$

Since $\alpha_i(x, y) > 0$, $i = 1, ..., k$, above inequalities yield

$$G'_{f_i}(f_i(y))\, f_i^+(y; \eta(x, y)) < 0,\ i = 1, ..., k. \tag{52}$$

Multiplying each inequality (52) by the associated Lagrange multiplier λ_i, we get

$$\lambda_i G'_{f_i}(f_i(y))\, f_i^+(y; \eta(x, y)) \leqq 0,\ i = 1, ..., k, \tag{53}$$

and for the least one $i^* \in I$,

$$\lambda_{i^*} G'_{f_{i^*}}(f_i(y))\, f_{i^*}^+(y; \eta(x, y)) < 0. \tag{54}$$

Adding both sides of (53) and (54), we obtain

$$\sum_{i=1}^{k} \lambda_i G'_{f_i}(f_i(y))\, f_i^+(y; \eta(x, y)) < 0. \tag{55}$$

Multiplying each inequality (50) by the associated Lagrange multiplier ξ_j, we get

$$-\xi_j G_{g_j}(g_j(y)) \\ \geqq \beta_j(x, y)\, \xi_j G'_{g_j}(g_j(y))\, g_j^+(y; \eta(x, y)), \\ j = 1, ..., m.$$

The second constraint of dual problem (G-VMWD) implies

$$\beta_j(x, y)\, \xi_j G'_{g_j}(g_j(y))\, g_j^+(y; \eta(x, y)) \leqq 0, \\ j = 1, ..., m. \tag{56}$$

Since $\beta_j(x, y) > 0$, $j = 1, ..., m$, (56) yields

$$\xi_j G'_{g_j}(g_j(y))\, g_j^+(y; \eta(x, y)) \leqq 0,\ j = 1, ..., m. \tag{57}$$

Adding both sides of (57), we obtain

$$\sum_{j=1}^{m} \xi_j G'_{g_j}(g_j(y))\, g_j^+(y; \eta(x, y)) \leqq 0. \tag{58}$$

Combining (55) and (58), we get that the following inequality

$$\sum_{i=1}^{k} \lambda_i G'_{f_i}(f_i(y))\, f_i^+(y; \eta(x, y)) \\ + \sum_{j=1}^{m} \xi_j G'_{g_j}(g_j(y))\, g_j^+(y; \eta(x, y)) < 0$$

holds, contradicting the first constraint of dual problem (G-VMWD). Thus, the proof of this theorem is completed. □

It is possible to prove a stronger result if we assume stronger semi-G-V-type I assumptions imposed on the functions constituting the considered vector optimization problem (VP).

Theorem 7. *(G-weak duality): Let* x *and* (y, λ, ξ) *be any feasible solutions in the considered multiobjective programming problem (VP) and its G-Mond-Weir vector dual problem (G-VMWD) (with respect to* η, G_f *and* G_g*), respectively. Further, assume that* (f, g) *is semi-strictly-G-V-type I objective and constraint functions at* y *on* $D \cup pr_X W$ *with respect to* η, G_f *and* G_g. *Then*

$$f(x) \nleq f(y).$$

Proof. Since proof of this theorem is similar to the proof of Theorem 6, therefore, it is omitted in the paper. □

Theorem 8. *(G-strong duality). Let $\overline{x} \in D$ be a (weak) Pareto solution in the primal multiobjective programming problem (VP), the G-constraint qualification (G-CQ) be satisfied at $\overline{x}$. Then there exist $\overline{\lambda} \in R^k_+$, $\overline{\xi} \in R^m_+$, $\overline{\lambda} \geq 0$, $\overline{\xi} \geqq 0$ such that $(\overline{x}, \overline{\lambda}, \overline{\xi})$ is feasible in the G-Mond-Weir vector dual problem (G-VMWD) (with respect to η, G_f and G_g). If also G-weak duality (Theorem 6 or Theorem 7, respectively) holds, then $(\overline{x}, \overline{\lambda}, \overline{\xi})$ is a (weakly) efficient solution of a maximum type in (G-VMWD), and the objective functions values are equal in problems (VP) and (G-VMWD).*

Proof. By assumption, $\overline{x}$ is a (weak) Pareto solution in the primal multiobjective programming problem (VP). Then, there exist $\overline{\lambda} \in R^k_+$, $\overline{\xi} \in R^m_+$, $\overline{\lambda} \geq 0, \overline{\xi} \geqq 0$ such that the G-Karush-Kuhn-Tucker conditions (23)-(25) hold with functions G_f and G_g. Hence, the feasibility of $(\overline{x}, \overline{\lambda}, \overline{\xi})$ in dual problem (G-VMWD) (with respect to η, G_f and G_g) follows from the G-Karush-Kuhn-Tucker conditions (23)-(25). Moreover, if weak duality (Theorem 6 or Theorem 7, respectively) holds, then $(\overline{x}, \overline{\lambda}, \overline{\xi})$ is a (weak) efficient solution of a maximum type in (G-VMWD). □

Theorem 9. *(G-Converse duality): Let $(\overline{y}, \overline{\lambda}, \overline{\xi})$ be a feasible solution of the G-Mond-Weir vector dual problem (G-VMWD) (with respect to η, G_f and G_g) with $\overline{y} \in D$. Further, assume that one of the following hypotheses is fulfilled:*

a) *(f, g) is (semi-G-V-type I) semi-strictly-G-V-type I objective and constraint functions at $\overline{y}$ on $D \cup pr_X W$ with respect to η, G_f and G_g*
b) *$(f_{I(\overline{y})}, g_{J(\overline{y})})$ is (semi-pseudo-$\widetilde{G}$-V-type I) semi-strictly-pseudo-$\widetilde{G}$-V-type I objective and constraint functions at $\overline{y}$ on $D \cup pr_X W$ with respect to η, where $\widetilde{G}_{f_i} = \overline{\lambda}_i G_{f_i}$, $i \in I(\overline{y})$, $\widetilde{G}_{g_j} = \overline{\xi}_j G_{g_j}$, $j \in J(\overline{y})$,*
c) *$(f_{I(\overline{y})}, g_{J(\overline{y})})$ is (semi-pseudo-quasi-$\widetilde{G}$-V-type I) semi-strictly-pseudo-quasi-$\widetilde{G}$-V-type I objective and constraint functions at $\overline{y}$ on $D \cup pr_X W$ with respect to η, where $\widetilde{G}_{f_i} = \overline{\lambda}_i G_{f_i}$, $i \in I(\overline{y})$, $\widetilde{G}_{g_j} = \overline{\xi}_j G_{g_j}$, $j \in J(\overline{y})$. Then $\overline{y}$ is (a weakly efficient solution) an efficient solution in problem (VP).*

Proof. Proof of this theorem under hypothesis a) follows directly from weak duality (see, Theorem 6 or Theorem 7, respectively). □

Theorem 10. *(G-Strict converse duality): Let $\overline{x}$ and $(\overline{y}, \overline{\lambda}, \overline{\xi})$ be feasible solutions in the considered multiobjective programming problem (VP) and its G-Mond-Weir vector dual problem (G-VMWD) (with respect to η, G_f and G_g), respectively, such that*

$$f(\overline{x}) = f(\overline{y}). \tag{59}$$

Further, assume that one of the following hypotheses is fulfilled:

a) *(f, g) is semi-strictly-G-V-type I objective and constraint functions at $\overline{y}$ on $D \cup pr_X W$ with respect to η, G_f and G_g*
b) *$(f_{I(\overline{y})}, g_{J(\overline{y})})$ is semi-strictly-pseudo-$\widetilde{G}$-V-type I objective and constraint functions at $\overline{y}$ on $D \cup pr_X W$ with respect to η, where $\widetilde{G}_{f_i} = \overline{\lambda}_i G_{f_i}$, $i \in I(\overline{y})$, $\widetilde{G}_{g_j} = \overline{\xi}_j G_{g_j}$, $j \in J(\overline{y})$,*
c) *$(f_{I(\overline{y})}, g_{J(\overline{y})})$ is semi-strictly-pseudo-quasi-$\widetilde{G}$-V-type I objective and constraint functions at $\overline{y}$ on $D \cup pr_X W$ with respect to η, where $\widetilde{G}_{f_i} = \overline{\lambda}_i G_{f_i}$, $i \in I(\overline{y})$, $\widetilde{G}_{g_j} = \overline{\xi}_j G_{g_j}$, $j \in J(\overline{y})$.*

Then $\overline{x} = \overline{y}$. If hypothesis a) is fulfilled, then $\overline{x}$ is an efficient solution in problem (VP) and $(\overline{y}, \overline{\lambda}, \overline{\xi})$ is an efficient solution in problem (G-VMWD).

Proof. Now, we prove this theorem under hypothesis a). Let $\overline{x}$ and $(\overline{y}, \overline{\lambda}, \overline{\xi})$ be feasible solutions in the considered multiobjective programming problem (VP) and its G-Mond-Weir vector dual problem (G-VMWD) (with respect to η, G_f and G_g), respectively, such that the relation (59) is fulfilled. By assumption, (f, g) is semi-G-V-type I objective and constraint functions at $\overline{y}$ on $D \cup pr_X W$ with respect to η, G_f and G_g. Using the feasibility of $(\overline{y}, \overline{\lambda}, \overline{\xi})$ in problem (G-VMWD) with respect to η, G_f and G_g, we have

$$-\overline{\xi}_j G_{g_j}(g_j(\overline{y})) \leqq 0, \ j = 1, ..., m. \tag{60}$$

Hence, by the definition of semi-strictly-G-V-type I objective and constraint functions, $\beta_j(\overline{x}, \overline{y}) > 0$, $j \in J$, and, therefore, (60) yields

$$-\beta_j(\overline{x}, \overline{y}) \overline{\xi}_j G_{g_j}(g_j(\overline{y})) \leqq 0, \ j \in J.$$

Adding both sides of the inequalities above, we get

$$-\sum_{j=1}^{m} \beta_j(\overline{x}, \overline{y}) \overline{\xi}_j G_{g_j}(g_j(\overline{y})) \leqq 0. \tag{61}$$

Thus, by the definition of semi-G-V-type I objective and constraint functions, (61) implies

$$\sum_{j=1}^{m} \overline{\xi}_j G'_{g_j}(g_j(\overline{y})) \, g_j^+(u; \eta(\overline{x}, \overline{y})) \leqq 0. \tag{62}$$

By $\overline{x} \in D$ and $\left(\overline{y}, \overline{\lambda}, \overline{\xi}\right) \in W$, the first constraint of (G-VMWD) and (62) yield

$$\sum_{i=1}^{k} \overline{\lambda}_i G'_{f_i}\left(f_i(\overline{y})\right) f_i^{+}\left(\overline{x}; \eta(\overline{x}, \overline{y})\right) \geqq 0. \qquad (63)$$

Hence, by the definition of semi-strictly-G-V-type I objective and constraint functions, (63) gives

$$\sum_{i=1}^{k} \alpha_i\left(\overline{x}, \overline{y}\right) \overline{\lambda}_i \left[G_{f_i}\left(f_i\left(\overline{x}\right)\right) - G_{f_i}\left(f_i\left(\overline{y}\right)\right)\right] > 0.$$

By definition, $\alpha_i\left(\overline{x}, \overline{y}\right) > 0$, $i \in I\left(\overline{y}\right)$. Since $\overline{\lambda}_i > 0$, $i \in I\left(\overline{y}\right)$, the inequality above implies that $f\left(\overline{x}\right) \neq f\left(\overline{y}\right)$, contradicting the assumption (59). Since the hypotheses of G-weak duality are also satisfied, by Theorem 6, it follows that $\overline{x}$ is a weak efficient solution in problem (VP) and $\left(\overline{y}, \overline{\lambda}, \overline{\xi}\right)$ is a weak efficient solution in problem (G-VMWD). Hence, the proof of this theorem under hypothesis a) is completed.

Now, we prove this theorem under hypothesis b) Let $\overline{x}$ and $\left(\overline{y}, \overline{\lambda}, \overline{\xi}\right)$ be feasible solutions in the considered multiobjective programming problem (VP) and its G-Mond-Weir vector dual problem (G-VMWD) (with respect to η, G_f and G_g), respectively. Suppose that $\overline{x} \neq \overline{y}$ and exhibit a contradiction. By assumption, (f, g) is semi-strictly-pseudo-$\widetilde{G}$-V-type I objective and constraint functions at $\overline{y}$ on $D \cup pr_X W$ with respect to η, G_f and G_g. By definition, there exist functions β_j, $j \in J$, such that $\beta_j\left(\overline{x}, \overline{y}\right) > 0$. Thus, by $\left(\overline{y}, \overline{\lambda}, \overline{\xi}\right) \in W$, it follows that

$$-\beta_j\left(\overline{x}, \overline{y}\right) \overline{\xi}_j G_{g_j}\left(g_j\left(\overline{y}\right)\right) \leqq 0,\ j \in J.$$

Adding both sides of the inequalities above, we get

$$-\sum_{j=1}^{m} \beta_j\left(\overline{x}, \overline{y}\right) \overline{\xi}_j G_{g_j}\left(g_j\left(\overline{y}\right)\right) \leqq 0 \qquad (64)$$

Since G_{g_j}, $j \in J(\overline{y})$, are strictly increasing functions on their domains, therefore $\widetilde{G}_{g_j} = \overline{\xi}_j G_{g_j}$, $j \in J(\overline{y})$, are also strictly increasing functions on the same sets. Then, by the definition of semi-strictly-pseudo-$\widetilde{G}$-V-type I functions, (64) implies

$$\sum_{j \in J(\overline{y})} \overline{\xi}_j G'_{g_j}\left(g_j(\overline{y})\right) g_j^{+}\left(\overline{y}; \eta(\overline{x}, \overline{y})\right) \leqq 0. \qquad (65)$$

By $\overline{x} \in D$ and $\left(\overline{y}, \overline{\lambda}, \overline{\xi}\right) \in W$, the first constraint of (G-VMWD) and (65) yield

$$\sum_{i \in I(\overline{y})} \overline{\lambda}_i G'_{f_i}\left(f_i(\overline{y})\right) f_i^{+}\left(\overline{y}; \eta(\overline{x}, \overline{y})\right) \geqq 0. \qquad (66)$$

Hence, by hypothesis b), the inequality (66) yields

$$\sum_{i \in I(\overline{y})} \alpha_i\left(\overline{x}, \overline{y}\right) \overline{\lambda}_i \left[G_{f_i}\left(f_i\left(\overline{x}\right)\right) - G_{f_i}\left(f_i\left(\overline{y}\right)\right)\right] > 0.$$

By definition, $\alpha_i\left(\overline{x}, \overline{y}\right) > 0$, $i \in I\left(\overline{y}\right)$. Then, by $\overline{\lambda}_i > 0$, $i \in I\left(\overline{y}\right)$, the inequality above implies that $f\left(\overline{x}\right) \neq f\left(\overline{y}\right)$, contradicting the assumption (59). Hence, the proof of this theorem under hypothesis b) is completed.

Proof of this theorem under hypothesis c) is similar and, therefore, it is omitted in the paper. □

5. Conclusion

This paper represents the study concerning the new class of directionally differentiable multiobjective programming problems with nonconvex functions. The so-called class of semi-G-V-type I objective and constraint functions and its various generalizations are introduced in the case of directional differentiability of the functions constituting the considered nonconvex multiobjective programming problem. The importance of the generalized G-invex functions is because, similarly to Craven's work [10], the transformations of functions do not destroy properties invex functions. We have proved new necessary and sufficient optimality conditions for directionally differentiable multiobjective programming problems. It is pointed out that our statements of the so-called G-Fritz John type necessary optimality conditions and the G-Karush-Kuhn-Tucker type necessary optimality conditions established in this work are more general than the classical Fritz John type necessary optimality conditions and the classical Karush-Kuhn-Tucker necessary optimality conditions found in the literature. Furthermore, we have proved the sufficiency of the introduced G-Karush-Kuhn-Tucker necessary optimality conditions for the considered nonconvex directionally differentiable multiobjective programming problem. Further, we have defined a new vector dual problem for the considered directionally differentiable multiobjective programming problem. The so-called G-Mond-Weir dual problem is a generalization of a well-known vector dual problem in the sense Mond-Weir. This work extends results obtained in literature by many authors (see, for example, [3], [6], [7], [18], [22], [23], [27], and others). Hence, the sufficiency of the Karush-Kuhn-Tucker necessary optimality conditions and various duality results in the sense of Mond-Weir have been proved for a new class of nonconvex directionally differentiable multiobjective programming problems.

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