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RESEARCH ARTICLE

An application of fuzzy linear modeling: prediction of uncertainty for beta-glucan content

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ABSTRACT

Beta-glucan (BG) has positive health effects for the mamalians. However, the BG sources have limited content of it. Besides, the production of the BG has stringent procedures with low productivity. Economical production of the BG needs the improvement of the BG production steps. In this study, it is aimed to improve the BG content during the first step of the BG production, microorganism growth step, by obtaining the optimal values of additive materials (EDTA, $CaCl_2$ and Sorbitol). For this purpose, the experimental data sets with replicated response measures (RRM) are obtained at spesific levels of EDTA, $CaCl_2$ and Sorbitol. Fuzzy modeling, a flexible modeling approach, is applied on the experimental data set because of the small sized data set and diffulty of satisfying probabilistic modeling assumptions. The predicted fuzzy function is obtained according to the fuzzy least squares approach. In order to get the optimal values of EDTA, $CaCl_2$ and Sorbitol, the predicted fuzzy function is maximized based on multi-objective optimization (MOO) approach. By using the optimal values of EDTA, $CaCl_2$ and Sorbitol, the uncertainty for predicted BG content is evaluated from the economic perspective.

1. Introduction

Beta glucan (BG) is an active ingredient which was approved by FDA (Food Drug Administration) in USA and EFSA in Europe (European Food Safety Administration) due to its positive effects on health. The BG has potential application in medicine and pharmacy, food, cosmetic and chemical industries, in feed production and veterinary medicine [1-4]. Production of the BG from different sources such as yeast, fungi, bacteria, and cereals is possible by extraction, isolation and purification technologies. Among these sources yeasts are the most used in industrial production because they have plentiful of BG about 8-16 %. Yeast cell wall contains the glucans, mannoproteins and chitin. Economically production of the BG from the yeast cell wall is depend on both the microbial growth and the extraction conditions. Growth conditions influence the morphology and composition of the cell wall during growth process.

Major factors, which affect the yeast cell wall composition, include yeast strain [5], growth conditions [6, 7] and the time of harvesting [8, 9]. Extraction of BG from yeast generally consists of two main steps: (i) yeast cell lysis (separation of cell wall from cytoplasm) and, (ii) BG extraction (extraction from insoluble cell wall) [10]. Several studies reported about chemical [11-13], physical [14, 15] and enzymatic lysis [16] of yeast cells. There are lots of medicinal studies about the health effects of the BG however there is still necessity of researches to enhance the BG content.

In many real life problems, e.g. engineering, health, business, economics, the researchers aim to obtain mathematical models of the problems. For this purpose, functional relationship between input (factor, independent) variable/variables and response (output, dependent) variable/variables is wanted to be defined. Generally, statistical regression analysis is considered as a basic modeling tool to define the analytical relationship between the variables [17, 18]. This procedure needs some assumptions to apply, e.g. adequate number of observations, certain relationship between variables, zero mean and uncorrelated errors, and normally distributed errors to make statistical inferences. However, there are some cases where the statistical modeling assumptions can not be satisfied. Modeling of the replicated response measured small sized data set can be one of the examples for this situation.

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When the data set has replicated response measures (RRM), the qualification of the response has uncertainty different than randomness. In this case, the uncertainty of the RRM can be defined as fuzzy number, firstly introduced by Zadeh [19] since it will be hard to define replicated values as a single numerical quantity. In this study, Triangular Type-1 Fuzzy Numbers (TT1FNs) are used to present the RRM due to sake of simplicity. In order to transform RRM to TT1FNs, some descriptive statistics of replicates, e.g. minimum, median, maximum, are used. Then, it is aimed to obtain fuzzy linear model with fuzzy model parameters for fuzzy response valued data set. It is assumed here that the input variables are crisp. In order to estimate the unknown fuzzy linear model parameters, fuzzy least squares (FLS) approach is used. The FLS is based on minimizing the sum of squared fuzzy errors.

In the literature, there have been several studies about modeling of the replicated response measured data sets through FLS approach. In the studies of Bashiri and Hosseininezhad [20], Bashiri and Hosseininezhad [21], Türkşen and Apaydın [22], Türkşen and Güler [23], Türkşen [24, 25], the RRM are transformed to TT1FNs for each observation unit. Therefore, the natural structure of replicated measures are taken into account to represent the uncertainty of replicated values.

The main aim of the study is to propose a flexible way of modeling, using FLS approach, for the small sized data set with RRM to obtain maximum amount of BG content from the yeast cell wall. Thus, the uncertainty of BG content should be predicted without using any strong modeling assumptions. In order to optimize predicted fuzzy response function, weighted optimization is applied. By maximizing the predicted fuzzy function, optimal values of additive materials are defined.

The paper is organized as follows: In Section 2, detailed information about BG production is given. Section 3 gives a brief information about data set with RRM and fuzzy linear regression modeling. And also, optimization methodology of predicted fuzzy response function is presented in Section 3. In Section 4, a real data set about BG content is used for application purpose of the fuzzy modeling approach with comparison results. Finally, conclusion is given in Section 5.

2. Beta-glucan production

Beta glucans (BGs) are polysaccharides that are composed of glucose units. The BGs exist together with the mannoproteins and chitin in the cell wall. In order to produce the BG, three steps process be followed; (1) microorganism growth, (2) cell wall lysis and, (3) extraction of BG from the cell wall components. For these three steps process, each step can be accomplished by various type of procedures. In the microorganism growth step, growth conditions (microorganism type, temperature, pH, aeration level, growth medium, operation mode etc.) are the most important factors. These growth factors must be determined according to selected microorganism strain to enhance the BG content. In general, yeast strains of *Saccharomyces cerevisiae* which is known as baker's yeast due to higher BG content. The *S. cerevisiae* has advantageous for producing high quality BG.

The usage of some additive materials in the growth medium results the enhancement of yeast BG content [6]. EDTA, Sorbitol and $CaCl_2$ are considered as additive materials in the growth medium. Determining the optimal values of these additive materials has importance to obtain maximum amount of BG content from the yeast cell wall. For this purpose, the experimental studies should be done at specifically defined conditions which are the growth factors (e.g. pH, T), the cell wall lysis and the extraction procedures of BG. These conditions were explained in detail from the previous studies [7].

3. Fuzzy modeling and optimization

3.1. Design of experimental data set with fuzzy response

A basic and fundamental step of an experiment is the experimental design which enables researchers to find the most valuable information about the problem. The design of the experimental data can be composed of replicated response measured data set as given in Table 1.

Table 1. Experimental design with RRM.

No	Input levels				Response			
	$\mathbf{X}_1$	$\mathbf{X}_2$	…	$\mathbf{X}_p$	$\mathbf{Y}$			
1	x_{11}	x_{12}	…	x_{1p}	y_{11}	y_{12}	…	y_{1r}
2	x_{21}	x_{22}	…	x_{2p}	y_{21}	y_{22}	…	y_{2r}
.	.	.		.	.	.		.
.	.	.	…	.	.	.	…	.
.	.	.		.	.	.		.
n	x_{n1}	x_{n2}	…	x_{np}	y_{n1}	y_{n2}	…	y_{nr}

Table 2. Experimental design with fuzzy response.

No	Input levels				Response
	$\mathbf{X}_1$	$\mathbf{X}_2$	…	$\mathbf{X}_p$	$\tilde{\mathbf{Y}}$
1	x_{11}	x_{12}	…	x_{1p}	$\tilde{y}_1$
2	x_{21}	x_{22}	…	x_{2p}	$\tilde{y}_2$
.	.	.		.	.
.	.	.	…	.	.
.	.	.		.	.
n	x_{n1}	x_{n2}	…	x_{np}	$\tilde{y}_n$

In Table 1, n denotes the number of experimental units and r is the number of replications for the response. The replicated values of the response are obtained for each setting of a group of p input variables. It is clear here that the RRM have uncertainty different than randomness for each unit. The uncertainty of the nature of the RRM should be taken into account to model the replicated response measured data set properly. For this

purpose, the RRM are transformed to TT1FNs instead of using a single numerical quantity for RRM. The design of the experimental data with fuzzy observed reponse is presented in Table 2.

3.2. Triangular type-1 fuzzy numbers

A type-1 fuzzy set A is a set function on universe D_A into [0,1], e.g. $\mu_A : X \to [0,1]$. The membership function (MF) of A is denoted and is called a type-1 MF, e.g. $A = \{(x, \mu_A(x)) : x \in X\}$ in which $0 \le \mu_A(x) \le 1$. When the uncertainty is modeled using a type-1 fuzzy set it is called a type-1 fuzzy number (TT1FNs). Let *A* be a fuzzy set in R. *A* is called a type-1 fuzzy number if: (*i*) *A* is normal, (*ii*) *A* is convex, and (*iii*) *A* has a bounded support [26].

A TT1FN is a fuzzy number represented with three points $(a_1, a_2, a_3) \in A$. A presentation of a TT1FN can be seen in Figure 1.

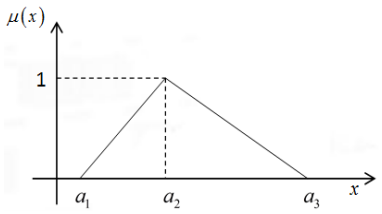

Figure 1. A presentation of TT1FN $x = (a_1, a_2, a_3)$

The MF formula for TT1FN is given below

$$\mu_A(x) = \begin{cases} (x - a_1)/(a_2 - a_1) & , \quad a_1 \le x < a_2 \\ (a_3 - x)/(a_3 - a_2) & , \quad a_2 \le x \le a_3 \\ 0 & , \quad x > a_3 \ \ or \ \ x < a_1 \end{cases} \tag{1}$$

where the *A* can be denoted as $\tilde{A} = (a_1, a_2, a_3)$. Suppose that $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ be two TT1FNs. Some elementary arithmetic operations for TT1FNs can be given basically as follows:

- $\tilde{A} + \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$
- $\tilde{A} - \tilde{B} = (a_1 - b_3, a_2 - b_2, a_3 - b_1)$
- $\tilde{A} \times \tilde{B} = (a_1 b_1, a_2 b_2, a_3 b_3)$
- Let λ be a scalar.

$$\lambda \tilde{A} = \begin{cases} (\lambda a_1, \lambda a_2, \lambda a_3) & , \quad \lambda > 0 \\ (-\lambda a_3, -\lambda a_2, -\lambda a_1) & , \quad \lambda < 0 \end{cases}$$

Detailed information about TT1FNs can be seen in the study of [27].

3.3. Transforming the RRM to TT1FNs

The observed replicated response measures can be represented in matrix form as below

$$\mathbf{Y} = \begin{bmatrix} y_{11} & y_{12} & \dots & y_{1r} \\ y_{21} & y_{22} & \dots & y_{2r} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & & \cdot \\ y_{n1} & y_{n2} & \dots & y_{nr} \end{bmatrix} \tag{2}$$

and the fuzzy presentation of the response for each unit can be given as

$$\mathbf{Y} = \begin{bmatrix} \tilde{y}_1 \\ \tilde{y}_2 \\ \cdot \\ \cdot \\ \cdot \\ \tilde{y}_n \end{bmatrix} = \begin{bmatrix} (y_1^l, y_1^c, y_1^u) \\ (y_2^l, y_2^c, y_2^u) \\ \cdot \\ \cdot \\ \cdot \\ (y_n^l, y_n^c, y_n^u) \end{bmatrix} \tag{3}$$

where $\tilde{y}_i = (y_i^l, y_i^c, y_i^u)$, $i = 1,2,...,n$ are obtained by using the following fuzzification rule:

$$\begin{aligned} y_i^l &= y_{i(1)} \\ y_i^c &= med(y_{ij}) \ , \ i = 1,2,...,n \ , \ j = 1,2,...,r \\ y_i^l &= y_{i(r)} \end{aligned} \tag{4}$$

in which $y_{i(1)}$ and $y_{i(r)}$ are the smallest and the largest order statistics, respectively, and the $med(y_{ij})$ is the median of the RRM for each unit.

3.4. Fuzzy linear regression model

The general form of the fuzzy linear regression model can be given as

$$\tilde{\mathbf{Y}} = \mathbf{X}\tilde{\boldsymbol{\beta}} + \tilde{\boldsymbol{\varepsilon}} \tag{5}$$

in which observed fuzzy response values ($\tilde{\mathbf{Y}}$), fuzzy model coefficients ($\tilde{\boldsymbol{\beta}}$), and fuzzy errors ($\tilde{\varepsilon}$) are assumed as TT1FNs whereas the input variables are considered as crisp values. In this study, it is assumed that the predicted fuzzy response function has second order polinomial form given as

$$\tilde{\mathbf{Y}} = \hat{\tilde{\boldsymbol{\beta}}}_0 + \sum_{j=1}^{p} \hat{\tilde{\boldsymbol{\beta}}}_j \mathbf{X}_j + \sum_{j=1}^{p} \sum_{j<w}^{p} \hat{\tilde{\boldsymbol{\beta}}}_{jw} \mathbf{X}_j \mathbf{X}_w + \sum_{j=1}^{p} \hat{\tilde{\boldsymbol{\beta}}}_{jj} \mathbf{X}_j^2 . \tag{6}$$

It is clear from the Eq. (6) that the predicted fuzzy response model is linear according to the fuzzy model parameters, $\tilde{\boldsymbol{\beta}}$.

3.5. Fuzzy least squares approach

The estimates of triangular fuzzy model coefficient vector, $\hat{\tilde{\boldsymbol{\beta}}}$, is calculated by minimizing the following sum of squared error function, called fuzzy least squares (FLS), with respect to Diamond's distance metric

$$\min_{\tilde{\boldsymbol{\beta}}} \phi(\tilde{\boldsymbol{\beta}}) = \min_{\tilde{\boldsymbol{\beta}}} (\tilde{\boldsymbol{\varepsilon}}' \tilde{\boldsymbol{\varepsilon}}) = \min_{\tilde{\boldsymbol{\beta}}} \left(d^2 \left(\tilde{\mathbf{Y}}, \hat{\tilde{\mathbf{Y}}} \right) \right) \tag{7}$$

where

$$d^2\left(\tilde{\mathbf{Y}},\hat{\tilde{\mathbf{Y}}}\right)=\frac{1}{3}\left(\left(\mathbf{Y}^l-\hat{\mathbf{Y}}^l\right)^2+\left(\mathbf{Y}^c-\hat{\mathbf{Y}}^c\right)^2+\left(\mathbf{Y}^u-\hat{\mathbf{Y}}^u\right)^2\right). \quad (8)$$

The root mean of sum of squared error (RMSE) is preferred to use as a criteria to evaluate the prediction performances of the fuzzy predicted regression model. In order to minimize the objective function given in Eq. (7), derivative-based optimization approach is applied as follows:

$$\begin{aligned}\phi\left(\tilde{\boldsymbol{\beta}}\right)&=\tilde{\boldsymbol{\varepsilon}}'\tilde{\boldsymbol{\varepsilon}}\\&=\left(\tilde{\mathbf{Y}}-\mathbf{X}\tilde{\boldsymbol{\beta}}\right)'\left(\tilde{\mathbf{Y}}-\mathbf{X}\tilde{\boldsymbol{\beta}}\right)\\&=\tilde{\mathbf{Y}}'\tilde{\mathbf{Y}}-\tilde{\mathbf{Y}}'\mathbf{X}\tilde{\boldsymbol{\beta}}-\tilde{\boldsymbol{\beta}}'\mathbf{X}'\tilde{\mathbf{Y}}+\tilde{\boldsymbol{\beta}}'\mathbf{X}'\mathbf{X}\tilde{\boldsymbol{\beta}}.\end{aligned}$$

By using derivative calculation

$$\frac{\partial\varphi}{\partial\tilde{\boldsymbol{\beta}}}=0$$

the fuzzy model coefficients are obtained as

$$\hat{\tilde{\boldsymbol{\beta}}}=\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{X}'\tilde{\mathbf{Y}}.$$

where $\mathbf{X}$ is design matrix composed with crisp input values and $\tilde{\mathbf{Y}}$ is fuzzy observed response vector.

3.6. Optimization of the predicted fuzzy response function

The main goal of the study is to obtain the optimal additive material values which maximize the BG content of the yeast. For this purpose, it is aimed to maximize the predicted fuzzy response function. The optimization can be achieved by solving the following problem:

$$\max_{x\in R}\hat{\tilde{Y}}=\max_{x\in R}(\hat{Y}^l,\ \hat{Y}^c,\ \hat{Y}^u). \quad (9)$$

In order to optimize the problem given in Eq. (9), it is possible to consider the problem as multi-objective optimization (MOO) problem since each element of the triplet should be maximized simultaneously. The MOO problem can be presented as

$$\begin{aligned}&\max_{x\in R}\hat{Y}^l\\&\max_{x\in R}\hat{Y}^c.\\&\max_{x\in R}\hat{Y}^u\end{aligned} \quad (10)$$

It is clear that the MOO problem, given in Eq. (10), is hard to solve. To make the calculations easier, the objective functions are aggregated in a single objective function with weighted approach as below

$$\max_{x\in R}\frac{1}{3}\left(\hat{Y}^l+\hat{Y}^c+\hat{Y}^u\right). \quad (11)$$

It should be noted here that the weighted values are chosen equal since each member of fuzzy predicted response triplet, denoted as $\hat{\tilde{\mathbf{Y}}}=\left(\hat{\mathbf{Y}}^l,\hat{\mathbf{Y}}^c,\hat{\mathbf{Y}}^u\right)$, has equal importance. The solution of the weighted objective function, given in Eq. (11), will give optimal values of additive materials for maximization of the BG content.

4. Application

In this section, a real RRM data set is used to illustrate the flexible modeling procedure to obtain optimal additive material values for maximizing the BG content which is considered as response variable. Here, EDTA (X_1), $CaCl_2$ (X_2) and Sorbitol (X_3) are considered as crisp input variables. The individual experiments are done for X_1, X_2 and X_3 by using optimal values of temperature (T) and pH which are dealed with growth factors. The optimal values of T and pH are considered as 34.7 ^oC and 4.8, respectively, from the previous studies. The levels of X_1, X_2 and X_3 are defined according to literature knowledge. And also, the second and third steps of the BG production process are done by following of the study [7].

The experimental results are presented as in Tables 3-5. It can be seen from the Tables 3-5 that the BG content is obtained with three replicates for each additive material value.

Table 3. Data set for EDTA and BG.

No	EDTA (μg / ml)	Beta-glucan (μg / ml)		
		Rep-1	Rep-2	Rep-3
1	0	17.4	17.8	17.7
2	10.1367	23.3	24.4	24.8
3	25	27.3	26.8	27.5
4	39.8633	16.7	16.3	16.8
5	50	15.5	15.2	15.8

Table 4. Data set for $CaCl_2$ and BG.

No	$CaCl_2$ (mmol / L)	Beta-glucan (μg / ml)		
		Rep-1	Rep-2	Rep-3
1	0.15	20.3	20.2	20.2
2	0.3223	16.2	16.3	16.3
3	0.575	17.1	17.1	17.1
4	0.8277	17.4	17.2	17.4
5	1.00	16.1	16.2	16.3

Table 5. Data set for Sorbitol and BG.

No	Sorbitol (mmol / L)	Beta-glucan (μg / ml)		
		Rep-1	Rep-2	Rep-3
1	0	14.3	14.8	15.2
2	4.0547	20.1	21.1	20.8
3	10	23.2	23.7	23.7
4	15.9453	17.9	17.8	17.4
5	20	17.4	17.3	17.2

In this study, fuzzy linear modeling approach is preferred to use to obtain functional relationship between BG and individual additive material since each data set has small sized and composed of RRM. In order to transform the replicated BG measures to TT1FNs, the fuzzification rule given in Eq. (4) is used. The data sets with fuzzy response values are given in Tables 6-8 for additive materials.

The predicted fuzzy regression models are obtained as following:

Table 6. Fuzzy data set for EDTA and BG.

No	EDTA (μg / ml)	Beta-glucan (μg / ml)
1	0	(17.4, 17.7, 17.8)
2	10.1367	(23.3, 24.4, 24.8)
3	25	(26.8, 27.3, 27.5)
4	39.8633	(16.3, 16.7, 16.8)
5	50	(15.2, 15.5, 15.8)

Table 7. Fuzzy data set for $CaCl_2$ and BG.

No	$CaCl_2$ (mmol / L)	Beta-glucan (μg / ml)
1	0.15	(20.2, 20.2, 20.3)
2	0.3223	(16.2, 16.3, 16.3)
3	0.575	(17.1, 17.1, 17.1)
4	0.8277	(17.2, 17.4, 17.4)
5	1	(16.1, 16.2, 16.3)

Table 8. Fuzzy data set for Sorbitol and BG.

No	Sorbitol (mmol / L)	Beta-glucan (μg / ml)
1	0	(14.3, 14.8, 15.2)
2	4.0547	(20.1, 20.8, 21.1)
3	10	(23.2, 23.7, 23.7)
4	15.9453	(17.4, 17.8, 17.9)
5	20	(17.2, 17.3, 17.4)

$$\hat{\tilde{Y}}_{EDTA} = (17.911, 18.5689, 18.9163) + (0.5761, 0.6421, 0.6881)X_1 + (-0.0158, -0.0148, -0.0133)X_1^2 \quad (12)$$

$$\hat{\tilde{Y}}_{CaCl_2} = (20.8605, 20.9247, 21.1967) + (-13.1142, -11.8605, -11.5751)X_2 + (7.4608, 7.7845, 8.8298)X_2^2 \quad (13)$$

$$\hat{\tilde{Y}}_{Sorbitol} = (14.8212, 15.4796, 16.0275) + (1.2092, 1.3939, 1.4936)X_3 + (-0.0728, -0.0684, -0.0594)X_3^2. \quad (14)$$

In Tables 9-11, the predicted fuzzy response values of the BG content are presented for EDTA, $CaCl_2$ and Sorbitol, respectively. The deviations between observed and predicted response values are calculated, denoted with **e**, as follows:

$$\mathbf{e} = \frac{1}{6}\left(\left|\mathbf{Y}^l - \hat{\mathbf{Y}}^l\right| + 4*\left|\mathbf{Y}^c - \hat{\mathbf{Y}}^c\right| + \left|\mathbf{Y}^u - \hat{\mathbf{Y}}^u\right|\right) \quad (15)$$

Table 9. Predicted BG content and deviations for EDTA

EDTA (μg / ml)	Observed BG (μg / ml)	Predicted BG (μg / ml)	Deviation (e)
0	(17.4, 17.7, 17.8)	(17.9, 18.6, 18.9)	0.8667
10.1367	(23.3, 24.4, 24.8)	(22.1, 23.6, 24.5)	0.7833
25	(26.8, 27.3, 27.5)	(22.4, 25.4, 27.8)	2.05
39.8633	(16.3, 16.7, 16.8)	(15.8, 20.6, 25.2)	4.0833
50	(15.2, 15.5, 15.8)	(7.2, 13.7, 20)	3.2333

It can be said from the Table 10 that the deviations for $CaCl_2$ are considerably small. Besides, it is seen from Table 9 and Table 11 that the deviations are getting larger for the larger values of EDTA and Sorbitol, respectively.

Table 10. Predicted BG content and deviations for $CaCl_2$

$CaCl_2$ (mmol/L)	Observed BG (μg / ml)	Predicted BG (μg / ml)	Deviation (e)
0.15	(20.2, 20.2, 20.3)	(19.1, 19.3, 19.7)	0.8833
0.3223	(16.2, 16.3, 16.3)	(17.4, 17.9, 18.4)	1.6167
0.575	(17.1, 17.1, 17.1)	(15.8, 16.7, 17.5)	0.55
0.8277	(17.2, 17.4, 17.4)	(15.1, 16.4, 17.7)	1.0667
1.00	(16.1, 16.2, 16.3)	(15.2, 16.8, 18.4)	0.9

Table 11. Predicted BG content and deviations for Sorbitol

Sorbitol (mmol/L)	Observed BG (μg / ml)	Predicted BG (μg / ml)	Deviation (e)
0	(14.3, 14.8, 15.2)	(14.8, 15.5, 16)	0.6833
4.0547	(20.1, 20.8, 21.1)	(18.5, 20, 21.1)	0.8
10	(23.2, 23.7, 23.7)	(19.6, 22.6, 25)	1.55
15.9453	(17.4, 17.8, 17.9)	(15.6, 20.3, 24.7)	3.1
20	(17.2, 17.3, 17.4)	(9.9, 16.9, 22.1)	2.2667

After flexible modeling, the optimal value of each additive material is to be obtained. For this purpose, each predicted fuzzy response functions given in Eq. (12)-(14) are maximized by the way of given in subsection 3.6. The calculations are performed by using Matlab programme.

The optimal values of additive materials are $X_1^* = 21.71\ \mu g / mL$, $X_2^* = 0.7591\ mmol/L$, and $X_3^* = 10.2111\ mmol/L$. The predicted fuzzy BG values for these optimal values are calculated as

$$\hat{\tilde{Y}}_{EDTA}(21.71) = (22.9712, 25.5333, 27.5863)\ \mu g / ml$$

$$\hat{\tilde{Y}}_{CaCl_2}(0.7591) = (15.2047, 16.4071, 17.4981)\ \mu g / ml$$

$$\hat{\tilde{Y}}_{Sorbitol}(10.2111) = (19.5779, 22.581, 25.0854)\ \mu g / ml.$$

If the minimum values of these additive materials are used then the predicted fuzzy BG values are obtained as

$$\hat{\tilde{Y}}_{EDTA}(0) = (17.911, 18.5689, 18.9163)\ \mu g / ml$$

$$\hat{\tilde{Y}}_{CaCl_2}(0.15) = (19.0612, 19.3208, 19.6591)\ \mu g / ml$$

$$\hat{\tilde{Y}}_{Sorbitol}(0) = (14.8212, 15.4796, 16.0275)\ \mu g / ml\ .$$

The fuzzy intervals of the BG content can be determined as [22.9712, 27.5863], [15.2047, 17.4981] and [19.5779, 25.0854] for optimal values of the EDTA, $CaCl_2$ and Sorbitol, respectively.According to the obtained results, if the researcher does not use any EDTA for the yeast growth, it is possible to get BG content approximately between [18-19] μg/ml. It is clear from the results that $CaCl_2$ may not be considered as an additive material whereas optimal value of Sorbitol can be used as an additive material for maximizing BG content. Besides, it seems from the results that the EDTA and the Sorbitol are

interchangeable. However, economically, it will be proper to prefer to use the EDTA for the yeast growth since the BG has high added value.

5. Conclusion

In this study, with the aim of enhancing the BG content of *S. cerevisiae*, concentration of each additive material (EDTA, $CaCl_2$ and Sorbitol) in the growth medium was optimized. This investigation is important due to the BG has incremental value in so many industries. It is clear that the BG content of the yeast cell wall increased when the yeast was cultured in the growth medium containing the EDTA and Sorbitol. Conversely usage of $CaCl_2$ more than the minimum concentration in the growth medium resulted with decrease the BG content of the yeast. Results show that the optimal concentration of EDTA (21.71 mg/mL) and Sorbitol (10.211 mmol/mL), cause to an increament in the BG content about 28-46 % and 32-56 %, respectively. In point of the economical aspects usage of optimal amount of EDTA has more profitable. It was proposed a flexible modeling way using the FLS approach for the production process of BG, which inherently have uncertainties. By considering this problem as MOO one, predicted fuzzy response functions were maximized simultaneously to determine the optimal values of additive materials. Thus, modeling and optimization procedures for a real engineering problem which have small sized and contain RRM data, were presented and applied successfully.

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