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A simulation algorithm with uncertain random variables

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ABSTRACT

In many situations, uncertainty and randomness concurrently occur in a system. Thus this paper presents a new concept for uncertain random variable. Also, a simulation algorithm based on uncertain random variables is presented to approximate the chance distribution using pessimistic value and optimistic value. An example is also given to illustrate how to use the presented simulation algorithm.



1. Introduction

Liu [1] introduced the uncertain random variable for modeling complex systems. In other words; uncertain random variable is improved to illustrate the phenomenon which mixes uncertainty with randomness. If we receive historical data from the sample, we can estimate the probability distribution. But if we have a new product, we can not achieve the probability distribution of this new product owing to lack of data. In this case, we run across both randomness and human uncertainty. Human uncertainty is investigated by some scholars. As a branch of mathematics based on normality, duality, subadditivity and product axioms, uncertainty theory was introduced by Liu [2] in 2007. Gao [3] presented uncertain bimatrix game. Yang and Gao [4] studied uncertain differential game. Gao and Qin [5] introduced the degree connectivity of uncertain graph. Dalman [6] presented a model for the uncertain multi-item solid transportation problem. Dalman [7] constructed models of uncertain random multi-item solid transportation problem. To model uncertain random event, Liu [8] introduced

the chance theory to networks optimization problem. Some scholars derived properties of uncertain random entropy [9, 10].

By employing chance theory, an uncertain random project scheduling problem is presented by Ke et al. [11]. They introduced an uncertain random simulation which randomly produces the sample points. But, this algorithm is ambivalent due to produces different values at different time. A simulation algorithm is presented to solve uncertain random shortest path problem by Sheng and Gao [12].

Therefore this paper presents an algorithm which includes the inverse uncertainty distribution and uniformly produces the sample points. It has powerful performances on the reliability and stability than the algorithms in [11, 12]. The paper is built as follows: some basic knowledge of uncertainty theory and chance theory is presented in section II. Section III shows some formulas for uncertain random variables presents an uncertain random simulation algorithm. A numerical example is presented in Section IV. Finally, this paper closes in Section V.

2. Preliminaries

2.1. Uncertainty theory

Let Γ be a nonempty set, \mathcal{L} be a σ -algebra over Γ and \mathcal{M} be an uncertain measure. Then $(\Gamma, \mathcal{L}, \mathcal{M})$ is a measurable space. A set function $\mathcal{M} : \mathcal{L} \rightarrow [0, 1]$ is called an uncertain measure if it satisfies the following four axioms:

Axiom 1. (*Normality Axiom*)(Liu [2]): $\mathcal{M}\{\Gamma\} = 1$ for the universal set Γ .

Axiom 2. (*Duality Axiom*)(Liu [2]): $\mathcal{M}\{\Lambda\} + \mathcal{M}\{\Lambda^c\} = 1$ for any event Λ .

Axiom 3. (*Subadditivity Axiom*)(Liu [2]): For every countable sequence of events $\Lambda_1, \Lambda_2, \dots$, we have

$$\mathcal{M}\left\{\bigcup_{i=1}^{\infty} \Lambda_i\right\} \leq \sum_{i=1}^{\infty} \mathcal{M}\{\Lambda_i\}.$$

Axiom 4. (*Product Axiom*)(Liu [13]): Let $(\Gamma_k, \mathcal{L}_k, \mathcal{M}_k)$ be uncertainty spaces for $k = 1, 2, \dots$. The product uncertain measure \mathcal{M} is an uncertain measure satisfying

$$\mathcal{M}\left\{\prod_{k=1}^{\infty} \Lambda_k\right\} = \bigwedge_{k=1}^{\infty} \mathcal{M}_k\{\Lambda_k\}$$

where Λ_k are arbitrarily chosen events from \mathcal{L}_k for $k = 1, 2, \dots$, respectively.

Definition 1. (see [2]): An uncertain variable is a function ξ from an uncertainty space $(\Gamma, \mathcal{L}, \mathcal{M})$ to the set of real numbers such that $\{\xi \in B\}$ is an event for any Borel set B of real numbers.

Remark 1: Note that the event $\{\xi \in B\}$ is a subset of the universal set $\{\xi \in B\} = \{\gamma \in \Gamma | \xi(\gamma) \in B\}$.

Definition 2. (Liu [14]): An uncertainty distribution $\Phi(x)$ is said to be regular if it is a continuous and strictly increasing function with respect to x at which $0 < \Phi(x) < 1$, and

$$\lim_{x \rightarrow -\infty} \Phi(x) = 0, \lim_{x \rightarrow +\infty} \Phi(x) = 1.$$

Definition 3. (Liu [14]): Let ξ be an uncertain variable with regular uncertainty distribution $\Phi(x)$. Then the inverse function $\Phi^{-1}(\alpha)$ is called the inverse uncertainty distribution of ξ .

Theorem 1. (Liu [14]): Let $\xi_1, \xi_2, \dots, \xi_n$ be independent uncertain variables with regular uncertainty distributions $\Phi_1, \Phi_2, \dots, \Phi_n$, respectively. If $f(\xi_1, \xi_2, \dots, \xi_n)$ is strictly increasing with respect to $\xi_1, \xi_2, \dots, \xi_m$ and strictly decreasing with respect to $\xi_{m+1}, \xi_{m+2}, \dots, \xi_n$, then

$$\xi = f(\xi_1, \xi_2, \dots, \xi_n) \tag{1}$$

has an inverse uncertainty distribution.

$$\Psi^{-1}(\alpha) = f(\Phi_1^{-1}(\alpha), \dots, \Phi_m^{-1}(\alpha), \Phi_{m+1}^{-1}(1 - \alpha), \dots, \Phi_n^{-1}(1 - \alpha)). \tag{2}$$

2.2. Chance theory

Definition 4. (Liu [1]): Let $(\Gamma, \mathcal{L}, \mathcal{M})$ be an uncertainty space and let $(\Omega, \mathcal{A}, \Pr)$ be a probability space. Then the product $(\Gamma, \mathcal{L}, \mathcal{M}) \times (\Omega, \mathcal{A}, \Pr)$ is called a chance space

$$(\Gamma, \mathcal{L}, \mathcal{M}) \times (\Omega, \mathcal{A}, \Pr) = (\Gamma \times \Omega, \mathcal{L} \times \mathcal{A}, \mathcal{M} \times \Pr).$$

Definition 5. (see [1]): An uncertain random variable is a function ξ from a chance space $(\Gamma, \mathcal{L}, \mathcal{M}) \times (\Omega, \mathcal{A}, \Pr)$ to the set of real numbers such that $\{\xi \in B\}$ is an event in an event in $\mathcal{L} \times \mathcal{A}$ for any Borel set B of real numbers.

Theorem 2. (see [14]): Let $\xi_1, \xi_2, \dots, \xi_n$ be uncertain random variables on the chance space $(\Gamma, \mathcal{L}, \mathcal{M}) \times (\Omega, \mathcal{A}, \Pr)$, and let f be a measurable function. Then

$$\xi = f(\xi_1, \xi_2, \dots, \xi_n)$$

is an uncertain random variable determined by

$$\xi(\gamma, \omega) = f(\xi_1(\gamma, \omega), \xi_2(\gamma, \omega), \dots, \xi_n(\gamma, \omega))$$

for all $(\gamma, \omega) \in \Gamma \times \Omega$.

Theorem 3. (Liu [15]): Let $\eta_1, \eta_2, \dots, \eta_m$ be independent random variables with probability distributions $\Psi_1, \Psi_2, \dots, \Psi_m$, respectively, and let $\tau_1, \tau_2, \dots, \tau_n$ be independent uncertain variables. Assume f is a measurable function. Then the uncertain random variable

$$\xi = f(\eta_1, \eta_2, \dots, \eta_m, \tau_1, \tau_2, \dots, \tau_n)$$

has a chance distribution

$$\Phi(x) = \int_{\mathfrak{R}^m} F(x; y_1, y_2, \dots, y_m) d\Psi_1(y_1) d\Psi_2(y_2) \dots d\Psi_m(y_m). \tag{3}$$

where $F(x; y_1, y_2, \dots, y_m)$ is the uncertainty distribution of the variable

$$f(y_1, y_2, \dots, y_m, \tau_1, \tau_2, \dots, \tau_n).$$

Definition 6. (Liu [1]): Let ξ be an uncertain random variable. Then its chance distribution is defined by

$$\Phi(x) = \text{Ch}\{\xi \leq x\} \tag{4}$$

for any $x \in \mathfrak{R}$.

Theorem 4. Let ξ be an uncertain random variable. Then its expected value is

$$E[\xi] = \int_0^{+\infty} \text{Ch}\{\xi \geq r\} dr - \int_{-\infty}^0 \text{Ch}\{\xi \leq r\} dr \tag{5}$$

provided that at least one of the two integrals is finite.

Theorem 5. Let ξ be an uncertain random variable with regular chance distribution Φ . If the expected value exists, then

$$E[\xi] = \int_0^1 \Phi^{-1}(\alpha) d\alpha. \quad (6)$$

In order to obtain the chance distribution, we prove following theorems and develop a simulation algorithm.

3. A simulation algorithm with uncertain random variables

In this section, by employing the above definition and theorems, the chance distribution will be obtained. To do this, the following theorems is proved and besides a simulation algorithm is developed.

In fact, Formula (3) is a theoretical formula, which is not easy to use in most cases due to the complexity of chance distribution function. To overcome the difficulty, an uncertain random simulation is proposed to evaluate the chance distribution. First, we introduce the concepts of α -pessimistic value and α -optimistic value for an uncertain random variable. Then, we approximate the chance distribution, α -pessimistic value and α -optimistic value by using a numerical integration method.

Definition 7. Let ξ be an uncertain random variable on chance space $(\Gamma, \mathcal{L}, \mathcal{M}) \times (\Omega, \mathcal{A}, \Pr)$ and $\alpha \in (0, 1]$. Then,

$$\xi_{\inf}(\alpha) = \inf\{r | \text{Ch}\{\xi \leq r\} \geq \alpha\} \quad (7)$$

and

$$\xi_{\sup}(\alpha) = \sup\{r | \text{Ch}\{\xi \geq r\} \geq \alpha\} \quad (8)$$

are called the α -pessimistic value and the α -optimistic value of ξ , respectively.

Note that Random variables and uncertain variables are special uncertain random variables. The α -pessimistic value and the α -optimistic value of linear uncertain variable $\mathcal{L}(a, b)$ are $\xi_{\inf}(\alpha) = (1 - \alpha)a + \alpha b$ and $\xi_{\sup}(\alpha) = \alpha a + (1 - \alpha)b$.

Theorem 6. Let ξ be an ordinary uncertain random variable and $\alpha \in (0, 1]$. Then, we have

$$\text{Ch}\{\xi \leq \xi_{\inf}(\alpha)\} = \alpha. \quad (9)$$

Proof. Since the chance distribution is continuous, it follows from the definition of the α -pessimistic value for each $\alpha \in (0, 1]$, we have $\text{Ch}\{\xi \leq \xi_{\inf}(\alpha)\} = \lim_{n \rightarrow \infty} \text{Ch}\{\xi \leq \xi_{\inf}(\alpha) - 1/n\} \leq \alpha$, and $\text{Ch}\{\xi \leq \xi_{\inf}(\alpha)\} = \lim_{n \rightarrow \infty} \text{Ch}\{\xi \leq \xi_{\inf}(\alpha) + 1/n\} \geq \alpha$, which imply that $\text{Ch}\{\xi \leq \xi_{\inf}(\alpha)\} = \alpha$ holds. The theorem is proved. \square

Theorem 7. Let ξ be an ordinary uncertain random variable and $\alpha \in (0, 1]$. Then, we have

$$\text{Ch}\{\xi \geq \xi_{\sup}(\alpha)\} = \alpha. \quad (10)$$

Proof. Since the continuity of chance distribution, it follows from the definition of the α -optimistic value for each $\alpha \in (0, 1]$ that $\text{Ch}\{\xi \geq \xi_{\sup}(\alpha)\} = \lim_{n \rightarrow \infty} \text{Ch}\{\xi \geq \xi_{\sup}(\alpha) - 1/n\} \geq \alpha$ and $\text{Ch}\{\xi \geq \xi_{\sup}(\alpha)\} = \lim_{n \rightarrow \infty} \text{Ch}\{\xi \geq \xi_{\sup}(\alpha) + 1/n\} \leq \alpha$, which imply that $\text{Ch}\{\xi \geq \xi_{\sup}(\alpha)\} = \alpha$ holds. The theorem is proved. \square

Theorem 8. Let ξ be an uncertain random variable and $\alpha \in (0, 1]$. Then, we have

$$\xi_{\inf}(\alpha) = \Phi^{-1}(\alpha). \quad (11)$$

Proof. It follows from Definition 7 immediately. \square

Theorem 9. Let ξ be an uncertain random variable and $\alpha \in (0, 1]$. Then, we have

$$\xi_{\inf}(\alpha) = \xi_{\sup}(1 - \alpha). \quad (12)$$

Proof. It follows from Equation (8) that

$$\text{Ch}\{\xi \geq \xi_{\sup}(1 - \alpha)\} = 1 - \alpha.$$

Thus,

$$\begin{aligned} \text{Ch}\{\xi \leq \xi_{\sup}(1 - \alpha)\} &= 1 - \text{Ch}\{\xi \geq \xi_{\sup}(1 - \alpha)\} \\ &= 1 - (1 - \alpha) \\ &= \alpha \end{aligned}$$

Thus, we have $\xi_{\inf}(\alpha) = \xi_{\sup}(1 - \alpha)$. The theorem is proved. \square

Theorem 10. Let ξ be an uncertain random variable and $\alpha \in (0, 1]$. Then, we have

$$\xi_{\sup}(\alpha) = \Phi^{-1}(1 - \alpha) \quad \text{and} \quad \xi_{\sup}(1 - \alpha) = \Phi^{-1}(\alpha). \quad (13)$$

Proof. It follows from Theorems 8 and 9. \square

Theorem 11. Let ξ be an ordinary uncertain random variable. Then, we have

$$E[\xi] = \int_0^1 \xi_{\inf}(\alpha) d\alpha. \quad (14)$$

Proof. Since $\Phi(x)$ is strictly increasing and continuous, we get

$$\begin{aligned} \xi_{\inf}(\alpha) &= \inf\{r | \text{Ch}\{\xi \leq r\} \geq \alpha\} \\ &= \inf\{r | \Phi(r) \leq \alpha\} \\ &= \Phi^{-1}(\alpha). \end{aligned}$$

According to Definition (), we have $E[\xi] = \int_0^1 \xi_{\inf}(\alpha) d\alpha$. The Theorem is proved. \square

Theorem 12. Let ξ be an ordinary uncertain random variable. Then, we have

$$E[\xi] = \int_0^1 \xi_{\text{sup}}(\alpha) d\alpha. \tag{15}$$

Proof. Since $\Phi(x)$ is strictly increasing and continuous, we get

$$\begin{aligned} \xi_{\text{sup}}(\alpha) &= \sup\{r | \text{Ch}\{\xi \geq r\} \geq \alpha\} \\ &= \sup\{r | \Phi(r) \leq 1 - \alpha\} \\ &= \Phi^{-1}(1 - \alpha). \end{aligned}$$

Thus, for all $\alpha \in (0, 1)$, we have

$$\begin{aligned} \int_0^1 \xi_{\text{sup}}(\alpha) d\alpha &= \int_0^1 \Phi^{-1}(1 - \alpha) d\alpha \\ &= \int_0^1 \Phi^{-1}(\alpha) d\alpha = E[\xi] \end{aligned}$$

The theorem is proved. □

Theorem 13. Let ξ be an ordinary uncertain random variable. Then, we have

$$E[\xi] = \frac{1}{2} \int_0^1 [\xi_{\text{inf}}(\alpha) + \xi_{\text{sup}}(\alpha)] d\alpha. \tag{16}$$

Proof. It follows directly from Theorem 11 and Theorem 12. □

Theorem 14. Let $\eta_1, \eta_2, \dots, \eta_m$ be independent random variables with probability distributions $\Psi_1, \Psi_2, \dots, \Psi_m$, and let $\tau_1, \tau_2, \dots, \tau_n$ be independent uncertain variables with regular uncertainty distributions $\Upsilon_1, \Upsilon_2, \dots, \Upsilon_n$. Assume $f(\eta_1, \eta_2, \dots, \eta_m, \tau_1, \tau_2, \dots, \tau_n)$ is strictly increasing with respect to $\tau_1, \tau_2, \dots, \tau_k$ and strictly decreasing with respect to $\tau_{k+1}, \tau_{k+2}, \dots, \tau_n$. Then

$$\xi = f(\eta_1, \eta_2, \dots, \eta_m, \tau_1, \tau_2, \dots, \tau_n)$$

has a chance distribution

$$\Phi(x) = \int_{\mathbb{R}^m} F(x; y_1, y_2, \dots, y_m) d\Psi_1(y_1) d\Psi_2(y_2) \dots d\Psi_m(y_m). \tag{17}$$

where $F(x; y_1, y_2, \dots, y_m)$ is determined by its inverse uncertainty distribution

$$\begin{aligned} &F^{-1}(y_1, y_2, \dots, y_m, \Upsilon_1^{-1}(\alpha), \dots, \\ &\Upsilon_2^{-1}(\alpha), \dots, \Upsilon_n^{-1}(\alpha)), \end{aligned} \tag{18}$$

$$\begin{aligned} &F^{-1}(y_1, y_2, \dots, y_m, (\xi_1)_{\text{sup}}(1 - \alpha), \dots, \\ &(\xi_k)_{\text{sup}}(1 - \alpha), (\xi_{k+1})_{\text{sup}}(\alpha), \dots, (\xi_n)_{\text{sup}}(\alpha)). \end{aligned} \tag{19}$$

Proof. It follows from Theorem 1 that inverse uncertainty distribution of $F(x; y_1, y_2, \dots, y_m)$ is determined by

$$\begin{aligned} &F^{-1}(y_1, y_2, \dots, y_m, \Upsilon_1^{-1}(\alpha), \dots, \Upsilon_k^{-1}(\alpha), \\ &\Upsilon_{k+1}^{-1}(1 - \alpha), \dots, \Upsilon_n^{-1}(1 - \alpha)). \end{aligned} \tag{20}$$

According to Theorem 10, we substitute $\Upsilon_1^{-1}(\alpha), \dots, \Upsilon_k^{-1}(\alpha)$ with $(\xi_1)_{\text{sup}}(1 - \alpha), \dots, (\xi_k)_{\text{sup}}(1 - \alpha)$ and $\Upsilon_{k+1}^{-1}(1 - \alpha), \dots, \Upsilon_n^{-1}(1 - \alpha)$ with $(\xi_{k+1})_{\text{sup}}(\alpha), \dots, (\xi_n)_{\text{sup}}(\alpha)$. Thus, Formula (19) holds. The theorem is completed. □

Theorem 15. Let $\eta_1, \eta_2, \dots, \eta_m$ be independent random variables with probability distributions $\Psi_1, \Psi_2, \dots, \Psi_m$, and let $\tau_1, \tau_2, \dots, \tau_n$ be independent uncertain variables with regular uncertainty distributions $\Upsilon_1, \Upsilon_2, \dots, \Upsilon_n$. Assume $f(\eta_1, \eta_2, \dots, \eta_m, \tau_1, \tau_2, \dots, \tau_n)$ is strictly increasing with respect to $\tau_1, \tau_2, \dots, \tau_k$ and strictly decreasing with respect to $\tau_{k+1}, \tau_{k+2}, \dots, \tau_n$. Then

$$\xi = f(\eta_1, \eta_2, \dots, \eta_m, \tau_1, \tau_2, \dots, \tau_n)$$

has a chance distribution

$$\Phi(x) = \int_{\mathbb{R}^m} F(x; y_1, y_2, \dots, y_m) d\Psi_1(y_1) d\Psi_2(y_2) \dots d\Psi_m(y_m). \tag{21}$$

where $F(x; y_1, y_2, \dots, y_m)$ is determined by its inverse uncertainty distribution

$$\begin{aligned} &F^{-1}(y_1, y_2, \dots, y_m, \Upsilon_1^{-1}(\alpha), \dots, \\ &\Upsilon_2^{-1}(\alpha), \dots, \Upsilon_n^{-1}(\alpha)) \end{aligned} \tag{22}$$

$$\begin{aligned} &F^{-1}(y_1, y_2, \dots, y_m, (\xi_1)_{\text{inf}}(\alpha), \dots, (\xi_k)_{\text{inf}}(\alpha), \\ &(\xi_{k+1})_{\text{inf}}(1 - \alpha), \dots, (\xi_n)_{\text{inf}}(1 - \alpha)). \end{aligned} \tag{23}$$

Proof. The proof is similar to that of Theorem 14. □

According to Theorems 1, 17, 8, 9 and 10, we design the following uniform discretization algorithm to simulate $\Phi(x)$, α -pessimistic value and α -optimistic value. The presented algorithm is very flexible. Because it can even simulate the empirical distribution.

Algorithm 1 (Uniform Discretization Algorithm)

Step 1. Discretize the range of the random variable η_i into N_i equally spaced points.

Step 2. Discretize α into K equally spaced points.

Step 3. Calculate $F^{-1}(\alpha; y_1, y_2, \dots, y_m)$.

Step 4. Calculate $F(x; y_1, y_2, \dots, y_m) =$

$$\begin{cases} 0 & \text{if } x \leq x_1, \\ \alpha_i + (\alpha_{i+1} - \alpha_i) \frac{x - x_i}{x_{i+1} - x_i} & \text{if } x_i \leq x \leq x_{i+1}, \\ 1 & \text{if } x \geq x_K, \end{cases} \quad 1 \leq i \leq K,$$

Step 5. Apply numerical integration to compute $\Phi(x)$, α -pessimistic value and α -optimistic value.

The uniform discretization algorithm is illustrated by the following example.

4. A numerical example

Example Suppose that η_1 and η_2 are independent random variables with probability distributions $U(1, 2)$ and $U(2, 4)$, and suppose that τ_1 and τ_2 are independent uncertain variables with uncertainty distributions $\mathcal{L}(1, 5)$ and $\mathcal{L}(1, 3)$. Then $\xi = \eta_1 + \eta_2 + \tau_1 - \tau_2$ is an uncertain random variable. Assume that ξ has chance distribution $\Phi(x)$.

For the sake of simplicity, we set $N_1 = 10, N_2 = 10, K = 10$. We also can assign a large number to N_1, N_2 and K . The probability distribution function of η_1 and η_2 are

$$\Psi_1(y_1) = \begin{cases} 0 & \text{if } y_1 \leq 1, \\ y_1 - 1 & \text{if } 1 \leq y_1 \leq 2, \text{ and} \\ 1 & \text{if } y_1 \geq 2, \end{cases}$$

$$\Psi_2(y_2) = \begin{cases} 0 & \text{if } y_2 \leq 2, \\ \frac{y_2 - 2}{2} & \text{if } 2 \leq y_2 \leq 4, \\ 1 & \text{if } y_2 \geq 4. \end{cases}$$

Then, we can have discrete forms of $\Psi_1(y_1)$ and $\Psi_1(y_2)$ in which $y_1 = 1 + 0.1 \cdot i$ and $y_2 = 2 + 0.2 \cdot j$ for $i = 1, 2, \dots, 10, j = 1, 2, \dots, 10$. The inverse uncertainty distribution function of τ_1 and τ_2 are $(\tau_1)_{\text{inf}}(\alpha) = (1 - \alpha) \cdot 1 + \alpha \cdot 5 = 1 + 4 \cdot \alpha$ and $(\tau_2)_{\text{inf}}(1 - \alpha) = \alpha \cdot 1 + (1 - \alpha) \cdot 3 = 3 - 2 \cdot \alpha$.

The chance distribution of ξ is

$$\Phi(x) = \frac{1}{2} \int_2^4 \int_1^2 F(x; y_1, y_2) dy_1 dy_2 \quad (24)$$

where $F(x; y_1, y_2)$ is obtained by the inverse uncertainty distribution function $F^{-1}(\alpha; y_1, y_2) = y_1 + y_2 + (1 + 4 \cdot \alpha) - (3 - 2 \cdot \alpha)$ for each $\alpha \in (0, 1]$. This implies that $F^{-1}(\alpha; y_1, y_2) = x_k, 1 \leq k \leq 10$ for each $\alpha \in (0, 1]$. The value of $F^{-1}(\alpha; y_1, y_2)$ is listed on the Table I.

Then, according to Step 4, we obtain $F(x; y_1, y_2)$. Now, we get the chance distribution function of ξ .

$$\begin{aligned} \Phi(x) &= \frac{1}{2} \int_2^4 \int_1^2 F(x; y_1, y_2) dy_1 dy_2 \\ &= \frac{1}{2} \sum_{i=1}^{10} \sum_{j=1}^{10} F(x; 1 + 0.1 \cdot i, 2 + 0.2 \cdot j) \cdot 0.1 \cdot 0.2 \end{aligned}$$

Applying the above presented algorithm, the chance distribution of ξ is obtained as (Figure 1): $\xi_{\text{inf}}(0.2) = 3.9$ and $\xi_{\text{sup}}(0.2) = 7.4$.

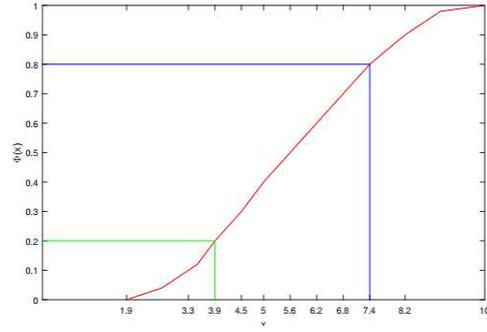


Figure 1. The chance distribution of ξ .

5. Conclusion

Here, an uncertain random simulation algorithm is presented to illustrate chance distribution. The presented simulation method can be generally applied to uncertain random optimization by approximating chance constraints. The results obtained show that the presented algorithm is the successful for the uncertain random simulation.

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