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RESEARCH ARTICLE

On the exact solutions of the fractional (2+1)- dimensional Davey - Stewartson equation system

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ABSTRACT

In this work, we construct the exact traveling wave solutions of the fractional (2+1)-dimensional Davey-Stewartson equation system (D-S) that is complex equation system using the Modified Trial Equation Method (MTEM). We obtained trigonometric function solutions by this method that are new in literature.

1. Introduction

In recent years, fractional differential equations and equation systems have been studied in various fields such as physics, chemistry, engineering and certainly mathematics. To solve these fractional differential equations several analytic and numerical techniques have been used by many researchers [1-3].One of these is Extended Trial Equation Method (ETEM). The method based on the fractional derivative in the sense of modified Riemann-Liouville derivative and traveling wave transformation, the fractional partial differential equation can be turned into the nonlinear nonfractional ordinary differential equation [4]. Bekir et al. constructed the exact solutions of the time fractional Fitzhugh-Nagumo and KdV equations with Exp-function Method [5]. The tanh method and the sine–cosine method have been used for investigate the two-dimensional Kadomtsev–Petviashivilli-modified equal width equation's solitary wave solutions and periodic solutions [6].

In recent years, studying on fractional differentials have been increased. The new fractional derivatives are presented in the literature [7-9]. The new definitions have been submitted to the literature as follows:

Definition 1. Let $f \in H^1(a,b), a<b, \alpha \in [0,1]$ then, the definition of the new Caputo fractional derivative is,

$$D_t^{\alpha}\left[f(t)\right] = \frac{M(\alpha)}{1-\alpha}\int_a^t f'(x)\exp\left(-\alpha\frac{t-x}{1-\alpha}\right)dx. \tag{1}$$

Where $M(\alpha)$ denotes a normalization function obeying $M(0)=M(1)=1.$ However, if the function does not belong to $H^1(a,b)$ then, the derivative has the form:

$$D_t^{\alpha}\left[f(t)\right] = \frac{\alpha M(\alpha)}{1-\alpha}\int_a^t \left[f(t)-f(x)\right] \exp\left(-\alpha\frac{t-x}{1-\alpha}\right)dx. \tag{2}$$

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If $\sigma = (1-\alpha)/\alpha \in [0,\infty], \alpha = 1/(1+\sigma) \in [0,1]$ then Eq. (2) assumes the form:

$$D_t^\alpha [f(t)] = \frac{N(\sigma)}{\sigma}\int_a^t f'(x)\exp\left(-\alpha\frac{t-x}{\sigma}\right)dx,$$
$$N(0) = N(\infty) = 1. \tag{3}$$

Definition 2. (Atangana-Baleanu fractional derivative in Riemann-Liouville sense). Let $f \in H^1(a,b)$, $a < b, \alpha \in [0,1]$ and not necessary differentiable then, the definition of the new fractional derivative is given as:

$${}_a^{ABR}D_t^\alpha (f(t)) = \frac{B(\alpha)}{1-\alpha}\frac{d}{dt}\int_a^t f(x)E_\alpha\left(-\alpha\frac{(t-x)^\alpha}{1-\alpha}\right)dx. \tag{4}$$

Definition 3. (Atangana-Baleanu fractional derivative in Caputo sense). Let $f \in H^1(a,b)$, $a < b, \alpha \in [0,1]$ then, the definition of the new fractional derivative is given as:

$${}_a^{ABC}D_t^\alpha (f(t)) = \frac{B(\alpha)}{1-\alpha}\int_a^t f'(x)E_\alpha\left(-\alpha\frac{(t-x)^\alpha}{1-\alpha}\right)dx. \tag{5}$$

The B has the same properties as in Caputo and Fabrizio case.

Definition 4. (Atangana-Baleanu fractional integral). The fractional integral associate to the new fractional derivative with non-local kernel is defined as:

$${}_a^{AB}I_t^\alpha [f(t)] = \frac{1-\alpha}{B(\alpha)}f(t) + \frac{\alpha}{B(\alpha)\Gamma(\alpha)}\int_a^t f(j)(t-j)^{\alpha-1}dj. \tag{6}$$

When alpha is zero we recover the initial function and if also alpha is 1, we obtain the ordinary integral.

Davey-Stewartson equation is higher dimensional generalizations of the nonlinear Schrodinger (NLS) equation. Davey and Stewartson first derived their model in the context of water waves, from purely physical considerations. In the context, $q(x, y, t)$ is the amplitude of a surface wave packet, while $\varphi(x, y)$ represents the velocity potential of the mean flow interacting with the surface wave [10].

Although the classical Davey-Stewartson equation had been studied many times, the studies for fractional Davey-Stewartson equation was started newly [11-13].

2. Fundamental facts of the modified trial equation method

The modified trial equation method (MTEM) consists of four steps. In this sub-section we present this steps.

Step 1. We consider partial differential equation in two variables and a dependent variable u :

$$P(u, u_t, u_x, u_{xx}, u_{xxx}, ...) = 0, \tag{7}$$

and take the wave transformation,

$$u(x,t) = u(\xi), \xi = kx - ct, \tag{8}$$

where k and c are constants can be determined later. By substituting Eq. (8) into Eq. (7), a nonlinear ordinary differential equation (NODE) is converted as following:

$$N(U, U', U'', U''', ...) = 0. \tag{9}$$

Step 2. Take trial equation as follows:

$$U' = \frac{F(u)}{G(u)} = \frac{\sum_{i=0}^{n} a_i u^i}{\sum_{j=0}^{l} b_j u^j} = \frac{a_0 + a_1 u + a_2 u^2 + \quad + a_n u^n}{b_0 + b_1 u + b_2 u^2 + \quad + b_l u^l}, \tag{10}$$

$$U'' = \frac{F(u)\left(F'(u)G(u) - F(u)G'(u)\right)}{G^3(u)}, \tag{11}$$

where $F(u)$ and $G(u)$ are polynomials. Substituting above relations into Eq. (9) yields an equation of polynomial $\Omega(u)$ of u :

$$\Omega(u) = \rho_s u^s + \quad + \rho_1 u + \rho_0 = 0. \tag{12}$$

According to the balance principle, we can get a relationship between n and l. We can compute some values of n and l.

Step 3. Let the coefficients of $\Omega(u)$ all be zero will yield an algebraic equations system:

$$\rho_i = 0, i = 0,1,2, \quad ,s. \tag{13}$$

By solving this system, we will thus determine the values of $a_0, a_1, \quad , a_n$ and $b_0, b_1, \quad , b_l$.

Step 4. Reduce Eq.(10) to the elementary integral form,

$$\pm(\mu - \mu_0) = \int \frac{G(u)}{F(u)}du. \tag{14}$$

Using a complete discrimination system for polynomial of $F(u)$, we solve Eq.(14) with the help of Mathematica 9 and classify the exact solutions to Eq. (9). For better explications of results obtained in this way, we can plot two and three dimensional surfaces of the solutions obtained by using suitable parameters.

3. Application

We consider the fractional (2+1)-dimensional Davey-Stewartson equation system defines as [14],

$$i\frac{\partial^{\alpha}u}{\partial t^{\alpha}}+\frac{\partial^{2\beta}u}{\partial x^{2\beta}}-\frac{\partial^{2\gamma}u}{\partial y^{2\gamma}}-2|u|^2u-2uv=0,$$

$$\frac{\partial^{2\beta}v}{\partial x^{2\beta}}+\frac{\partial^{2\gamma}v}{\partial y^{2\gamma}}+2\frac{\partial^{2\beta}|u|^2}{\partial x^{2\beta}}=0, \tag{15}$$

where $0<\alpha,\beta,\gamma\le 1, x,y>0$, u and v are the functions of (x,y,t). We apply following transformations then the Eq. (15) can be reduced to ordinary differential equation.

$$u(x,y,t)=e^{i\theta}U(\eta), v(x,y,t)=V(\eta),$$

$$\theta=\left(\frac{px^{\beta}}{\Gamma(1+\beta)}+\frac{qy^{\gamma}}{\Gamma(1+\gamma)}+\frac{ct^{\alpha}}{\Gamma(1+\alpha)}\right)+r,$$

$$\eta=\left(\frac{wx^{\beta}}{\Gamma(1+\beta)}+\frac{\varepsilon y^{\gamma}}{\Gamma(1+\gamma)}+\frac{\lambda t^{\alpha}}{\Gamma(1+\alpha)}\right)+\sigma, \tag{16}$$

where $p,q,c,w,\varepsilon,\lambda$ are nonzero constants and $a\neq 1, b\neq 1, f\neq 1, h\neq 1, \phi\neq 1$ are fractal indexes.

$$\left(h^2q^2-f^2p^2-\phi c\right)U+\left(a^2w^2-b^2\varepsilon^2\right)U''$$
$$-2U^3-2UV=0, \tag{17}$$

$$\left(a^2w^2+b^2\varepsilon^2\right)V''+2\left(U^2\right)''=0. \tag{18}$$

The Eq. (18) is integrated two times and Eq. (1711) is rearranged as this requirement, we have,

$$\left(h^2q^2-f^2p^2-\phi c\right)U+\left(a^2w^2-b^2\varepsilon^2\right)U''$$
$$-2\left(\frac{a^2w^2+b^2\varepsilon^2-2}{a^2w^2+b^2\varepsilon^2}\right)U^3=0. \tag{19}$$

According to balance principle highest order nonlinear terms of U'' and U^3 in Eq.(19), we get the following relationship

$$2n-2l-1=3\Rightarrow n=l+2. \tag{20}$$

This resolution procedure is performed and we can obtain some analytical solutions as follows.

Case 1. If we take $l=0$ and $n=2$ in Eq.(20), we can write following equations:

$$U'=\frac{F(u)}{G(u)}=\frac{\sum_{i=0}^{n}a_iu^i}{\sum_{j=0}^{l}b_ju^j}=\frac{a_0+a_1u+a_2u^2}{b_0}, \tag{21}$$

$$U''=\frac{F(u)\left(F'(u)\,G(u)-F(u)\,G'(u)\right)}{G^3(u)}=$$
$$\frac{\left(a_1+2a_2u\right)\left(a_0+a_1u+a_2u^2\right)}{b_0^2}, \tag{22}$$

where $a_2\neq 0$ and $b_0\neq 0$. When we use Eq. (22) in Eq. (19), we get a system of equations from the coefficients of polynomial of Ω. Solving this algebraic system of equations with the help of Wolfram Mathematica 9 yields the following coefficients,

$$a_1=0, a_2=-\frac{i\sqrt{-2+a^2w^2+b^2\varepsilon^2}\,b_0}{\sqrt{-a^4w^4+b^4\varepsilon^4}},$$
$$a_0=-\frac{i\left(\phi c+f^2p^2-h^2q^2\right)\left(a^2w^2+b^2\varepsilon^2\right)b_0}{2\sqrt{\left(-2+a^2w^2+b^2\varepsilon^2\right)\left(-a^4w^4+b^4\varepsilon^4\right)}}. \tag{23}$$

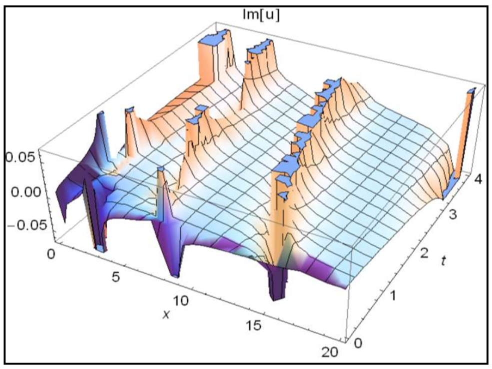
By substituting these coefficients into Eq.(14), we have,

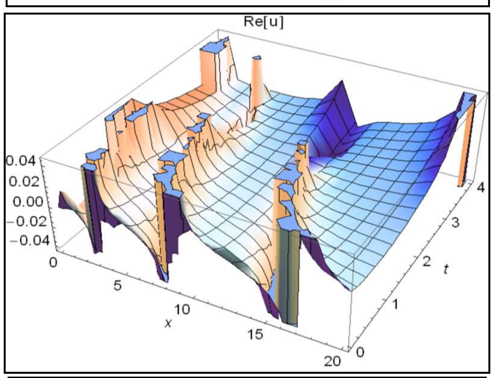
$$\pm\left(\mu-\mu_0\right)=\int\frac{b_0}{a_2u^2+a_1u}du. \tag{24}$$

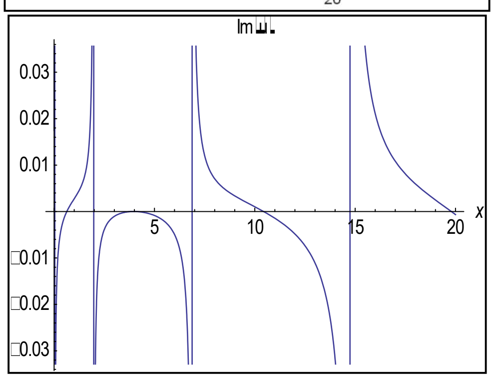
Integrating Eq.(24) by using Wolfram Mathematica 9 and solve equation we obtain the following trigonometric solution of $u(x,t)$ ve $v(x,t)$,

$$u(x,t)=\sqrt{\frac{\left(a^2w^2+b^2\varepsilon^2\right)\left(\phi c+f^2p^2-h^2q^2\right)}{2\left(-2+a^2w^2+b^2\varepsilon^2\right)}}$$
$$e^{i\theta}Tan\left[\sqrt{-\frac{\left(\phi c+f^2p^2-h^2q^2\right)}{2\left(b^2\varepsilon^2-a^2w^2\right)}}\eta\right], \tag{25}$$

$$v(x,t) = -\frac{\left(\phi c + f^2 p^2 - h^2 q^2\right)}{\left(-2 + a^2 w^2 + b^2 \varepsilon^2\right)} \mathrm{Tan}\left[\sqrt{-\frac{\left(\phi c + f^2 p^2 - h^2 q^2\right)}{2\left(b^2 \varepsilon^2 - a^2 w^2\right)}}\eta\right]^2. \quad (26)$$







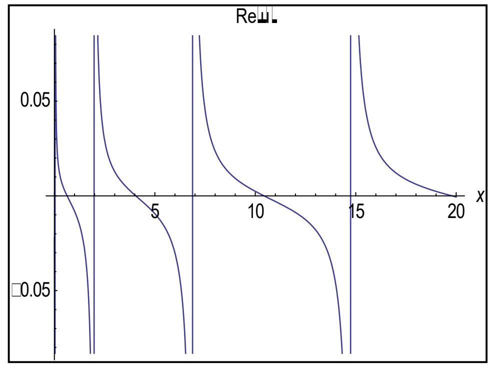
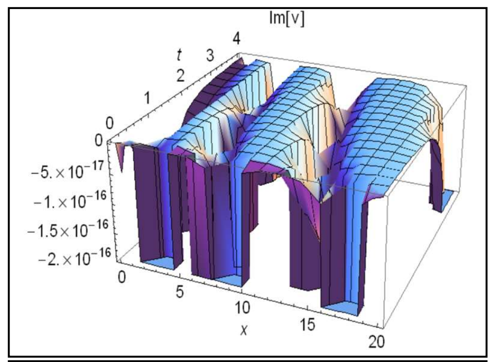
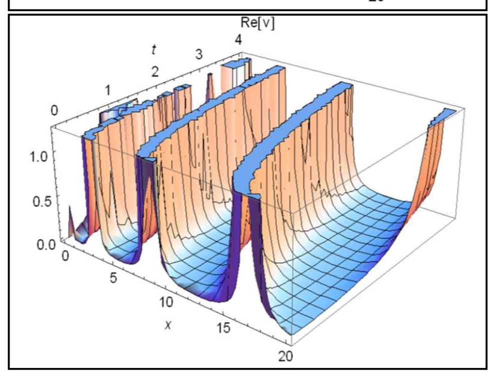
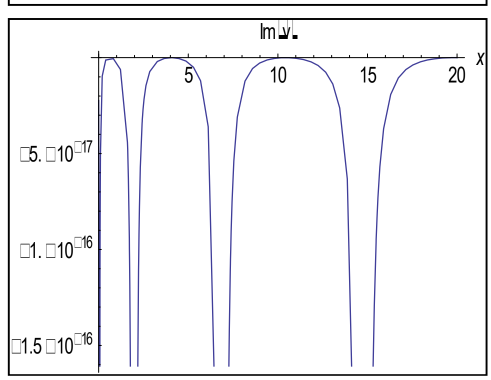


Figure 1. The graphics that has been hold by Eq.(25), $u(x,t)$ corresponding to the values $b_0 = 2, c = 1, \lambda = 0.4, \ w = 0.9, \varepsilon = 0.2, k = 0.04,$ $\alpha = \beta = \gamma = 0.5, \ q = 0.4, \ p = 0.3,$ $\sigma = r = a = b = h = f = \phi = 0.1, t = 0.4,$ $0 < x < 20, 0 < t < 4.$







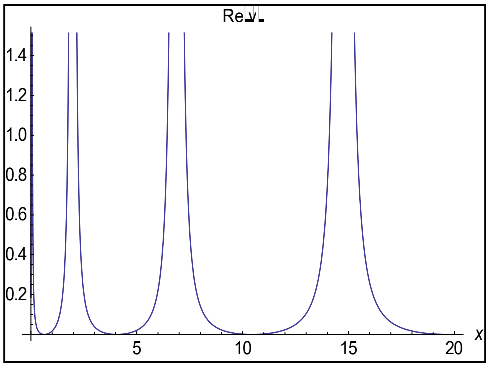


Figure 2. The graphics that has been hold by Eq.(26), $v(x,t)$ corresponding to the values

$$b_0 = 2, c = 1, \lambda = 0.4,\ w = 0.9, \varepsilon = 0.2, k = 0.04,$$
$$\alpha = \beta = \gamma = 0.5,\ t = 0.4\ q = 0.4,\ p = 0.3,$$
$$\sigma = r = a = b = h = f = \phi = 0.1.$$
$$0 < x < 20, 0 < t < 4.$$

4. Conclusion

In this article, the modified trial equation method has been succesfully applied to the fractional (2+1)-dimensional Davey-Stewartson equation system. We have found new trigonometric solution to the Eq.(15). We have given some figures describe the behavior of the obtained solutions of Eq.(15). According to these results, this method is reliable and effective for like this complex equation systems.

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