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A Modified Cellular Automaton Model in Lagrange Form with Velocity Dependent Acceleration Rate

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Abstract. Road traffic micro simulations based on the individual motion of all the involved vehicles are now recognized as an important tool to describe, understand and manage road traffic. With increasing computational power, simulating traffic in microscopic level by means of Cellular Automaton becomes a real possibility. Based on Nasch model of single lane traffic flow, a modified Cellular Automaton traffic flow model is proposed to simulate homogeneous and mixed type traffic flow. The model is developed with modified cell size, incorporating different acceleration characteristics depending upon the speed of each individual vehicle. Comparisons are made between Nasch model and modified model. It is observed that slope of congested branch is changed for modified model as the vehicle that are coming out of jam having dissimilar acceleration capabilities, therefore there is not a sudden drop in throughput near critical density ρ_c .

Keywords: Cellular Automata, Nasch model, braking parameter, slow-to-start rule.

AMS subject classifications: 68Q80, 49M99, 90B99

1. Introduction

Traffic flow problems have attracted considerable attention of researchers because of manifold increase in traffic density in cities [1]. Broadly there are two different approaches for dealing with traffic flow. Macroscopic traffic simulation models incorporate analytical models that deal with average traffic stream characteristics such as flow, speed, density etc. On the other hand microscopic traffic simulation models consider the characteristics of individual vehicles, and their interactions with other vehicles in traffic stream. These models required a large computational power to deal with the realistic traffic flow. Cremer and Ludwing introduced a new type of microscopic model for vehicular traffic, which is capable of reproducing measured macroscopic behavior [2].

This is known as Cellular Automata (CA) model. Evolutionary properties of CA are the properties that are affected by rules. Out of 256 different rules, the rule-184 CA, which is one of the elementary CA, was proposed by Wolfram [3]. Rule-184 CA model is known to represent the minimal model for movement of vehicles in one lane and shows a simple phase transition from free to congested state of traffic flow.

The first traffic cellular automaton model, Nagel-Schrekenberg model known as Nasch model, successfully reproduces typical properties of real traffic [4]. There has been continuous

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evolution of CA models for traffic flow to examine and study the various traffic features under realistic conditions [5]. In recent year CA models have been used to model complex traffic systems such as ramps and crossings [6] and signal controlled traffic system [7, 8]. In the recent years some attempts have made to implement CA models for heterogeneous traffic by modifying cell size and updating CA rules for traffic flow [9]. Mallikarjuna and Rao studied the suitability of different available CA based models for mixed traffic [10]. Reduced cell size is used to incorporate real traffic situations in a single lane traffic CA model in our previous paper [11]. The study shows the effect of s-t-s rule along with anticipation rule over throughput in Nasch model with reduced cell size and variable acceleration rate.

In the present paper single lane traffic Cellular Automaton model based on Nasch model is discussed. Cell size is reduced and acceleration rate is changed such that it depends upon the speed of each individual vehicle. Under this fine discretization we can describe the vehicle moving process more properly. The cell size is actual vehicle dimension plus the safe distance with the leading vehicle in jam condition. Physical length of these vehicles is given in Table 1. Slow-to-Start rule used in Lagrange model for single lane traffic simulation is implemented to velocity dependent acceleration rate CA model [12]. S-t-s rule in Lagrange model is applicable to all vehicles in traffic stream. We investigate the effect of s-t-s rule over throughput and a comparison between Nash model and modified model is carried out using simulation.

Table 1. Physical length of vehicle

S.No.	Type of vehicle	Actual length in meters	Design length in meters	Cell size in cells
1.	Light vehicle	3.72	6	10
2.	Heavy vehicle	7.5	10	20

2. Basic concept and early work

2.1. Cellular Automata

CA consists of finite, regular grid of cells, each in one of the finite number of states. The grid can be of any number of finite dimensions. For each cell there is neighborhood that locally determines the evolution of the cell. The size of the neighborhood is the same for each cell in the lattice. A one-dimensional Cellular Automata consists of a line of sites with each site carrying a value 0 or 1. The site value evolves synchronously in discrete time steps according to the value of their nearest neighborhood. These values are updated in a sequence of discrete time space according to finite fixed rules. Each time the rules are applied to the whole grid and a new generation is produced. With the help of Cellular Automata microscopic and macroscopic traffic flow parameters and their interaction can be studied, driver's behavior can be incorporated properly through probability.

2.2. Nagel Schrekenberg model for traffic flow

Nagel-Schrekenberg model popularly known as Nasch model is one dimensional Cellular Automata model for single lane. This model explicitly includes a stochastic noise terms to it's rules. In Nasch model road is subdivided into cells of same size (δx=7.5 meters). Each cell is either empty or occupied by 1 vehicle with a discrete speed v varying from 0 to $V_{\max}$, with $V_{\max}$ the maximum speed of vehicle. In this model vehicles are assumed as anisotropic particles, i.e. they only respond to frontal stimuli. The motion of the vehicle is described by the following rules:

Rule1: Acceleration: $v_i^{(1)}=\min\{v_i^{(0)}+1, V_{max}\}$

Rule2: Deceleration: $v_i^{(2)}=\min\{v_i^{(1)}, x_{i+1}^t - x_i^t - 1\}$

Rule3: Randomization: $v_i^{(3)}=\max\{v_i^{(2)}-1, 0\}$

With randomization parameter p;

Rule4: Movement: $x_i^{t+1}= x_i^t+ v_i^{(3)}$

Where $x_{i+1}^t - x_i^t - 1$, the number of empty cells in front of i^{th} vehicle at time t and is called distance headway. x_i^t is the position of the i^{th} vehicle at time t. A time step of $\delta t = 1\sec$, the typical reaction time of driver with a maximum speed $V_{\max}$ = 5 cells/time step i.e.135 Km./Hour is taken in this model. Nasch model contains the rule of randomization that introduces stocasticity in the system. At each time step a random number between 0 and 1 is drawn from a uniform distribution. This number is then compared with a stochastic noise parameter p between 0 and 1; as a result there is a probability p, that a vehicle will slowdown it's velocity by 1.

2.3. Slow-to-start rule in Nasch model

In order to obtain a correct behavioral picture of traffic flow breakdown and stable jam, it is necessary that a vehicle's minimum headway or reaction time should be smaller than it's escape time from a jam. This reduced outflow can be accomplished by making vehicles wait a short while longer before accelerating again from stand still. As such they are said to be slow-to start.

2.3.1 Takayasu Takayasu (T^2) model

Takayasu and Takayasu proposed TCA model that incorporated a delay in acceleration for stopped vehicles [13]. According to s-t-s rule given in this model, a vehicle with a space gap of just one cell will remain stop in next time step with slow-to-start probability q.In T^2 model, Acceleration rule i.e. rule 1 of Nasch model is modified as:

If $v_i^{(0)}=0$ and $x_{i+1}^t - x_i^t - 1 = 1$

Rule1: Acceleration: $v_i^{(1)}=\min\{v_i^{(0)}+1, V_{\max}\}$

with probability $(1-q)$

The rest of the features of the T^2 model are exactly the same as NaSch model.

2.3.2 Benjamin-Johnson-Hui model

Benjamin, Johnson and Hui constructed another type of TCA model, using a s-t-s that is temporal in nature [14]. In this model Nasch model is extended with a rule that adds a small delays to a stopped vehicle that is pulling away from the downstream front of a queue. According to this model, only those vehicles which stopped due to a vehicle ahead of them and is not stopped due to randomization will remain stopped in next time step with a s-t-s probability q. Mathematicaly this rule is represented as:

If $v_i^{(0)}=0$ and $x_{i+1}^t - x_i^{t-1} - 1 = 0$

$v_i^{(1)}=v_i^{(2)}=v_i^{(3)}=0$ with s-t-s probability q

2.3.3 Velocity Dependent Randomization (VDR) Model

In VDR model, the s-t-s rule is generalized by applying an intuitive s-t-s rule for stopped vehicles [15]. According to VDR model only those vehicles which had 0 speed in previous time step either blocked by leading vehicle or due to random deceleration will remain stationary in next time step with s-t-s probability q. Depending on their speed, vehicles are subject to different randomizations. Typical metastable behavior results when s-t-s probability q is much higher than braking probability p, meaning that stopped vehicles have to wait longer before they can continue their journey. In VDR model, to implement slow-to-start effects, rule 3 i.e. randomization rule of Nasch model is modified as:

Rule3: Randomization: If $v_i^{(0)}(t-1)=0$

$v_i^{(3)}(t)=0$

Rest of the rules are same as in Nasch model.

3. Modified cell size and variable acceleration rate

In Nasch model and other previous models a definite cell size of 7.5 meter was taken for all type of vehicles and acceleration rate is assumed to be constant i.e. 1 cell/sec^2 for all type of vehicles. This means all the vehicles on the road have the same acceleration rate, which does not correspond with real situation. When modeling realistic traffic stream that consists of different type of vehicles, having variable speed and acceleration, finer discretisation is useful. Cell size is reduced and acceleration rate is chosen such that it depends upon the speed of each individual vehicle. Under this fine discretization we can describe the vehicle moving process more properly. Cell size is reduced to 0.5 meters and a light vehicle occupies 10 cells with $V_{max} = 60$ cells which correspond to 108 km/h whereas heavy vehicle occupies 20 cells with $V_{max} = 40$ cells which corresponds to 72 km/h. With these characteristics, distance headway for i^{th} light vehicle is $x_{i+1}^t - x_i^t - 10$ and for i^{th} heavy vehicle is $x_{i+1}^t - x_i^t - 20$.

Rule 1 i.e. acceleration rule of Nasch model is modified as:

$$\text{Rule1 : Acceleration : } v_i^{(t+\delta t/3)} = \min\{v_i^{(t)} + a, V_{max}\}$$

Where acceleration a is determined as follows:

If n^{th} vehicle is light vehicle:

$$a = \begin{cases} \dfrac{V_{max}}{15}, & \text{if } v_n <= \dfrac{V_{max}}{4} \\ \dfrac{V_{max}}{20}, & \text{if } \dfrac{V_{max}}{4} < v_n <= \dfrac{V_{max}}{2} \\ \dfrac{V_{max}}{30}, & \text{if } v_n > \dfrac{V_{max}}{2} \end{cases}$$

If n^{th} vehicle is heavy vehicle:

$$a = \begin{cases} \dfrac{V_{max}}{30}, & \text{if } v_n <= \dfrac{V_{max}}{4} \\ \dfrac{V_{max}}{60}, & \text{if } v_n > \dfrac{V_{max}}{4} \end{cases}$$

4. Velocity dependent acceleration rate CA model with implementation of s-t-s rule

In section 3 different type of s-t-s rules that have been implemented to Nasch model is discussed. Here we investigate the effect of implementation of s-t-s rule given in Lagrange model with anticipation parameter $S = 1$ over throughput in a stochastic CA model with reduced cell size and velocity dependent acceleration rate. We choose s-t-s rule described in Lagrange traffic flow model in the present study for the reason that it does not affect only stationary vehicles but all the vehicles with s-t-s probability q.

Rule1: Acceleration: $v_i^{(1)}=\min\{v_i^{(0)}+a, V_{max}\}$

Rule2(a): Slow to Start Rule:

$v_i^{(2)}=\min\{v_i^{(1)}, x_{i+1}^{t+1} - x_i^{t-1} - sz\}$

with s-t-s probability q

Rule2(b): Deceleration Rule:

$v_i^{(3)}=\min\{v_i^{(2)}, x_{i+1}^{t} - x_i^{t} - sz\}$

with braking probability p

Rule3: Randomization Rule:

$v_i^{(4)}=\max\{v_i^{(3)}-1, 0\}$

Rule4: Movement: $x_i^{t+\delta t} = x_i^t + v_i^{(4)}$

In these rules x_i^t is Lagrange variable that denotes the position of i^{th} vehicle at time t. sz is the size of vehicle in term of cells whether it is light or heavy vehicle and is given in table 1. We use parallel scheme where these rules are applied all vehicles simultaneously. $v_i^{(4)}$ becomes $v_i^{(0)}$ in the next time step. Rule 2(a) states that slow to start effect is on with probability q.

5. Numerical Simulation

The basic feature of this model is the relation between density and flow i.e. $q = \rho v$, where v is the average velocity. Under the periodic boundary conditions, the number of vehicles N is conserved. The road is divided into L identical cells. In the present model cell size is modified. The length of each cell is 0.5 meters. The time interval δt is taken 1 second, the typical driver's reaction time. In our simulation process, the number of cells is 10,000 i.e. equivalent to a 5 Km .road. When we started to perform numerical simulation, all vehicles with given density were initially arranged randomly on the whole lane. Figure 1 is the flow chart showing how the vehicles set their velocity according as the new updated rules. After a transient period of 10,000 time steps, we recorded value of traffic flow q (No. of vehicles moving ahead per unit time step) at different densities ρ for various values of q (slow to start probability) keeping braking probability $p = 0.1$.

The computational formulas used in numerical simulation are given as follows:

$$\bar{p}_i = \frac{1}{T}\sum_{t=1}^{T} n_i(t) \qquad (1)$$

$$\bar{q}_i = \frac{1}{T}\sum_{t=1}^{T} m_i(t) \qquad (2)$$

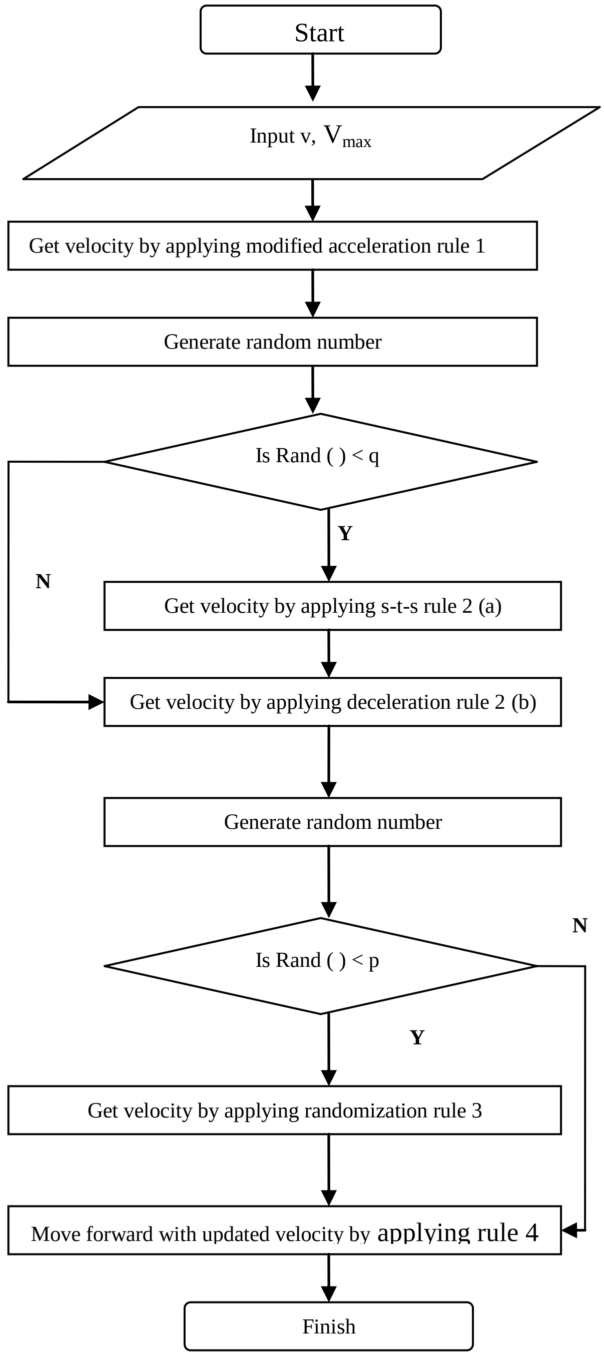

Figure 1. Flow chart for setting vehicle movement

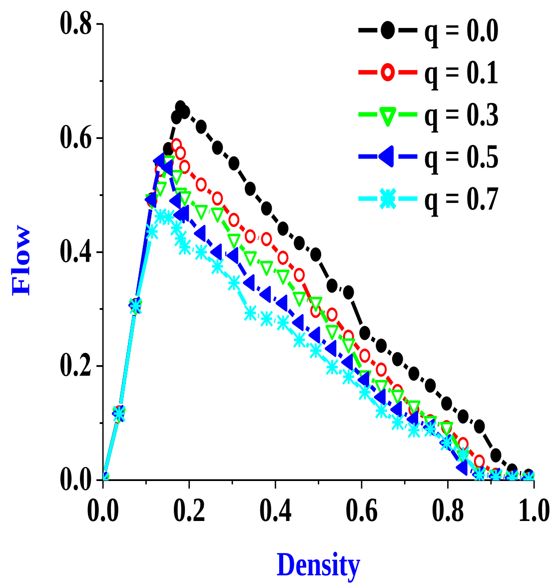

Figure 2 (a). Flow-density relationship of modified CA model at different values of s-t-s probability q .

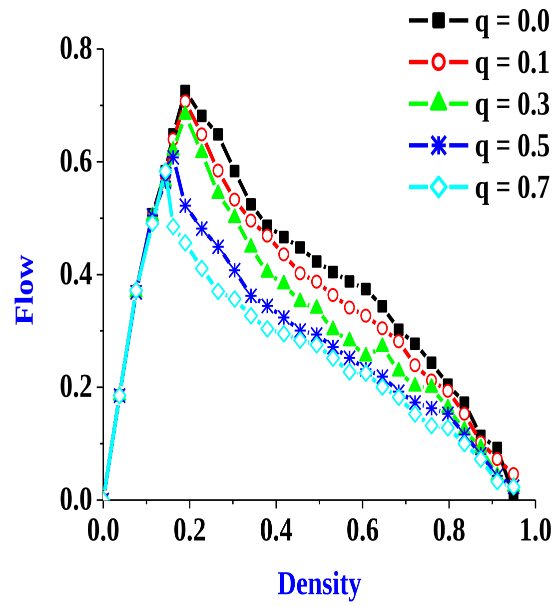

Figure 2 (b). Flow-density relationship of Nasch CA model at different values of s-t-s probability q .

Where $n_i(t) = 1$ if ith vehicle moves ahead at a given time step other wise 0. Density and flow is measured averaged over a time period of T .

6. Results and Discussions

Figure 2(a) is the fundamental diagram of modified CA model with homogeneous traffic which incorporates slow-to-start behavior in single lane traffic flow. For comparison fundamental diagram of Nasch model is shown in Figure 2(b). Parameter p, probability of stochastic braking that measures intrinsic fluctuations among vehicles is taken 0.1. When $q = 0.8$, free flow break down occurs at low density ($\rho \approx 0.14$). For s-t-s probability $q = 0.0$, the point of maximum throughput shifted to density $\rho \approx 0.18$. With higher values of parameter q, there will always be jam in high density region, which does not contribute to flux, as a result flow decreases linearly with density ρ. Figure 2(b) shows steeper free flow branches in comparison to Figure 1 (a) because of variable acceleration rate. Vehicles that are coming out of jam have variable acceleration rates depending upon their speed. It is also found by fundamental diagrams of two models, that modified model leads lower value of maximum flow than obtained from Nasch model. It is due to that for modified model maximum speed limit $V_{\max}$ is 60cells/s which corresponds to a speed limit of 108 km/h. Whereas in Nasch model this maximum speed limit $V_{\max}$ is 5 cells/s which corresponds to 135 km/h.

Figure 3 (a) and 3(b) are the plot of average velocity against density for modified model with homogeneous traffic and Nasch model respectively at different values of s-t-s probability q . In absence of s-t-s rule, average speed converges to maximum speed $V_{\max}$ in free flow regime. Once density ρ surpasses critical density ($\rho_c \approx 0.18$), average speed becomes decreasing function of density. Speed variance near critical density is observed more in case of Nasch model than in modified model. This is again dissimilar acceleration rate of vehicles coming out from a jam. Fall in average velocity near critical density occurs more drastically in Nasch model than in modified model. The spatio temporal pattern with different values of parameter q is presented in Figure 4(a)-4(f). It is found that as q increases, small jams on the road transform into wide jams. This is to say that more vehicles will stop forming successive jams and free flow transforms into stop and go jam.

With further increase in density, there are series of jam forming on the road increases. In high density region, as outflow from a jam is not very large, the small width jams can not dissolve and merge into a wide jam.

Throughput and maximum flow q_{max} of single lane decreases in presence of 10% heavy vehicle as shown in Figure 5(a). It is attributed to the large size of heavy vehicle and their low speed limit and therefore low acceleration rate. Fall in Average speed of traffic stream observed even in free flow regime as shown in Figure 5 (b). This effect is due to mixed type traffic. Same effect is observed in spatio temporal pattern given in Figure 6(a)-6(d). Jams become wider because of low speed limit of heave vehicles.

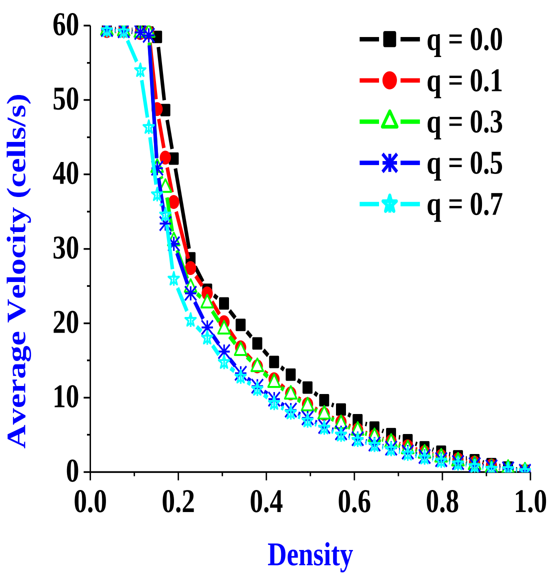

Figure 3 (a). Average velocity-density relationship of modified CA model at different values of s-t-s probability q .

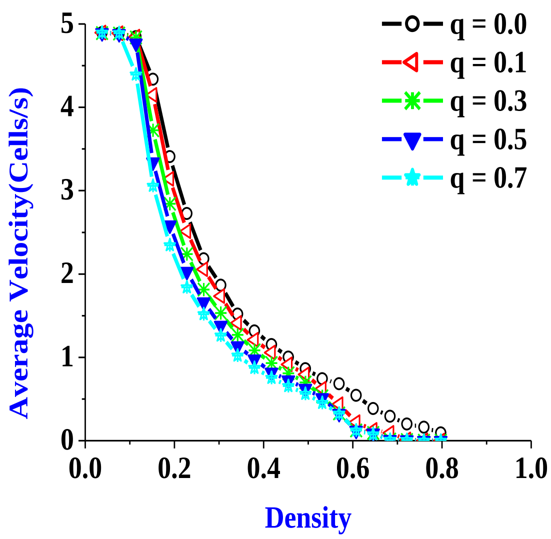

Figure 3 (b). Average velocity-density relationshipof Nasch model at different values of s-t-s probability q .

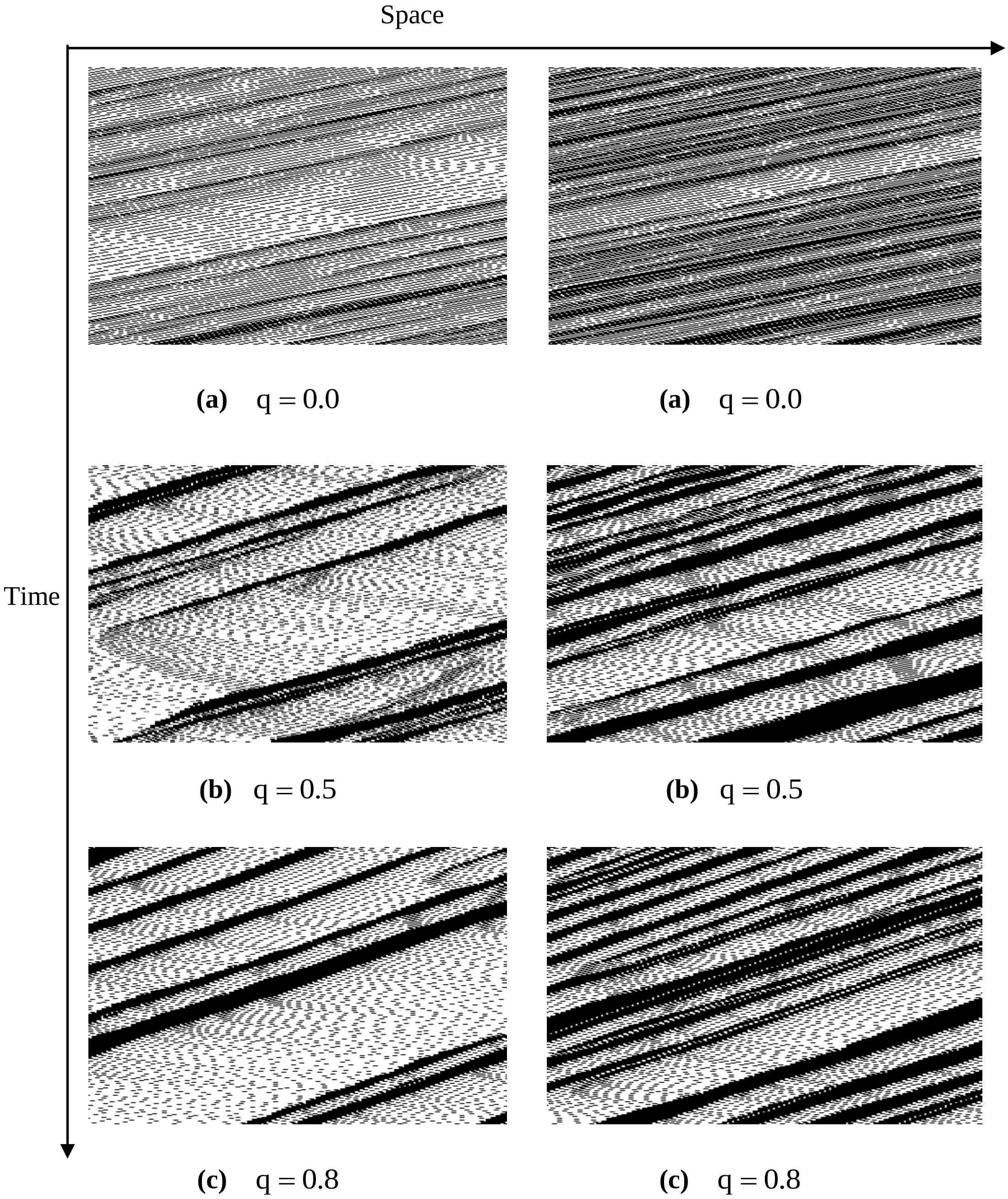

Figure 4. Spatio temporal pattern of simulation of velocity dependent acceleration rate CA model with one type of vehicle at $\rho = 0.34$

Figure 5. Spatio temporal pattern of simulation Of velocity dependent acceleration rate CA model with one type of vehicle.at $\rho = 0.57$

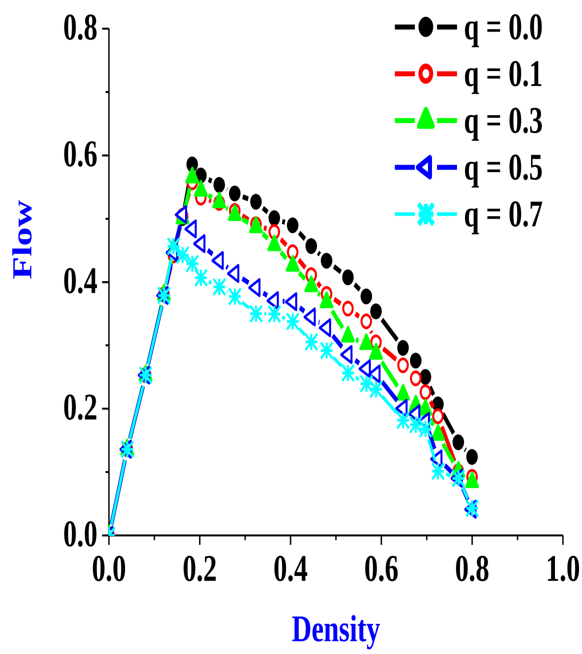

Figure 6 (a). Flow-Density Relationship of modified CA model with 10% heavy vehicle at different values of s-t-s probability q .

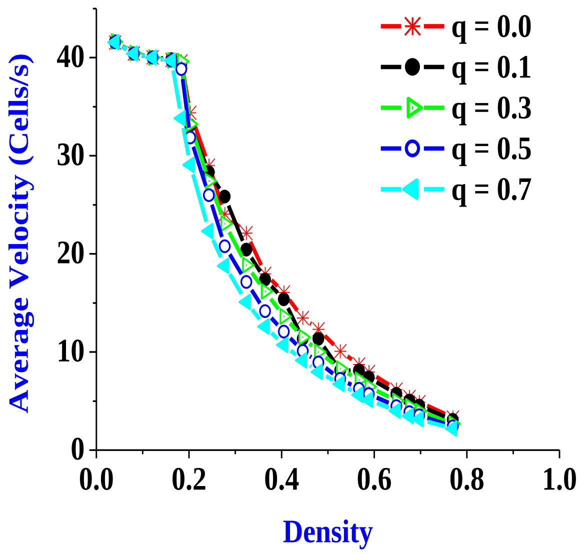

Figure 6 (b). Average velocity-Density relationship of modified CA model with 10% heavy vehicle at different values of s-t-s probability q .

7. Conclusion

Effect of slow-to-start behavior among vehicles on a single lane road using one dimensional TCA model based on Nasch model is discussed in present paper. Cell size is reduced and velocity dependent acceleration rate is taken into account to simulate homogeneous and mixed type traffic flow. Simulation result shows that S-t-s rule incorporated in present study along with variable acceleration rate can reproduce jammed flow in high density region. S-t-s effect over traffic flow is realistic in the manner that it affects all the vehicles. Comparisons have been made between modified model and Nasch model and traffic flow mechanism has been analyzed. Furthermore it is observed that fundamental diagram obtained by numerical simulation of Nasch model are steeper than that of modified model indicating that vehicles coming out from a jam have variable acceleration capabilities depending upon their speed, as a result there is not a sudden drop in throughput near critical density. Present model is rather powerful in dealing with realistic traffic flow phenomena, because it takes into account velocity dependent acceleration rate. Simulating traffic flow by small cell size CA model captures minute variability in real traffic flow.

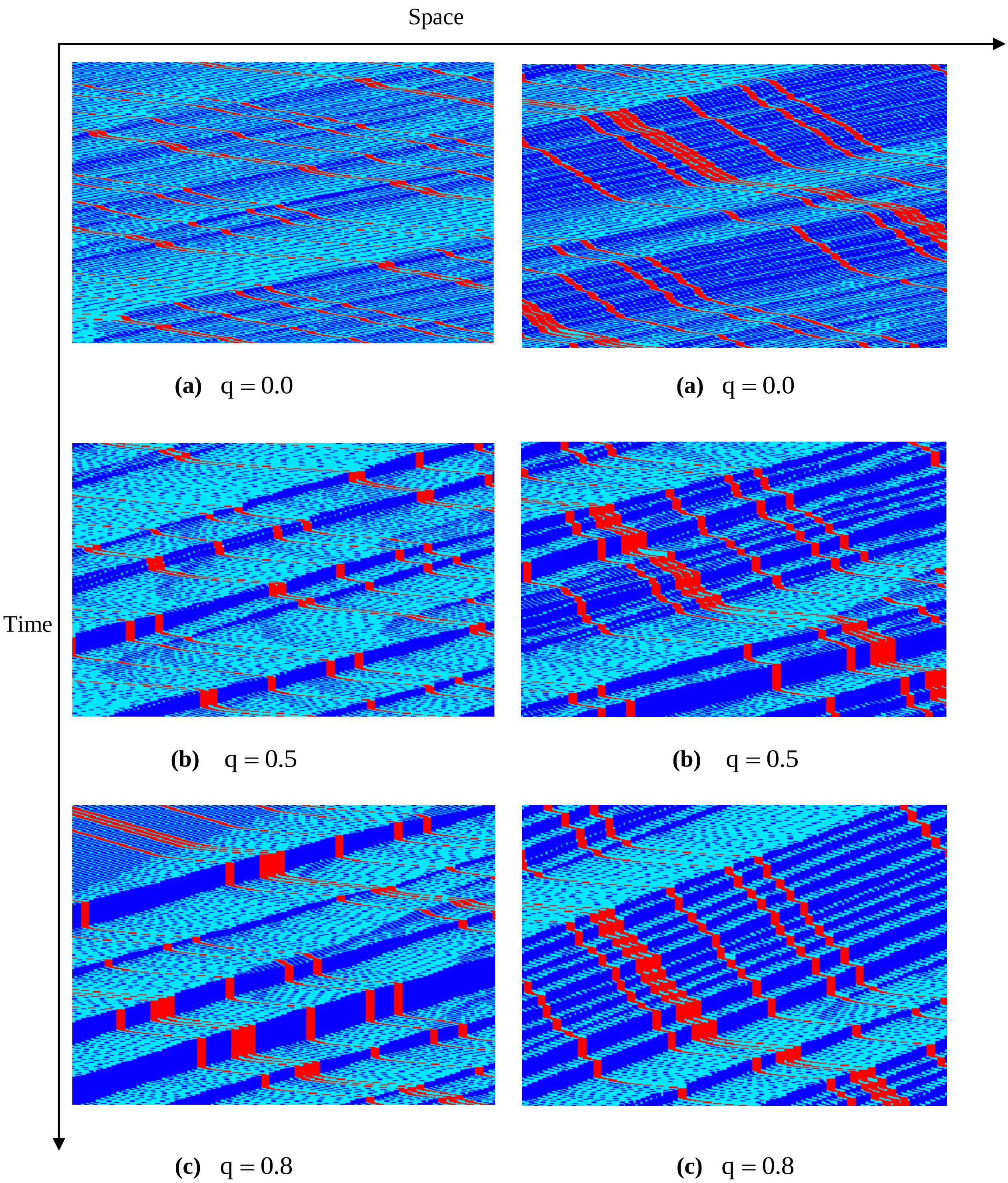

Figure 7. Spatio temporal pattern of simulation of modified CA model with 10% heavy vehicle at $\rho = 0.4$.

Figure 8. Spatio temporal pattern of simulation of modified CA model with 10% heavy vehicle at $\rho = 0.6$

Acknowledgments

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