An International Journal of Optimization and Control: Theories & Applications
ISSN: 2146-0957 eISSN: 2146-5703
Vol.7, No.1, pp.42-49 (2017)
https://doi.org/10.11121/ijocta.01.2017.00292

RESEARCH ARTICLE

New soliton solutions of the system of equations for the ion sound and Langmuir waves

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ARTICLE INFO

Article history:
Received: 14 January 2016
Accepted: 22 August 2016
Available Online: 29 November 2016

Keywords:
The system of equations
Ion sound wave
Langmuir wave
Generalized Kudryashov method
Dark soliton solutions
Mathematica Release 9

AMS Classification 2010:
35-04, 35C08, 35N05, 68N15

ABSTRACT

This study is based on new soliton solutions of the system of equations for the ion sound wave under the action of the ponderomotive force due to high-frequency field and for the Langmuir wave. The generalized Kudryashov method (GKM), which is one of the analytical methods, has been tackled for finding exact solutions of the system of equations for the ion sound wave and the Langmuir wave. By using this method, dark soliton solutions of this system of equations have been obtained. Also, by using Mathematica Release 9, some graphical simulations were designed to see the behavior of these solutions.

1. Introduction

Nonlinear evolution equations are widely used as models to define large numbers of physical phenomena [1-4]. The system of equations for the ion sound wave under the action of the ponderomotive force due to high-frequency field and for the Langmuir wave, which is one type of nonlinear evolution equations, will be handled in this work. The investigation of new soliton solutions for the ion sound wave and the Langmuir wave has a highly important position among the authors. A number of researchers have focused on the Langmuir solitons. L. M. Degtyarev et al. have tackled some properties of Langmuir solitons [5]. Then, they have considered the Langmuir wave energy dissipation [6]. Some scientists have found the numerical simulations of Langmuir collapse [7-10]. E. S. Benilov has indicated the stability of solitons by using the Zakharov equations which defines the interaction between Langmuir and ion-sound waves [11].

V. E. Zakharov et al. have presented the modelling of Langmuir turbulence [12]. A. I. Dyachenko et al. have done computer simulations of Langmuir collapse [13]. A. M. Rubenchik et al. have handled strong Langmuir turbulence in laser plasma [14]. S. L. Musher et al. have introduced weak Langmuir turbulence [15]. Also, some scholars have focused on Langmuir waves [16-18]. I. Y. Dodin et al. have investigated Langmuir wave evolution in nonstationary plasma [19]. A. Zavlavsky et al. have presented spatial localization of Langmuir waves [20]. Also, Langmuir wave spectral energy densities have been derived from the electric field and compared to the weak turbulence results by H. Ratcliffe et al. [21].

We introduce the system of equations for the ion sound wave under the action of the ponderomotive force due to high-frequency field and for the Langmuir wave [22],

$$i\frac{\partial E}{\partial t}+\frac{1}{2}\frac{\partial^2 E}{\partial x^2}-nE=0,$$
$$\frac{\partial^2 n}{\partial t^2}-\frac{\partial^2 n}{\partial x^2}-2\frac{\partial^2 |E|^2}{\partial x^2}=0, \qquad (1)$$

where $Ee^{-iw_p t}$ is the normalized electric field of the Langmuir oscillation and n is the normalized density perturbation. The spatial variable x and the time variable t are also normalized appropriately [22]. The system of equations Eq. (1) for the ion sound and Langmuir waves has been formulated by V. E. Zakharov [23].

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In this paper, the basic interest is to construct the new soliton solutions of the system of equations for the ion sound and Langmuir waves by performing GKM. In Sec. 2, we discuss general structure of GKM [24-29]. In Sec. 3, we get dark soliton solutions of the system of equations for the ion sound and Langmuir waves by implementing GKM.

2. Basic facts of the GKM

We survey a common nonlinear partial differential equation (NLPDE)

$$P\left(u,u_t,u_x,u_{xx},u_{xxx},\cdots\right)=0. \tag{2}$$

Step 1. Initially, we must perform the travelling wave solution of Eq.(2) as following form;

$$u\left(x,t\right)=e^{i\theta}u\left(\xi\right),\ \theta=kx+mt,\ \xi=px+rt, \tag{3}$$

where k,m,p and r are arbitrary constants. Eq.(2) was reduced to a nonlinear ordinary differential equation:

$$N(u,u',u'',u''',\cdots)=0, \tag{4}$$

where the prime denotes differentiation with regard to ξ.

Step 2. Suggest that the exact solutions of Eq.(4) can be tackled as follows;

$$u\left(\xi\right)=\frac{\sum_{i=0}^{N}a_iQ^i\left(\xi\right)}{\sum_{j=0}^{M}b_jQ^j\left(\xi\right)}=\frac{A\left[Q(\xi)\right]}{B\left[Q(\xi)\right]}, \tag{5}$$

where Q is $\frac{1}{1\pm e^{\xi}}$. We highlight that the function Q is solution Eq. (6)

$$Q_\xi=Q^2-Q. \tag{6}$$

Taking account of Eq.(5), we gain

$$u'\left(\xi\right)=\frac{A'Q'B-AB'Q'}{B^2}=Q'\left[\frac{A'B-AB'}{B^2}\right]=\left(Q^2-Q\right)\left[\frac{A'B-AB'}{B^2}\right], \tag{7}$$

$$u''\left(\xi\right)=\frac{Q^2-Q}{B^2}\left[(2Q-1)\left(A'B-AB'\right)\right]+\frac{\left(Q^2-Q\right)^2}{B^3}\left[B\left(A''B-AB''\right)-2B'A'B+2A\left(B'\right)^2\right], \tag{8}$$

$$\begin{aligned}u'''\left(\xi\right)&=\left(Q^2-Q\right)^3\left[\frac{\left(A'''B-AB'''-3A''B'-3B''A'\right)B}{B^3}\right]\\&+\left(Q^2-Q\right)^3\left[\frac{6\left(AB''+B'A'\right)}{B^2}-\frac{6A\left(B'\right)^3}{B^4}\right]\\&+3\left(Q^2-Q\right)^2\left(2Q-1\right)\left[\frac{B\left(A''B-AB''\right)-2B'A'B+2A\left(B'\right)^2}{B^3}\right]\\&+\left(Q^2-Q\right)\left(6Q^2-6Q+1\right)\left[\frac{A'B-AB'}{B^2}\right].\end{aligned} \tag{9}$$

Step 3. The solution of Eq.(4) can be expressed as follows:

$$u\left(\xi\right)=\frac{a_0+a_1Q+a_2Q^2+\cdots+a_NQ^N+\cdots}{b_0+b_1Q+b_2Q^2+\cdots+b_MQ^M+\cdots}. \tag{10}$$

To compute the values M and N in Eq.(10) that is the pole order for the general solution of Eq.(4), we develop comparably as in the classical Kudryashov method on balancing the highest order nonlinear terms in Eq.(4) and we can establish a relation of M and N. We can find values of M and N.

Step 4. Substituting Eq.(5) into Eq.(4) ensures a polynomial $R(Q)$ of Q. Extracting the coefficients of $R(Q)$ to zero, we get a system of algebraic equations. Solving this system, we can identify c and the variable coefficients of $a_0,a_1,a_2,\cdots,a_N,b_0,b_1,b_2,\cdots,b_M$. Thus, we gain the exact solutions to Eq.(4).

3. GKM for the system of equations for the ion sound and the Langmuir waves

In this section, we seek the exact solutions of the system of equations for the ion sound and Langmuir waves by using GKM.

In an effort to find travelling wave solutions of the Eq. (1), we get the transformation by use of the wave variables

$$E\left(x,t\right)=e^{i\theta}u\left(\xi\right),\ n\left(x,t\right)=v\left(\xi\right),\quad \theta=kx+mt,\ \xi=px+rt, \tag{11}$$

where k,m,p and r are arbitrary constants.

Inserting Eqs. (12-14) into Eq. (1),

$$iE_t=-me^{i\theta}u+ir\,e^{i\theta}u', \tag{12}$$

$$E_{xx}=-k^2e^{i\theta}u+2ipk\,e^{i\theta}u'+p^2e^{i\theta}u'', \tag{13}$$

$$n_{tt}=r^2v'',\ n_{xx}=p^2v'',\ \left(\left|E\right|^2\right)_{xx}=p^2\left(u^2\right)'', \tag{14}$$

we obtain the following system

$$i(r+pk)u'(\xi)=0, \tag{15}$$

$$p^2u''-\left(2m+k^2\right)u-2uv=0, \tag{16}$$

$$\left(r^2-p^2\right)v''-2p^2\left(u^2\right)''=0. \tag{17}$$

By setting the integration constant to zero, we integrate function v with respect to ξ, we find

$$v(\xi)=\frac{2p^2}{r^2-p^2}u^2(\xi). \tag{18}$$

Putting Eq.(18) into Eq.(16) and by using Eq. (15), we gain

$$p^2\left(k^2-1\right)u''-\left(k^2-1\right)\left(2m+k^2\right)u-4u^3=0, \tag{19}$$

where the prime remarks the derivative with respect to ξ.

Substituting Eqs. (5) and (8) into Eq. (19) and balancing the highest order nonlinear terms of u'' and u^3 in Eq. (19), then the following formula is found

$$N-M+2=3N-3M\Rightarrow N=M+1. \tag{20}$$

If we take $M=1$ so $N=2$,then

$$u(\xi)=\frac{a_0+a_1Q+a_2Q^2}{b_0+b_1Q}, \tag{21}$$

$$u'(\xi)=\left(Q^2-Q\right)\left[\frac{\left(a_1+2a_2Q\right)}{\left(b_0+b_1Q\right)}\right]-\left(Q^2-Q\right)\left[\frac{b_1\left(a_0+a_1Q+a_2Q^2\right)}{\left(b_0+b_1Q\right)^2}\right], \tag{22}$$

$$\begin{aligned}u''(\xi)=&\frac{Q^2-Q}{\left(b_0+b_1Q\right)}(2Q-1)\left(a_1+2a_2Q\right)\\&-\frac{Q^2-Q}{\left(b_0+b_1Q\right)^2}(2Q-1)\left[b_1\left(a_0+a_1Q+a_2Q^2\right)\right]\\&+\frac{\left(Q^2-Q\right)^2}{\left(b_0+b_1Q\right)^2}\left[2a_2\left(b_0+b_1Q\right)-2b_1\left(a_1+2a_2Q\right)\right]\\&+\frac{\left(Q^2-Q\right)^2}{\left(b_0+b_1Q\right)^3}\left[2b_1^2\left(a_0+a_1Q+a_2Q^2\right)\right],\end{aligned} \tag{23}$$

$$\begin{aligned}u'''(\xi)=&\left(Q^2-Q\right)\left(6Q^2-6Q+1\right)\left[\frac{\left(a_1+2a_2Q\right)}{\left(b_0+b_1Q\right)}\right]\\&-\left(Q^2-Q\right)\left(6Q^2-6Q+1\right)\left[\frac{b_1\left(a_0+a_1Q+a_2Q^2\right)}{\left(b_0+b_1Q\right)^2}\right]\\&+3\left(Q^2-Q\right)^2(2Q+1)\left[\frac{2a_2\left(b_0+b_1Q\right)-2b_1\left(a_1+2a_2Q\right)}{\left(b_0+b_1Q\right)^2}\right]\\&+3\left(Q^2-Q\right)^2(2Q+1)\left[\frac{2b_1^2\left(a_0+a_1Q+a_2Q^2\right)}{\left(b_0+b_1Q\right)^3}\right]\\&+\left(Q^2-Q\right)^3\left[\frac{-6a_2b_1\left(b_0+b_1Q\right)+6b_1^2\left(a_1+2a_2Q\right)}{\left(b_0+b_1Q\right)^3}\right]\\&-\left(Q^2-Q\right)^3\left[\frac{6b_1^3\left(a_0+a_1Q+a_2Q^2\right)}{\left(b_0+b_1Q\right)^4}\right].\end{aligned} \tag{24}$$

The exact solutions of Eq.(1) is obtained as the following;

Case 1

$$\begin{aligned}a_0&=-\frac{\sqrt{\left(-1+k^2\right)p^2}\,b_0}{2\sqrt{2}},\\a_2&=-2a_1+\frac{\sqrt{2}\sqrt{\left(-1+k^2\right)p^2a_1^2}\,b_0}{a_1},\\b_1&=\frac{-2\left(\sqrt{2}\sqrt{\left(-1+k^2\right)p^2a_1^2}-\left(-1+k^2\right)p^2b_0\right)}{\left(-1+k^2\right)p^2},\\m&=\frac{1}{4}\left(-2k^2-p^2\right),\end{aligned} \tag{25}$$

When we substitute Eq.(25) into Eq.(21), we get dark soliton solutions of Eq.(1)

$$\begin{aligned}E_1(x,t)&=e^{i(kx+mt)}\left[A\tanh\left(p_1x+r_1t\right)\right],\\n_1(x,t)&=\left(\frac{2p^2}{r^2-p^2}\right)\left[A\tanh\left(p_1x+r_1t\right)\right]^2,\end{aligned} \tag{26}$$

where $A=-\frac{\sqrt{\left(-1+k^2\right)p^2}}{2\sqrt{2}}$, $p_1=\frac{p}{2}$, and $r_1=\frac{r}{2}$.

Case 2

$$a_0 = \frac{-a_1}{2},\ a_2 = -a_1,\ b_0 = \frac{-ia_1}{\sqrt{2p^2 - 2k^2 p^2}},$$
$$b_1 = \frac{2ia_1}{\sqrt{2p^2 - 2k^2 p^2}},\ m = -\frac{k^2}{2} - p^2, \tag{27}$$

If we substitute Eq.(27) into Eq.(21), we gain dark soliton solutions of Eq.(1)

$$E_2(x,t) = e^{i(kx+mt)}\left[B\left(\coth(p_1 x + r_1 t) + \tanh(p_1 x + r_1 t)\right)\right],$$
$$n_2(x,t) = \left(\frac{2p^2}{r^2 - p^2}\right)\left[B\left(\coth(p_1 x + r_1 t) + \tanh(p_1 x + r_1 t)\right)\right]^2, \tag{28}$$

where $B = -\frac{1}{4} i\sqrt{2p^2 - 2k^2 p^2}$.

Case 3

$$a_0 = 0,\ a_2 = -a_1,\ b_0 = \frac{a_1}{\sqrt{2}\sqrt{(-1+k^2)p^2}},$$
$$b_1 = \frac{-\sqrt{2}a_1}{\sqrt{(-1+k^2)p^2}},\ m = \frac{1}{2}\left(-k^2 + p^2\right), \tag{29}$$

When we substitute Eq.(30) into Eq.(21), we have dark soliton solutions of Eq.(1)

$$E_3(x,t) = e^{i(kx+mt)}\left[C\left(\tanh(p_1 x + r_1 t) - \coth(p_1 x + r_1 t)\right)\right],$$
$$n_3(x,t) = \left(\frac{2p^2}{r^2 - p^2}\right)\left[C\left(\tanh(p_1 x + r_1 t) - \coth(p_1 x + r_1 t)\right)\right]^2, \tag{30}$$

where $C = -\frac{1}{4\sqrt{2}\sqrt{(-1+k^2)p^2}}$.

In Figures 1-2, we plot two and three dimensional graphics of $E_1(x,t)$ in Eq. (26), which explain the vitality of solutions with suitable parameters. In Figure 3, we draw two and three dimensional graphics of $n_1(x,t)$ in Eq. (26), which indicate the dynamics of solutions with proper parameters. Also, in Figures 4-5, we plot two and three dimensional graphics of $E_3(x,t)$ in Eq. (30), which express the vitality of solutions with appropriate parameters. Finally, in Figure 6, we draw two and three dimensional graphics of $n_3(x,t)$ in Eq. (30), which show the dynamics of solutions with proper parameters.

Remark 1. The exact solutions of Eq. (1) were found via GKM, have been calculated by using Mathematica 9. As far as we know, the solutions of Eq. (1) obtained in this study, are new and are not observable in former literature.

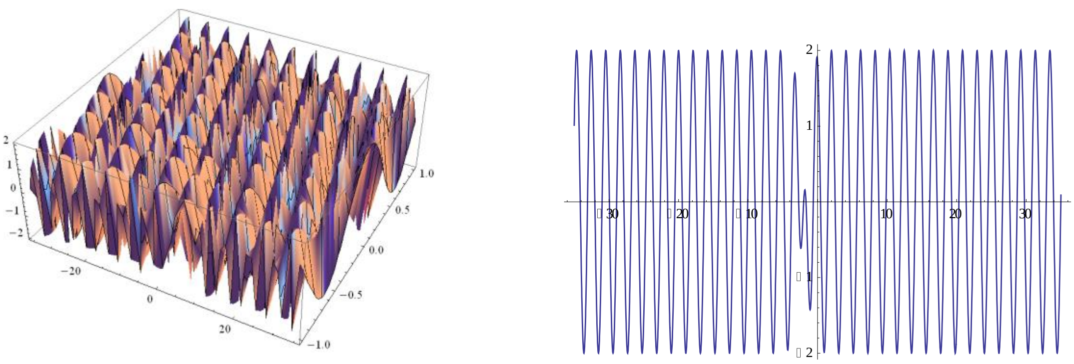

Figure 1. Graph of imaginary values of $E_1(x,t)$ in Eq. (26) is shown at $k = 3, m = 5, p = 2, r = 4, -35 < x < 35, -1 < t < 1$ and the second graph represents imaginary values of $E_1(x,t)$ in Eq. (26) for $-35 < x < 35,\ t = 1$.

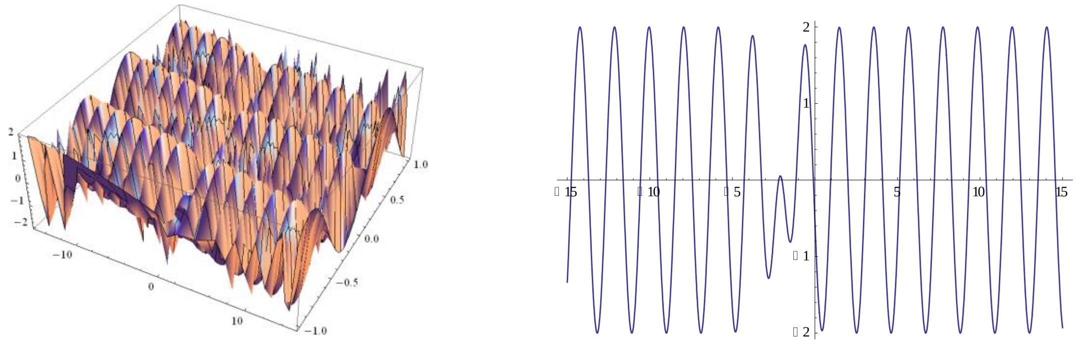

Figure 2. Graph of real values of $E_1\left(x,t\right)$ in Eq. (26) is indicated at $k=3, m=5, p=2, r=4, -15<x<15, -1<t<1$ and the second graph introduces real values of $E_1\left(x,t\right)$ in Eq. (26) for $-15<x<15, t=1.$

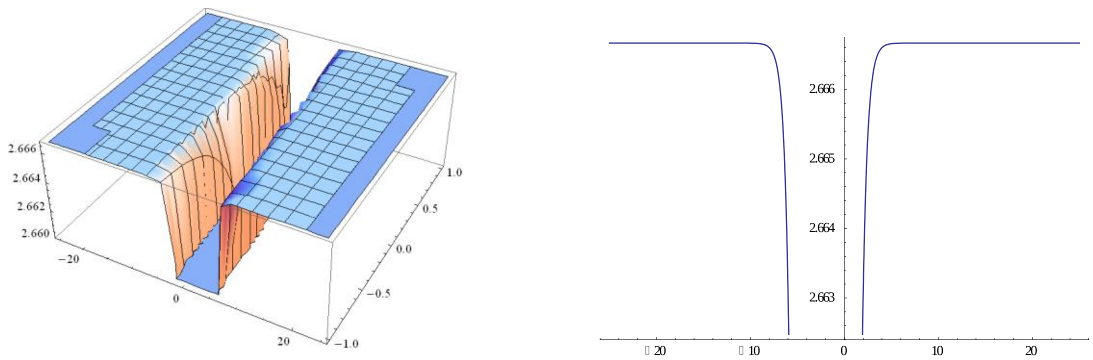

Figure 3. Graph of $n_1\left(x,t\right)$ in Eq. (26) is shown at $k=3, p=2, r=4, -25<x<25, -1<t<1$ and the second graph represents $n_1\left(x,t\right)$ in Eq. (26) for $-25<x<25, t=1.$

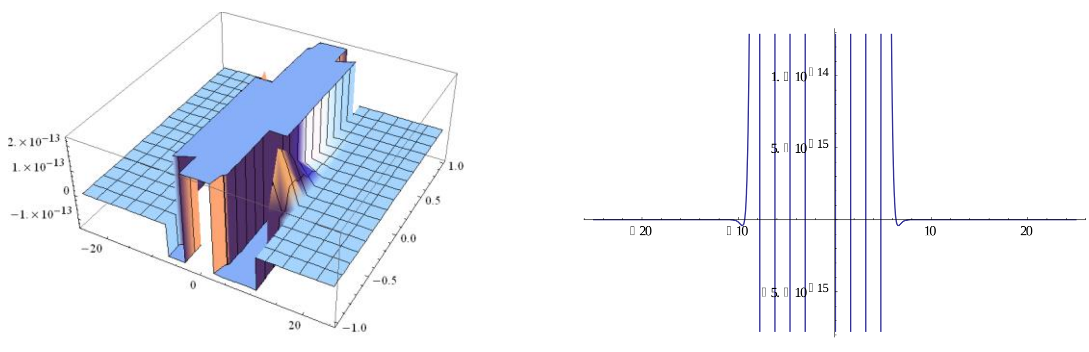

Figure 4. Graph of imaginary values of $E_3\left(x,t\right)$ in Eq. (30) is indicated at $k=2, m=3, p=4, r=6,$ $-25<x<25, -1<t<1$ and the second graph denotes imaginary values of $E_3\left(x,t\right)$ in Eq. (30) for $-25<x<25, t=1.$

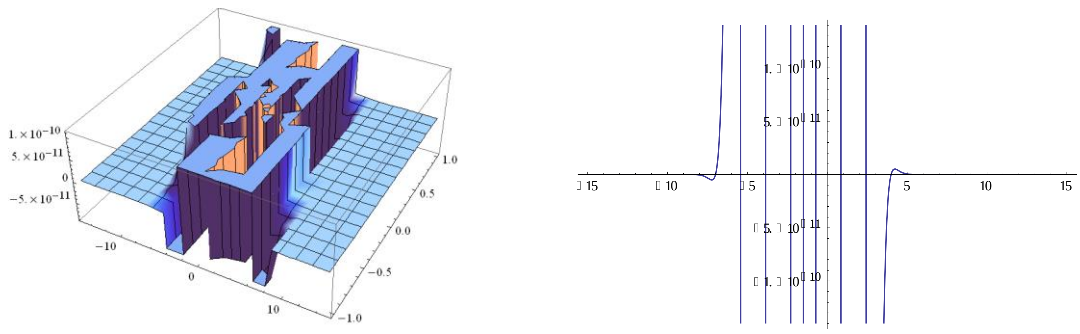

Figure 5. Graph of real values of $E_3(x,t)$ in Eq. (30) is shown at $k=2, m=3, p=4, r=6, -15<x<15, -1<t<1$ and the second graph remarks real values of $E_3(x,t)$ in Eq. (30) for $-15<x<15,\ t=1$.

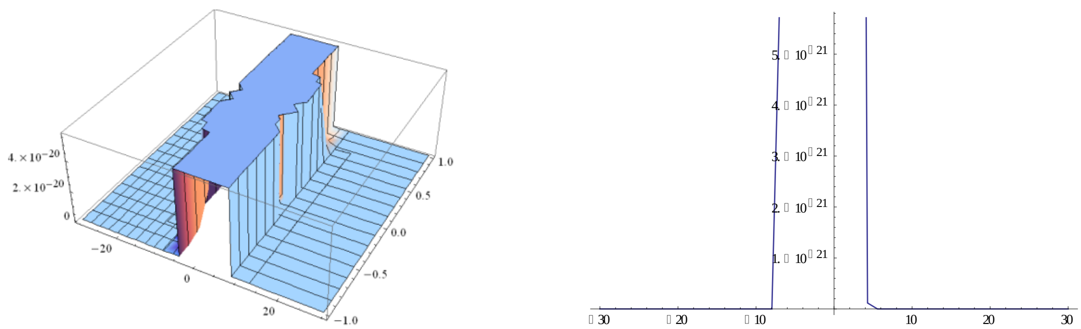

Figure 6. Graph of $n_3(x,t)$ in Eq. (30) is shown at $k=2, p=4, r=6, -30<x<30, -1<t<1$ and the second graph represents $n_3(x,t)$ in Eq. (26) for $-30<x<30, t=1$.

4. Physical explanation

In this section, we will present physical interpretation of the system of equations for the ion sound wave under the action of the ponderomotive force due to high-frequency field and for Langmuir wave.

Solitons are very special types of solitary waves. Soliton solutions occur in two kinds such as dark soliton and bright soliton. If the solution is in terms of sech function, the soliton is called bright soliton. But if the solution is in terms of tanh function, the soliton is called dark soliton. In the view of such information, the solutions Eqs. (26), (28) and (30) of Eq. (1) are dark soliton solutions.

5. Conclusion

In this paper, we obtain dark soliton solutions of the system of equations for the ion sound and Langmuir waves by using GKM. Then, for suitable parametric choices, we plot two and three dimensional graphics of some dark soliton solutions of this system of equations by using Mathematica Release 9. This method provides us to do complicated and tedious algebraic calculations. That is to say the availability of computer programmes such as Mathematica facilitates the tedious algebraic calculations.

The above results show that GKM has been efficient for the analytical solutions of the system of equations for the ion sound and Langmuir waves. Also, this method is a powerful mathematical tool in finding new dark and bright soliton solutions. Thus, we can point out that GKM has a key role to obtain analytical solutions of NLPDEs. The graphical demonstrations clearly indicate the effectiveness of the recommended method. We suggest that this method can also be applied to other NLPDEs.

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