An EPQ model with imperfect items using interval grey numbers

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Abstract. The classic economic production quantity (EPQ) model has been widely used to determine the optimal production quantity. However, the analysis for finding an EPQ model has many weaknesses which lead many researchers and practitioners to make extensions in several aspects on the original EPQ model. The basic assumption of EPQ model is that 100% of manufactured products are non-defective that is not valid for many production processes generally.

The purpose of this paper is to develop an EPQ model with grey demand rate and cost values with maximum backorder level allowed with the good quality items in units under an imperfect production process. The imperfect items are considered to be low quality items which are sold to a particular purchaser at a lower price and, the others are reworked and scrapped. A mathematical model is developed and then an industrial example is presented on the wooden chipboard production process for illustration of the proposed model.

Keywords: EPQ; grey system theory; inventory management; rework; imperfect items.
AMS Classification: 90B05

1. Introduction

Basically, the questions of when and how much to order must be answered for an inventory control system. The economic production quantity (EPQ) model was firstly presented by Taft to assist manufacturing firms in minimizing total production inventory costs [1]. The EPQ model is still widely accepted by many industries today, due to many unrealistic assumptions regarding to the model input parameters, especially setup cost, holding cost and demand rates, some practitioners and researchers were discussed to modify the EPQ model to match real-life situations and published hundreds of papers and books under various conditions and assumptions such as production rates, shortages,

backorders, imperfect items, inspection/with errors, learning effects, delay in payments and trade credit policies [2-8]. In literature, we faced up with several studies of EPQ models by the way extended these variations and assumptions. Imperfect items are also placed in these variations. At this phase, previous studies of EPQ with imperfect quality items production are given below.

Rosenblatt and Lee [9] derived an EPQ model with imperfect production process since the system reached out-of control point by producing 100% good items with an exponential distribution. Their model is extended with the assumption of general randomness during the good item production process by Kim and Hong [10]. Then, Hayek and Salameh [11] studied

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imperfect items with shortages and reworking under uniform distribution. Hayek and Salameh [11] model derived with random defective rate, reworking and backlogging for optimal lot size model by Chiu [12]. Papachristos and Konstantaras [13] examined on Salameh and Jaber [2] model closely and discussed many of its assumptions, and in particular with shortages [5]. Pan et al. [14] integrated an EPQ model to minimise the total expected production cost per production cycle while simultaneously determining the optimal parameters of control chart design by using pattern search for an imperfect production process. Some of the other studies, EPQ model is presented with scrap rates, stochastic machine breakdowns, maintenance, inflation effects etc. [15-19].

Other applications for EPQ models are derived under modern heuristics and artificial intelligence techniques, especially fuzzy logic. Chen et al. [20] derived a fuzzy EPQ model in which it involved the fuzzy opportunity cost, trapezoidal fuzzy costs and quantities with repairable defective items under crispness. Halim et al. [21] considered that the production process may shift from the in-control state to the out-of-control state at any random time for an imperfect production system by using fuzzy parameter of shift uncertainty with the centroid method. The similar papers of fuzzy EPQ are in remanufacturing [22], fuzzy production rate [23], fuzzy demand and defective rate [24, 25], flexibility and reliability [26], backorders with fuzzy parameters and decision variables [27] etc. The several heuristic papers of EPQ/Economic Order Quantity (EOQ) models are used genetic algorithms [28-32] and particle swarm optimization [33].

Whereas any EPQ model paper is not seen in literature by using grey system theory. The grey system theory was proposed in 1982 by Deng and its applications were widely used in several industrial areas under uncertainty and cybernetics of system approaches [34]. In this study, we expanded Eroglu et al. [35] model with greyness of demand rate and cost values and supported with a numerical example of wooden particleboard industry.

2. Grey System Theory

In the past century, tools of mathematics and statistics are also rapidly developed in order to solve many daily routine and complex problems in daily life and industry. Especially, after from

1950s the meta-heuristics and also algorithmic optimization techniques were became a quite tool for solving the n-dimensional problems as NP-Hard with development of computing concurrently. Then, we had been mentioned about linguistically information about many systems for modelling process of real life problems. The fuzzy set theory was introduced to us by Zadeh [36]. The most difficult part of doing a research was to analyse and classification of system data. The existence of information was important to have a first order under uncertainty. The information of a system is partly known and partly unknown is called as a grey system which was first introduced at the beginning of 1980s by Deng [34]. Since then, it has become a very popular technique with its applications on the partially unknown parameters, variables etc. According to Tseng [37], grey system theory is an interdisciplinary scientific research area that is a unique concept to deal with uncertainty with discrete data. One of the advantages of the technique is that it requires only a limited amount data with poor information for estimating the behaviour of the system for statistical techniques [38]. So, it can be said that it is an effective method on many real life systems. A general structure of a grey system is shown in Figure1.
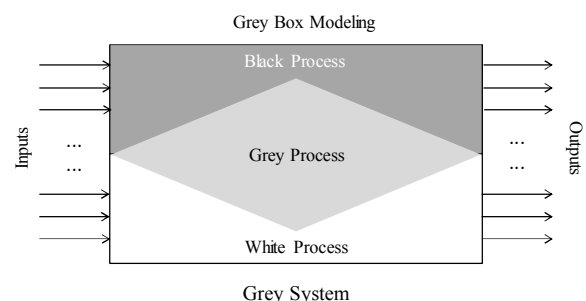

Figure 1. A general structure of grey system

In this paper, we apply basic definitions in grey theory. One of them is the black-grey-white box modelling system. A black box system is the relation between inputs and outputs which known but the behaviour of the system is unknown. If the system behaviour is completely known, the system is also called a white box system. In addition, if the system behaviour is partially known and/or partially unknown the system would be called a grey (box) system [39].

Yamaguchi et al. [40] presented that the grey system theory is an extension of fuzzy set theory or rough set theory. Such as, if the grey mathematics and fuzzy mathematics are compared, the grey number is an extension of the

fuzzy number, because a lower and an upper membership functions are clearly added to the fuzzy membership function (mf). The second one is grey system theory deals with interval grey algebraic, whereas fuzzy mf is a set of real numbers and most of information system in rough set theory includes nominal data. During the last three decades, the grey system theory has been widely and successfully applied to various systems such as social, economic and technical systems. Thus the many researchers have been developed new models of scientific and technological, industrial, mechanical, robotics, mechatronics, transportation, financial, military and, also natural events as meteorological, agricultural, ecological, hydrological, geological, bio-medical, etc. systems on transforming of cybernetics. In the following, this research briefly reviews some essential definitions of grey theory with Figure 2 [39]:
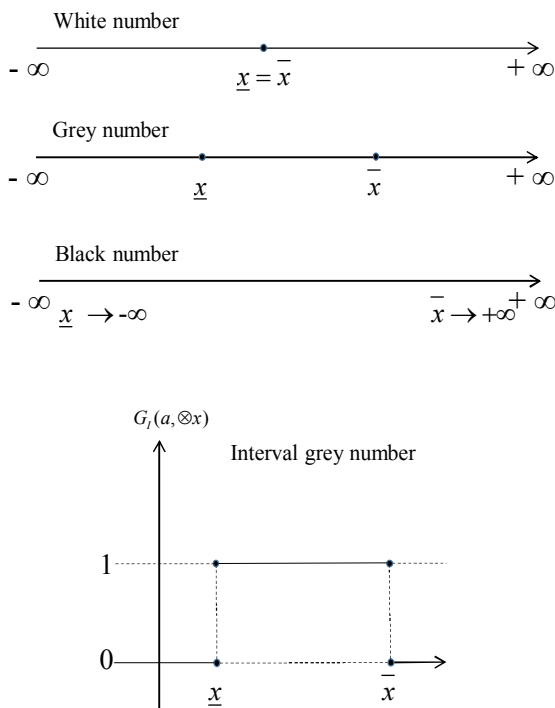

Figure 2. Definition of grey numbers with an interval grey number [40]

Definition 1. A grey number is such a number whose exact value is unknown but a range within that the value lies is known.

Definition 2. A grey number with both a lower limit $\underline{a}$ and an upper limit $\overline{a}$ is named as an interval grey number, denoted as $\otimes \in [\underline{a}, \overline{a}]$.

In this study, the interval grey numbers are used for making greyness of demand rate and inventory cost parameters to find maximum backorder level allowed with good quality items in units and economic production quantity.

2.1. Whitenization of Grey Numbers

The grey numbers are generally vibrated around a certain fixed value in applications. So, the grey numbers can be whitenized relatively easy and denoted $\otimes(a) = a + \delta_a$ and/or $\otimes(a) \in (-, a, +)$ where a stands for certain fixed value and δ_a stands for vibration. Then, the whitenizated grey number is shown as $\tilde{\otimes}$. In the following part, some definitions are given for whitenization of the grey numbers [41]:

Definition 3. A grey number with both a lower limit $\underline{a}$ and an upper limit $\overline{a}$ is an interval grey number, denoted as $\otimes \in [\underline{a}, \overline{a}]$. The whitenization of the form $\tilde{\otimes} = \alpha \underline{a} + (1 - \alpha)\overline{a}, \ \alpha \in [0,1]$ is called equal weight whitenization.

Definition 4. In an equal weight whitenization, the whitenization value, obtained when taking $\alpha = 1/2$, is called an equal weight mean whitenization.

Definition 5. The typical whitenization weight function is a continuous function with certain fixed starting and ending points (Figure 3a) and in the practical applications for computer programming and computing $L(x)$ and $R(x)$ functions are generally simplified into straight-lines (Figure 3b).

In Figure 3a,

$$f(x) = \begin{cases} L(x), & x \in [a_1, b_1) \\ 1, & x \in [b_1, b_2] \\ R(x), & x \in (b_2, a_2) \end{cases} \quad (1)$$

In Figure 3b,

$$f(x) = \begin{cases} L(x) = \dfrac{x - x_1}{x_2 - x_1} & , x \in [x_1, x_2) \\ 1 & , x \in [x_2, x_3] \\ R(x) = \dfrac{x_4 - x}{x_4 - x_3} & , x \in (x_3, x_4] \end{cases} \quad (2)$$
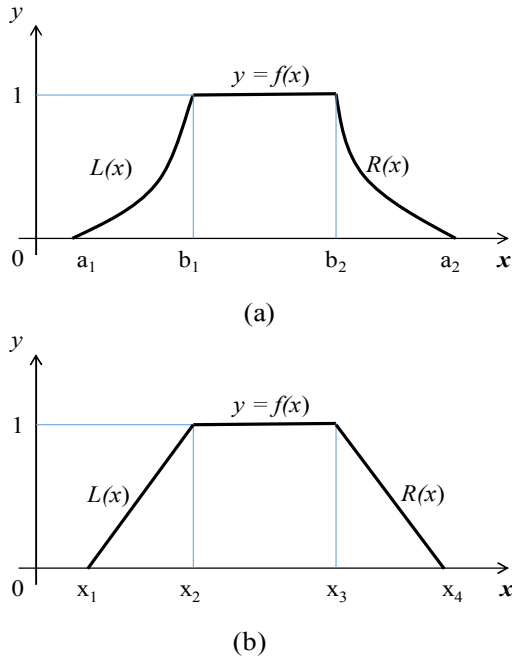
Figure 3. The typical whitenization weight functions

In general when the distribution information of an interval grey number is hardly known, we often use the equal weight mean whitenization [39]. But, we considered that $\alpha(k), k \in [0,1]$ coefficient is assumed adaptively and a framework is developed for the proposed grey EPQ model on Matlab® environment.

2.2. Degree of Greyness

Definition 6. A degree of greyness g^o is obtained from the typical whitenization weight function $f(x)$ in Figure 3a and given as follows [41]:

$$g^o = \frac{2|b_1 + b_2|}{b_1 + b_2} + \max\left\{\frac{|a_1 - b_1|}{b_1}, \frac{|a_2 - b_2|}{b_2}\right\} \quad (3)$$

The equation (3) is the sum of two parts, the first one is the peak area and the second one is $L(x)$ and $R(x)$ functions,

When $\max\left\{\frac{|a_1 - b_1|}{b_1}, \frac{|a_2 - b_2|}{b_2}\right\} = 0$,

$$g^o = \frac{2|b_1 + b_2|}{b_1 + b_2} \quad (4)$$

3. Greyness of Notations and Terminology

In this paper, we assumed that a good quality item and an imperfect quality item which is divided into low quality items, rework items and scrap items in imperfect production. The following notations are used in the proposed analysis:

Q : *production quantity/lot size in a cycle*
B : *maximum backorder level allowed with good quality items in units for each cycle*
x : *production quantity in units per unit time*
λ : *reworked item quantity in units per unit time*
P_g : *rate of good quality items*
p_1 : *rate of low quality items*
p_2 : *rate of rework items*
p_3 : *the rate of imperfect items that cannot be reworked (scrap items)*
    $p_g + p_1 + p_2 + p_3 = 1$
H_1 : *the maximum level of on-hand inventory in units, when the regular production process stops*
H : *the maximum level of on-hand inventory in units, when the rework process ends*
T : *cycle time*
TC : *the total inventory costs*
TCU : *the total inventory costs per unit time*

Grey notations with interval grey numbers are given as follows:

$\otimes(\beta) \in [\underline{\beta}, \overline{\beta}]$: *grey demand rate in units per unit time (β)*
$\otimes_1 \in [\underline{K}, \overline{K}]$: *grey setup cost for each production run (K)*
$\otimes_2 \in [\underline{C}, \overline{C}]$: *grey production cost per item ($/item, inspection cost per item is included; C)*
$\otimes_3 \in [\underline{C}_R, \overline{C}_R]$: *grey repair cost for each imperfect quality item reworked ($/item, C_R)*
$\otimes_4 \in [\underline{C}_S, \overline{C}_S]$: *grey disposal cost for each scrap item produced ($/ scrap item, C_S)*
$\otimes_5 \in [\underline{h}, \overline{h}]$: *grey holding cost per item per unit time ($/item/unit item, h)*
$\otimes_6 \in [\underline{h}_1, \overline{h}_1]$: *grey holding cost for each imperfect quality items being reworked per unit time ($/item/unit item, h_1)*
$\otimes_7 \in [\underline{\pi}, \overline{\pi}]$: *grey shortage cost per item per unit time (($/item/unit item, π)*

At this point, we preferred to use interval grey numbers given above for demand rate and inventory costs for exact values of them, only we have minimum (lower limit) and maximum (upper limit) values as a range. In addition, we didn't have any information sufficiently and the data haven't been classified with a statistical distribution in all grey parameters of the notations.

4. The Proposed Grey EPQ Model

4.1. Mathematical model

The imperfect items in a production process are generated due to process failures or other factors [12]. This paper studies the effect of the greyness on the backordering and reworking on an imperfect production model. Here, the production rate x is constant, and is much larger than the grey demand rate $\otimes(\beta)$. The inspection cost per item is involved when all items are screened in the production cost.

A portion p_g of good quality items is produced and then all of the imperfect items are not reworked and also are classified as low quality,

rework-able and scrap items in a portion 1-p_g of perfect quality items. Thus, low quality items are sold with a discount price at a rate of p_1 and scrapes must be discarded with a disposal cost per item at a rate of p_3 before the rework process starts. When the good production ends, the reworking of imperfect items starts immediately at a rate of p_2. In addition, the maximum backorder level is allowed with good quality items in units for each cycle. The behaviours of the inventory levels are illustrated in Figure 4.

A new grey-based EPQ model is proposed for the cycles, shown in Figure 4, which have included an interval grey numbers in a range and is shown in equation-1 for cycle T_1.

$$T_1 = \otimes(T_1) \in [\min(T_{1\_L}, T_{1\_U}) \quad , \quad \max(T_{1\_L}, T_{1\_U})] \quad (5)$$

In the cycle of T_1, if the demand rate is in upper limit, the maximum backorder level allowed with good quality items (B) has a slope steeper than lower limit. So, the cycle time will be reached with shorter (T_{1_U}). On the other hand, if it is in lower limit, the B has a slope lower than upper limit. So, the cycle time will be reached with longer (T_{1_L}).
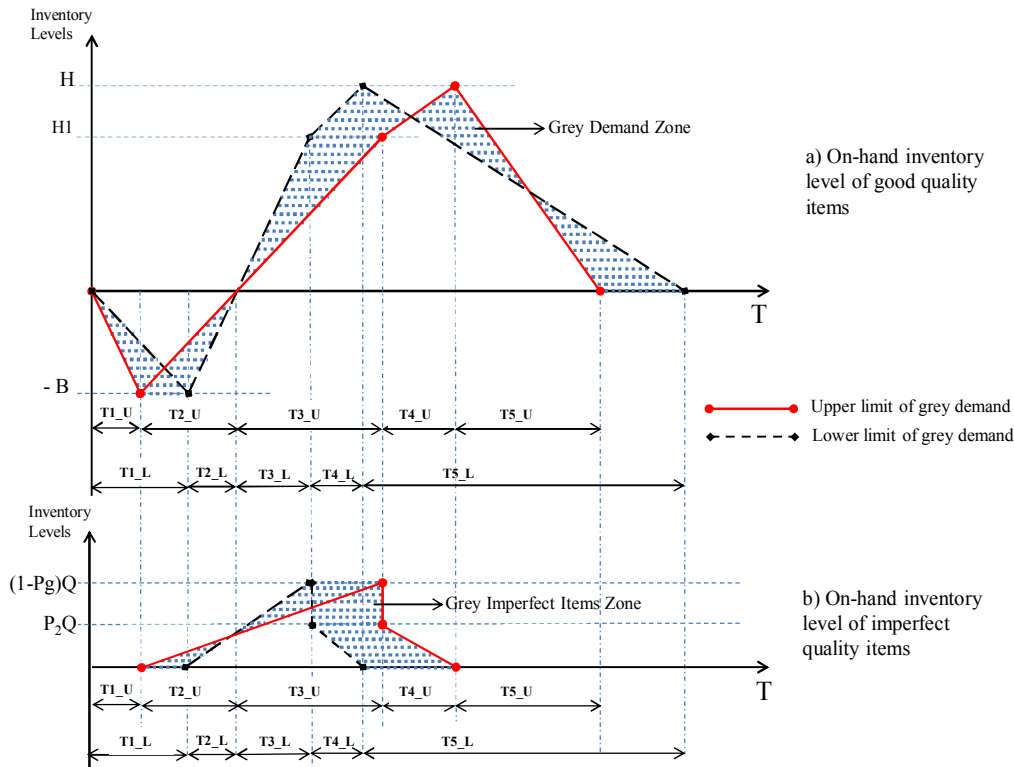

Figure 4. The behaviours of inventory levels on time

The maximum backorder level allowed with good quality items in units for T_1 cycle

$$T_1 = \frac{B}{\otimes(\beta)} \tag{6}$$

and the remaining part is used to eliminate backorders in T_2 cycle

$$T_2 = \frac{B}{A} . \tag{7}$$

We obtained hold on stock from good quality items when the production process starts as notated $A = xp_g - \otimes(\beta)$ and

$$T_2 + T_3 = \frac{Q}{x} \tag{8}$$

$$T_3 = \frac{Q}{x} - \frac{B}{xp_g - \otimes(\beta)} . \tag{9}$$

In addition, it is apparent from Fig. 1 that the following equations can be written;

$$T_3 = \frac{H_1}{xp_g - \otimes(\beta)} \tag{10}$$

$$T_4 = \frac{H - H_1}{\lambda - \otimes(\beta)} \tag{11}$$

$$T_4 = \frac{p_2 Q}{\lambda} \tag{12}$$

$$T_5 = \frac{H}{\otimes(\beta)} \tag{13}$$

From Eq. (9) and Eq. (10) H_1 is obtained as;

$$H_1 = (p_g - \otimes(\beta)/x)Q - B \tag{14}$$

From Eq. (11), Eq. (13) and Eq. (14) H is obtained as;

$$H = [(1 - \otimes(\beta)/x)p_2 + p_g - \otimes(\beta)/x]Q - B \tag{15}$$

Thus, total cost (TC) equation is obtained as follows;

$$TC = \otimes_1 + \otimes_2 Q + p_2 \otimes_3 Q + p_3 \otimes_4 Q + \otimes_5 [\frac{H_1 T_3}{2} + \frac{(H + H_1)T_4}{2} + \frac{HT_3}{2} + \frac{(T_2 + T_3)(1 - p_g)Q}{2}]$$

$$+ \otimes_6 [\frac{T_4 p_2 Q}{2}] + \otimes_7 [\frac{(T_1 + T_2)B}{2}]$$

$$TC = \otimes_1 + \otimes_2 Q + p_2 \otimes_3 Q + p_3 \otimes_4 Q + \frac{\otimes_5}{2}[\frac{1}{x} + 2p_2(\frac{p_g}{\otimes(\beta)} - \frac{1}{x}) + (1 - \frac{\otimes(\beta)}{\lambda})\frac{p_2^2}{\otimes(\beta)} + \frac{p_g^2}{\otimes(\beta)} - \frac{2p_g}{x}]Q^2$$

$$- \otimes_5 [\frac{(p_2 + p_g)BQ}{\otimes(\beta)}] + (\otimes_5 + \otimes_7)[\frac{p_g B^2}{2\otimes(\beta)(p_g - \frac{\otimes(\beta)}{x})}] + \otimes_6 [\frac{p_2^2 Q}{2\lambda}] \tag{16}$$

$$TC = \otimes_1 + Q(\otimes_2 + p_2 \otimes_3 + p_3 \otimes_4)$$

$$+ \frac{\otimes_5}{2\otimes(\beta)}[\frac{\otimes(\beta)}{x} + 2p_2(p_g - \frac{\otimes(\beta)}{x}) + p_2^2(1 + (\frac{\otimes_6}{\otimes_5} - 1)\frac{\otimes(\beta)}{\lambda}) + p_g^2 - \frac{2p_g \otimes(\beta)}{x}]Q^2$$

$$- \otimes_5 [\frac{(p_2 + p_g)BQ}{\otimes(\beta)}] + (\otimes_5 + \otimes_7)[\frac{p_g B^2}{2\otimes(\beta)(p_g - \frac{\otimes(\beta)}{x})}]$$

The cycle time T is given as;

$$T = \sum_{i=1}^{5} T_i = \frac{(p_2 + p_g)Q}{\otimes(\beta)} . \tag{17}$$

The total inventory costs per unit time (TCU) is obtained as;

$$TCU = \frac{TC}{T} = \frac{\otimes(\beta)\otimes_1}{(p_2 + p_g)Q}$$

$$+ \frac{\otimes(\beta)(\otimes_2 + p_2 \otimes_3 + p_3 \otimes_4)}{(p_2 + p_g)}$$

$$+ \frac{\otimes_5 FQ}{2(p_2 + p_g)} - \otimes_5 B + \frac{(\otimes_5 + \otimes_7)B^2}{2(p_2 + p_g)F_1 Q} \tag{18}$$

where,

$$F = \frac{\otimes(\beta)}{x} + 2p_2(p_g - \frac{\otimes(\beta)}{x})$$
$$+ p_2^2(1 + (\frac{\otimes_6}{\otimes_5} - 1)\frac{\otimes(\beta)}{\lambda}) + p_g^2 - \frac{2p_g \otimes(\beta)}{x}] \quad (19)$$

$$F_1 = 1 - \frac{\otimes(\beta)}{xp_g}. \quad (20)$$

Consequently, for finding the maximum backorder level allowed with good quality items B* and optimal production quantity level Q* in units for cycle T, Eq. (18) is derived partially Q and B variable at first order then Eq. (21) and Eq. (22) are obtained as follows;

$$B^* = \frac{\otimes_5(p_2 + p_g)F_1Q^*}{(\otimes_5 + \otimes_7)} \quad (21)$$

$$Q^* = \sqrt{\frac{2\otimes_1\otimes(\beta)}{\otimes_5(F - \frac{\otimes_5(p_2 + p_g)^2 F_1}{(\otimes_5 + \otimes_7)})}}. \quad (22)$$

TCU function is convex, a proof is given in Appendix. According to appendix, only one Q* and one B* is exist for minimizing of TCU function. But, when we considered about grey TCU, different solutions exist from any $\alpha(k), k \in [0,1]$ whitenization coefficient for grey demand and costs.

4.2. Conditions for grey EPQ model

In cycle T_2, to eliminate of backorders, good quality item production size must be larger than demand quantity. This situation is shown in Eq. (23) as;

$$A > 0 \quad \text{and/or} \quad xp_g > \otimes(\beta). \quad (23)$$

Then, the inventory level is reduced to the shortages during the rework process. Here,

$$H_1 + \lambda T_4 > \otimes(\beta)T_4 \quad \text{and/or}$$
$$\frac{(1 - \otimes(\beta))\otimes_7}{(\otimes_5 + \otimes_7)} > p_1 + p_3 + \frac{\otimes(\beta)p_2}{\lambda}. \quad (24)$$

Eq. (23) and Eq. (24) must be provided for proposed grey based EPQ model.

5. Experimental Study

A numerical example is selected from the paper by Eroglu et al. (2008) in accordance with today's demands and cost values. Accordingly, a chipboard manufacturing system which is bared and covered on 18 mm nominal size with thick melamine chipboards manufactures and analyzed for determining optimal production quantity in wooden industry.

The thickness size has a normal distribution with *N(18,0.32)* for chipboard technical specifications. According to TS EN 312 standards, the thickness size must be between 17.70-18.30 mm. When the thickness size is less than 17.40 mm and more than 18.70 mm, the items are classified as scraps. If the thickness size is between 17.40-17.70 mm, these items are classified as low quality items which are sold to particular customers at a lower price. If the other imperfect items are between 18.30-18.70 mm, these items are also reworked to specification limits and added to good quality items. The quantitative values are given as below for the developed grey system theory based EPQ model:

Q: production quantity/lot size in a cycle
B: maximum backorder level allowed with good quality items in units for each cycle

$x = 4000$ items, $\lambda = 500$ items,

The rates of low quality items, reworked items and scrapped items are determined as follows by using standard normal distribution:

$$p_g = P(17.70 \le X \le 18.30) =$$
$$P(\frac{17.70 - 18.00}{0.32} \le z \le \frac{18.30 - 18.00}{0.32}) = 0.6514 \quad (25)$$

$$p_1 = P(17.40 \le X \le 17.70) =$$
$$P(\frac{17.40 - 18.00}{0.32} \le z \le \frac{17.70 - 18.00}{0.32}) = 0.1439 \quad (26)$$

$$p_2 = P(18.30 \le X \le 18.70) =$$
$$P(\frac{18.30 - 18.00}{0.32} \le z \le \frac{18.70 - 18.00}{0.32}) = 0.1599 \quad (27)$$

We know that $p_g + p_1 + p_2 + p_3 = 1$ then we obtained $p_3 = 0.0448$. Then, the grey values with interval grey numbers with degree of greyness are given in Table1.

Table 1. Grey notations with degree of greyness for the numerical example

	Grey parameters	Degree of Greyness (g^o)
Demand rate	$\otimes(\beta) \in [2000, 2400]$	0.182
Setup cost	$\otimes_1 \in [90, 110]$	0.200
Production cost	$\otimes_2 \in [23, 27]$	0.160
Repair cost	$\otimes_3 \in [5, 7]$	0.333
Disposal cost	$\otimes_4 \in [3.5, 4.5]$	0.200
Holding cost	$\otimes_5 \in [0.3, 0.7]$	0.800
Holding cost	$\otimes_6 \in [0.55, 0.65]$	0.167
Shortage cost	$\otimes_7 \in [0.65, 0.75]$	0.143

Table 2. Experimental results for numerical example

No	$\alpha(k)$	Q*	B*	TCU*
1	0.00	1734.19	53.59	84154.60
2	0.05	1732.16	58.06	82810.17
3	0.10	1730.32	62.38	81476.50
4	0.15	1728.71	66.54	80153.59
5	0.20	1727.32	70.54	78841.43
6	0.25	1726.20	74.38	77540.04
7	0.30	1725.36	78.04	76249.39
8	0,35	1724.83	81.52	74969.50
9	0.40	1724.64	84.81	73700.35
10	0.45	1724.83	87.91	72441.95
11	0.50	1725.44	90.80	71194.29
12	0.60	1728.08	95.91	68731.19
13	0.70	1733.01	100.05	66311.00
14	0.80	1740.78	103.10	63933.70
15	0.90	1752.16	104.91	61599.23
16	1.00	1768.17	105.29	59307.53

That Table 2 includes the computational results is quite revealing in several ways. If the value of *α(k)* is 0.00, grey demand rate and costs are realized on upper limit. When *α(k)* value increases, the optimal production quantity and maximum backorder level also increase but the total quantity cost per unit time decreases. In the same way, If the value of *α(k)* is 1.00, grey demand rate and costs are realized on lower limit and so the total quantity cost per unit time is obtained minimum value as 61771.91 $/unit time. But, Eroglu et al. (2008) model has 1725.4 piece inventory level, 90.8 piece allowed backorder level and 71194.3 $ total cost per unit time as only one solution.

Then, Figure 5 shows the visualisation of results with surfaces graphically. We generally point out that when the value of grey coefficient $\alpha(k)$ increases, individually, the optimal total cost per unit time also decreases. In addition, the Table 2 and Figure 5b show that economic production quantity *(Q*)* and total cost per unit time *(TCU*)* *α(k)* value is between 1.00-0.40, *TCU** and *Q** are also decreased but after *α(k)* value is 0.40, as *TCU** value continues by decreasing, Q* value starts with increase.

6. Conclusions

The classical EPQ model is not valid sufficiently when ordered lots have some imperfect items. Therefore, new models are required for more realistic solutions with eliminating of some assumptions of EPQ in real-life problems. In this paper, a grey EPQ model is developed when each ordered lot contains some imperfect items, shortages backordered and reworked under grey demands and inventory costs by using interval grey numbers. This model assumed that defective rate is uniformly distributed and imperfect items are classified as reworks, scraps and lower quality. Lower quality items are sold on a discounted selling price as a single lot.

A numerical example is provided for the proposed grey EPQ model and its effects of individual changes in defective and scrap rates on optimal solution have been studied on a wooden chipboard manufacturing from greyness by Eroglu et al. (2008) model. One notices that, when grey coefficient $\alpha(k)$ increases individually, the optimal total cost per unit time also decreases. So, we have a major advantage by using grey system approach. This, when we don't know an exact value of demand rate and costs (setup, production, holding, repair, disposal, shortage etc.), only if we will define an interval values (lower and upper limit as range) for them, the grey system theory is presented a wide area to evaluate the EPQ model for inventory and backorder levels to monitor the process under an imperfect production environment in this paper. The grey system theory also suggests that we could express an uncertainty range of parameters and decision variables.
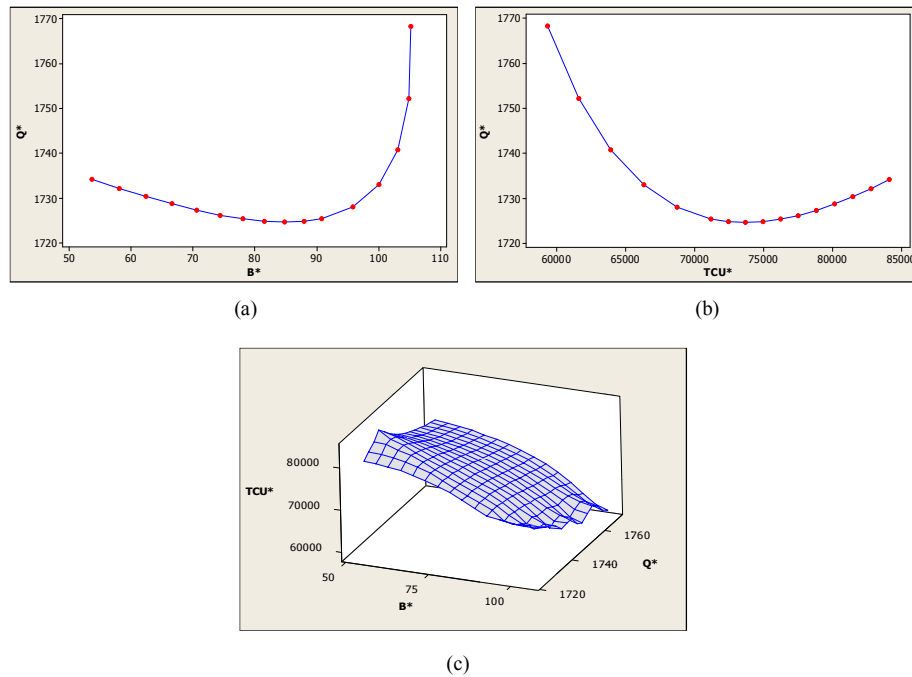
Figure 5. The visualization of experimental results

About the further research we are considering to eliminate the other assumptions and discuss learning curves, inflation effects, reliability, stochastic breakdowns, re-manufacturing and also inspection errors etc. by using approaches of grey system theory.

References

[1] Taft, E. W., The most economical production lot. Iron Age, 101, 1410-1412 (1918).

[2] Salameh, M. K., Jaber, M. Y., Economic production quantity model for items with imperfect quality. International Journal of Production Economics, 64, 59-64 (2000).

[3] Liao, J. J., On an EPQ model for deteriorating items under permissible delay in payments. Applied Mathematical Modelling, 31, 393-403 (2007).

[4] Darwish, M., EPQ model with varying setup cost. International Journal of Production Economics, 113, 297-306 (2008).

[5] Jaber, M. Y., Goyal, S. K., Imran, M., Economic production quantity model for items with imperfect quality subject to learning effects. International Journal of Production Economics, 115, 143-150 (2008).

[6] Pentico, D. W., Matthew, J. D., Toews, C., The deterministic EPQ with partial backordering: a new approach. Omega, 37, 624-636 (2009).

[7] Teng, J. T., Chang, C. T., Optimal manufacturer's replenishment policies in the EPQ model under two levels of trade credit policy. European Journal of Operational Research, 195, 358-363 (2009).

[8] Mahata, G. C., An EPQ-based inventory model for exponentially deteriorating items under retailer partial trade credit policy in supply chain. Expert Systems with Applications, 39, 3537–3550 (2012).

[9] Rosenblatt, M. J., Lee, H. L., Economic production cycles with imperfect production processes. IEE Transactions, 18, 48-55 (1986).

[10] Kim, C. H., Hong, Y., An optimal production run length in deteriorating production processes. International Journal of Production Economics, 58, 183-189 (1999).

[11] Hayek, P. A., Salameh, M. K., Production lot sizing with the reworking of imperfect quality items produced. Production Planning and Control, 12(6), 584-590 (2001).

[12] Chiu, Y. P., Determining the optimal lot size for the finite production model with random defective rate, the rework process, and backlogging. Engineering Optimization, 35(4),

427-437 (2003).

[13] Papachristos, S., Konstantaras, I., Economic ordering quantity models for items with imperfect quality. International Journal of Production Economics, 100, 148–154 (2006).

[14] Pan, J. C. H., Yang, M. F., Integrated inventory models with fuzzy annual demand and fuzzy production rate in a supply chain. International Journal of Production Research, 46(3), 753-770 (2008).

[15] Chiu, S. W., Wang, S. L., Chiu, Y. S. P., Determining the optimal run time for EPQ model with scrap, rework, and stochastic breakdowns. European Journal of Operational Research, 180(2), 664-676 (2007).

[16] Lo, S. T., Wee, H. M., Huang, W. C., An integrated production-inventory model with imperfect production processes and Weibull distribution deterioration under inflation. International Journal of Production Economics, 106, 248–260 (2007).

[17] Ojha, D., Sarker, B. R., Biswas, P., An optimal batch size for an imperfect production system with quality assurance and rework. International Journal of Production Research, 45:14, 3191-3214 (2007).

[18] Wee, H. M., Widyadana, G. A., Economic production quantity models for deteriorating items with rework and stochastic preventive maintenance time. International Journal of Production Research, 50 (11), 2940-2952 (2012).

[19] Chung, C. J., Widyadana, G. A., Wee, H. M., Economic production quantity model for deteriorating inventory with random machine unavailability and shortage, International Journal of Production Research, 49(3), 883-902 (2011).

[20] Chen, S. H., Wang, S. T., Chang, S. M., Optimization of fuzzy production inventory model with repairable defective products under crisp or fuzzy production quantity. International Journal of Operations Research, 2 (2), 31-37 (2005).

[21] Halim, K. A., Giri, B. C., Chaudhuri, K. S., Fuzzy EPQ models for an imperfect production system. International Journal of Systems Science, 40 (1), 45-52 (2009).

[22] Roy, A., Maity, K., Kar, S., Maiti, M., A production-inventory model with remanufacturing for defective and usable items in fuzzy-environment. Computers and

Industrial Engineering, 56, 87-96 (2009).

[23] Pan, E., Jin, Y., Wang, S., Cang, T., An integrated EPQ model based on a control chart for an imperfect production process. International Journal of Production Research, 50 (23), 6999-7011 (2012).

[24] Chang, H. C., An application of fuzzy sets theory to the EOQ model with imperfect quality items. Computers and Operation Research, 31, 2079-2092 (2004).

[25] Mula, J., Peidro, D., Poler, R., The effectiveness of a fuzzy mathematical programming approach for supply chain production planning with fuzzy demand. International Journal of Production Economics, 128, 136-143 (2010).

[26] Islam, S., Roy, T. K., A fuzzy EPQ model with flexibility and reliability consideration and demand dependent unit production cost under a space constraint: A fuzzy geometric approach. Applied Mathematics and Computation, 176, 531-544 (2006).

[27] Kazemi, N., Ehsani, E., Jaber, M. Y., An inventory model with backorders with fuzzy parameters and decision variables. International Journal of Approximate Reasoning, 51, 964-972 (2010).

[28] Mondal, S., Maiti, M., Multi-item fuzzy EOQ models using genetic algorithm. Computers and Industrial Engineering, 44 (1), 105-117 (2003).

[29] Pasandideh, S. H. R., Niaki, S. T. A., A genetic algorithm approach to optimize a multi-products EPQ model with discrete delivery orders and constrained space. Applied Mathematics and Computation, 195, 506-514 (2008).

[30] Pal, S., Maiti, M. K., Maiti, M., An EPQ model with price discounted promotional demand in an imprecise planning horizon via Genetic Algorithm. Computers and Industrial Engineering, 57 (1), 181-187 (2009).

[31] Bhunia, A. K., Kundu, S., Sannigrahi, T., Goyal, S. K., An application of tournament genetic algorithm in a marketing oriented economic production lot-size model for deteriorating items. International Journal of Production Economics, 119 (1), 112–121 (2009).

[32] Zhang, R. Q., Kaku, I., Xiao, Y. Y., Deterministic EOQ with partial back ordering and correlated demand caused by cross-selling.

European Journal of Operational Research, 210 (3), 537-551 (2011).

[33] Zhao, P., Wang, H., Gao, H., Improved particle swarm optimization algorithm for stochastic EOQ models with multi-item and multi-storehouse. Proceedings of the IEEE International Conference on Information Acquisition, August 20-23, Welhal, Shandong, China (2006).

[34] Deng, J., Control problems of grey systems. Systems and Control Letters, 1(5), 288–294 (1982).

[35] Eroglu, A., Sutcu, A., Sulak, H., An economic production quantity model with random defective rate in imperfect production processes. Journal of Faculty of Engineering and Architecture of Gazi University, 23(4), 923-929 (2008).

[36] Zadeh, L. A., Fuzzy Sets. Information Control, 8, 338-353 (1965).

[37] Tseng, M. L., A causal and effect decision making model of service quality expectation using grey-fuzzy DEMATEL approach. Expert Systems with Applications, 36, 7738-7748 (2009).

[38] Deng, J., Introduction to grey system theory. The Journal of Grey System, 1, 1–24 (1989).

[39] Liu, S., Lin, Y., Grey Information: Theory and Practical Applications. Springer, Germany (2006).

[40] Yamaguchi, D., Li, G. D., Chen, L. C., Nagai, M., Reviewing Crisp, Fuzzy, Grey and Rough Mathematical Models. Proceedings of 2007 IEEE International Conference on Grey Systems and Intelligent Service, Nov 18-20, Nanjing, China (2007).

[41] Liu, S., Lin, Y., Grey Systems: Theory and Applications. Springer, Germany (2010).

Appendix

Let us consider the following Hessian matrix (H):

$$H_e = \begin{bmatrix} \dfrac{\partial^2 TCU}{\partial Q^2} & \dfrac{\partial^2 TCU}{\partial Q \partial B} \\ \dfrac{\partial^2 TCU}{\partial B \partial Q} & \dfrac{\partial^2 TCU}{\partial B^2} \end{bmatrix}$$

If $[Q \ B] \times [H_e] \times \begin{bmatrix} Q \\ B \end{bmatrix} > 0, \ Q, B \neq 0,$, the function of TCU is strictly convex.

$$\frac{\partial^2 TCU}{\partial Q^2} = \frac{2 \otimes_1 \otimes (\beta) + \left[(\otimes_5 + \otimes_7) B^2 / F_1 \right]}{(P_2 + P_g) Q^3} \text{ and}$$

$$\frac{\partial^2 TCU}{\partial B^2} = \frac{\otimes_5 + \otimes_7}{(P_2 + P_g) F_1 Q}$$

and $\dfrac{\partial^2 TCU}{\partial Q \partial B} = \dfrac{\partial^2 TCU}{\partial B \partial Q} = \dfrac{(\otimes_5 + \otimes_7) B}{(P_2 + P_g) F_1 Q^2}$.

Consequently,

$$[Q \ B] \times [H_e] \times \begin{bmatrix} Q \\ B \end{bmatrix} = \frac{2 \otimes_1 \otimes (\beta)}{(P_2 + P_g) Q} > 0, \ Q, B \neq 0,$$

therefore, the function of TCU is strictly convex. Thus, Q* and B* which make TCU to minimum have single values.

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