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Application of a mathematical model to an advertisement reservation problem

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Abstract. Television networks provide TV programs free of charge to the public. However, they acquire their revenue by telecasting advertisements in the midst of continuing programs or shows. A key problem faced by the TV networks in Turkey is how to accept and televise the advertisements reserved by a client on a specified advertisement break which we called “Advertisement Reservation Problem” (ARP). The problem is complicated by limited time inventory, by different rating points for different target groups, competition avoidance and the relationship between TV networks and clients. In this study we have developed a mathematical model for advertisement reservation problem and extended this model for some cases encountered in real business life. We have also discussed how these cases affect the decisions of a TV network. Mixed integer linear programming approach is proposed to solve these problems. This approach has been implemented to a case taken from one of the biggest TV networks of Turkey.

Keywords: Advertisement reservation; media planning; mixed integer linear programming; revenue management; scheduling.
AMS Classification: 90C11, 90B35

1. Introduction

Television networks provide TV programs to the public free of charge. By this way, they attract a large variety of viewers who become somehow ready to watch commercial videos. Advertisers want to buy time periods, which is called advertising breaks and located within the TV programs, to persuade the audiences (viewers) to purchase or take some action upon their products or services.

It is known that TV networks gain more than 95% of their revenue from selling commercial time slots from advertising breaks that are inserted into the programs. Selling commercial times through an upfront and scatter market has been standard practice for US-Based broadcasting industries but this is not the only way of doing business for all over the world. The upfront market typically occurs at the end of the month of May. The TV networks announce their program schedule for the next broadcasting year and accept offers from the advertisers and/or their media agencies which include specific requests. TV networks are faced to pay penalties for unmet requests of advertisers whose offers are accepted. The problem of a TV network is to decide how much of the total time inventory to sell during the upfront market. The remaining inventory after the upfront market will be sold

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throughout the broadcast season which is called the scatter market, where the prices per unit time are higher than the upfront market and there are no penalty costs to be paid.

However, in Turkey, TV networks do not announce their program schedule for entire broadcasting year. Since television broadcasting business is highly competitive, TV networks want to control all the process week by week and want to interfere in program schedules fast. They do not sign long term contracts with program producers and they have power to cancel the contracts with producers whose programs do not attract enough audience. A TV network works based on requests that are taken from the advertisers or from their media agencies. These requests that are called reservations have not been scheduled yet. The reservations taken from an advertiser include the length of the commercial video (usually in seconds) and the target group to appeal. Since there are numerous of reservations taken for a show, the decision maker -- TV network -- should decide which reservations are more profitable to telecast.

Additionally, TV advertisements began to lose market share in advertising business. According to the report of VERITAS Media in 2008, TV advertising share in the market continues to decline both in USA and in Europe. The fundamental reason behind this is the growth in internet advertising and the uncertainty in the delivery of the advertisement (i.e. TV channels can't determine the exact number of people watching them.) World Advertising Research Center (WARC) had forecasted that the amount of TV advertisements will decline 17.2% in UK and at least 5% in USA, Japan, Australia, and in France, and most considerably internet advertising may capture TV advertising by 2020. Moreover, advertisers want to pay according to the amount of target client group watching them. In order to cope with this situation, selecting clients according to the gain in terms of the highest profits and placing the order of a client at optimum time could be an aid for TV networks.

In this study we have developed a mathematical model for advertisement reservation problem and extended this model for some cases encountered in real business life. We have also discussed how these cases affect the decisions of a TV network. In the following section, we provide a brief literature survey and section 3 provides a mathematical model for the selection problem and a detailed analysis of the problem is given. Section 4 ensures an application of our methodology to a real life case which is taken from one of Turkey's leading TV channels. Section 5 shows the outcomes and finally section 6 concludes and states the remarks of this study.

2. Literature Review

Media revenue management problem is a highly complex multi-level problem [1]. [1,2] both state that optimally managing the limited advertising space is one of the most important problems of media companies. Nonetheless these companies also take into account the valuation of the advertising space.

Current practice in US based TV Broadcasting Industry is to make decisions qualitatively such as upfront planning and operational planning or scatter planning [3,1]. Previously published papers on media revenue management have focused on developing mathematical models for scheduling commercials in advertisement breaks and generating sales plans, based on deterministic models [1,4-6].

The marketing literature has not focused on the issue of airtime capacity planning. It has extensively investigated the impact of TV advertising on consumer behavior and sales [7-9].

There are also several works on media planning which can be defined as the process of selecting time and space in various media for advertising. One of the core issues in media allocation, apportion the information to appropriate media vehicles and in determining the number of ads in each vehicle, is how to allocate the media budget [10]. Optimization studies on media planning were started by [11,12]. Day [13] and Engel and Warshaw [14] proposed linear programming models for media allocation. Stasch [15] and Brown and Warshaw [16] also proposed linear programming models for media selection. Such a problem is determined as a dynamic programming problem [17,18]. Aaker [19] and Zufryden [20] determined the issue as a probabilistic model. A differential game model for media budgeting and allocation is presented by Fruchter and Kalish [21]. The impact of differential lag effects on the allocation of advertising budgets across media is studied in [22] and Mihiotis and Tsakiris [23] use integer programming to solve the advertisement allocation problem. In their problem the best possible combination of placements of an advertisement is interrogated including the channel, the time, and frequency with the objective of the highest rating where the clients have budget limitation that is available for advertising.

Cetin and Esen [24] have studied media allocation problem by giving a good example of military operations research models that can be adapted to contemporary business world applications where they modeled the problem as a weapon-target model and solved it using integer nonlinear programming. In [25] they have also studied media allocation problem. They applied a linear time algorithm that finds a solution to the "maximum weight coloring" problem for an interval graph with interval weight to solve the problem that involves selecting different show slots telecast on different television channels in a day so as to reach the maximum number of viewers. These are all highlights of the literature review of the subject.

There has been some pioneering work on scheduling models using ratings data by [26-28]. A trigonometric time-series approach to forecast aggregate audience viewing for different times, and seasons is used in [26]. In [27] a two stage model is provided to develop program schedules. Finally, [28] develop a heuristic method for scheduling a TV network's programs.

In spite of its richness and complexity, the Media Revenue Management has not been studied largely in the operations literature. Chapter 10.5 of [29] provides a brief explanation of the media revenue management problem.

Previous papers on time inventory and revenue management focus on scheduling problems where they are using deterministic and combinatorial models. A math-programming-based algorithm to rapidly generate near-optimal sales plans that meet advertiser requirements have been developed by [4] where a sales plan consists of a complete schedule of advertisements to be aired for an advertiser during a broadcast year to meet its requirements. Bollapragada and Garbiras [5] have also developed an algorithm to schedule client videotapes in the advertisement slots which they purchased to meet certain specific client objectives. These two papers use linear penalties for violated constraints. A two-step hierarchical approach has been given by [1] first to select advertisers and match them with shows which are called winner determination, and then schedule their commercials to individual slots within a specific show which is called pod assignment. Kimms and Muller-Bungart [6] have proposed an integrated approach. These latter two papers try to meet all the constraints.

Bollapragada and Mallik [3] work on the decision process of a risk averse network, and try to show how they should allocate rating points between aggregate upfront and scatter markets. In this paper they work on uncertain and independent audience and scatter market revenues, and they have developed a one period model that aggregates demand from each market. In their model, the decision variable is the total number of rating points sold in the upfront market. They do not consider the actual capacity allocation and the realization of this allocation across clients and over time.

Araman and Popescu [30] developed stylized stochastic optimization models of airtime inventory planning and allocation across multiple clients under audience uncertainty. They devised a simple procedure for accepting upfront client contracts and estimating their overall inventory requirements. Their work also complements the work of [3] by emphasizing the actual capacity allocation. They proved that it is appropriate to aggregate demand across clients, and they explained how aggregate rating allocation can be mapped into client-level inventory allocation. Moreover, they studied the case where audience is revealed periodically which has not been studied by [3]. The characteristic that differentiated the media management problem worked by [30] from the above literature is the uncertain value of supply. This aspect makes their problem similar to production planning models under random yield, where reviews are provided by [31,32]. Their static model is an equivalent of the random yield model proposed by [33]. Stochastic Programming yield models are widely used in the random yield literature [34-37].

3. Problem Definition

A key problem faced by the TV networks in Turkey is how to accept and televise the advertisements reserved by a client on a specified advertisement break which we called "Advertisement Reservation Problem" (ARP). Therefore, the problem addressed here is how TV networks in countries like Turkey should select commercials to telecast from a set of reservation in order to maximize their revenues.

In Turkey, TV Networks still have limited information and knowledge about their own operations. The contribution we have made to the solution of their problems is to make TV Networks manage their revenues gained from advertisements better and more efficiently. Additionally, TV Networks will save both time and money when they assign the advertisements to the advertisement slots through an optimization model. The model will reduce the time needed to assign the ads to the slots, and bring away the possible errors caused by manual appointments. TV Networks will also make a lightning effort to their customers' demands about their ads, and briskly revise the customer reservations.

The selection commercials procedure has to consider various factors, such as the length of the commercial video, the target group of advertisers, inventory of available time per advertisement break, competing clients which do not want their advertisements to be telecasted in the same advertisement break, and relationship between TV network and advertisers.

TV networks unveil their weekly program schedule and tempt the advertisers and media buyers to buy airtime in bulk for the entire week. Clients (advertisers or their media agencies) send requests to be telecasted on an advertisement break of a specified program. That is, they make reservations for this specific advertisement break. Since an advertiser wants to reach a planned number of audiences with its existing advertising campaign, and all advertisers require this, there are numerous of reservations taken for an advertisement break of a show.

Another constraint is limiting time inventory. Times may vary according to the regulations or practice by country but there is a common time lag for the advertisement period called advertisement break which is 420 seconds in Turkey. Generally, the total duration of the taken reservations is longer than the regulated time lag, however the total length of the selected commercials must be within an interval, less than an upper limit and more than a lower limit.

Finally, all clients have the target group of advertisements that should be considered. There are always specific target groups that clients want to live up to. A target group can be defined in various ways- by household, by geographic market, by a given demographic group, such as men between age 18 and 49 or women between age 25and 54, or even by product usage or ownership, such as people have a digital camera, etc. The target group should clearly be set and identified according to the needs of the client and the assignment should be done suitably since clients generally prefer to pay the price according the amount of people reached. That is, the more people the TV network can reach by telecasting the request at specific time, the more price they can gain. In fact, in real life there is an extensively known media tool called cost per rating point (CPP) that can evaluate the price the clients have to pay for each rating point against a particular target group for their advertisements reached. The rating point is equal to one percent of all households who have a television set. In most countries the performance of TV networks is measured using these rating points.

Therefore, our model is formed by considering all these constraints which is described in the following section in detailed.

4. Model

Advertisement Reservation Problem (ARP) is a problem of selecting advertisements from a set of reservations that advertisers have specific target groups aimed to reach. Conclusive goal is generally to maximize the expected revenue. Client $i \in I$ informs his requirements for an advertisement $j \in J$ such as target audience group g $\left(g = 1, 2, ..., G\right)$, competing clients, duration of advertisements, etc. By considering these constraints, the most profitable order and selection of advertisements are decided to maximize the revenue.

Each client i specifies his target audience group set $R(i, j)$ for each advertisement j and to reach all these target groups in this set should be provided in the model. Furthermore, the order of telecasting advertisement is important since the first and the last advertisements are assumed to have higher CPPs than the others as usual. Because, the audience level for the first advertisement will be high since the break starts just now and the audience level for the last advertisement will be high since the program watched by audience will start immediately after that. Therefore we have three slots to telecast an advertisement. The advertisement that will be selected to telecast in slot $s = 1$ and $s = 3$ has a higher CPP and as a result these slots are also more expensive. We assumed that all rating points CPP_s for each slot s are known deterministically for this problem and are expected rating point $E[g]$ for target audience group g too. Client i should also determine the duration of each advertisement j denoted by t_{ij}. Moreover each advertisement break has an upper limit of total telecasting time T_U and a lower limit T_L. Under these constraints, our decision variable (1) is also which advertisement of which client will be telecasted at which slot order.

$$y_{ijs} = \begin{cases} 1, & \text{if advertisement } j \text{ of Client } i \text{ is telecasted at order } s \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

The problem is modelled by using mixed integer programming and solved by using GAMS software package. The total revenue (TR) that will be gained from advertisement *j* of client *i* is calculated as in (2).

$$TR_{ij} = \sum_{g \in R(i,j)} E[g].t_{ij}.CPP_s \quad (2)$$

Therefore, our objective function (3) aims to maximize the total expected revenue gained from telecasted advertisements of clients taking into account the relationship between TV network and clients. The contribution of a client to the total expected revenue, if it is selected to telecast, is the amount obtained from (2).

$$\max z = \sum_{i \in I} \sum_{j \in J} \sum_{s=1}^{3} y_{ijs} \left(\sum_{g \in R(i,j)} E[g].t_{ij}.CPP_s \right) \quad (3)$$

The above mentioned constraints are modeled as follows:

$$\sum_{i \in I} \sum_{j \in J} \sum_{s=1}^{3} y_{ijs}.t_{ij} \le T_U \quad (4)$$

$$\sum_{i \in I} \sum_{j \in J} \sum_{s=1}^{3} y_{ijs}.t_{ij} \ge T_L \quad (5)$$

$$\sum_{i \in I} \sum_{j \in J} y_{ij1} = 1 \quad (6)$$

$$\sum_{i \in I} \sum_{j \in J} y_{ij3} = 1 \quad (7)$$

$$\sum_{s=1}^{3} y_{ijs} \le 1 \qquad \forall i, j \quad (8)$$

$$\sum_{i \in I} \sum_{j \in J} \sum_{s=1}^{3} y_{ijs} \le 1 \quad (9)$$

Constraint set (4) and (5) restrict the total duration of the telecasted advertisements with the maximum and minimum time availability, respectively. The total duration of an advertisement break must be less than an upper limit and more than a lower limit. Constraint (6) and (7) demonstrates that the first and the last slot of the advertisement break is allocated only one advertisement. Constraint (8) ensures that all reserved advertisements of all clients are telecasted only once. That is, the same advertisement should not be telecasted more than once in the same advertisement break. Finally, constraint (9) specifies that advertisements of clients competing with each other will not be telecasted in the same advertisement break because some clients don’t want their advertisements are telecasted with their competing clients.

This is the basic reservation model for TV networks but some modifications can be applied on this model. Such modifications are mentioned in the following subsections.

4.1. ARP with clients paying fixed prices case (ARP-FP)

Clients, sometimes, do not want to pay per rating point. They prefer to make their reservations with a fixed price. In that case, TV networks have to take into account this fixed amount of revenues. There will be additional parameters for this modified model which are listed below:

P: Set of clients that make their reservations with a fixed price

Q: Set of advertisements of clients that make their reservations with a fixed price

Now, the modified objective function of ARP-FP can be modelled as in (10).

$$\max z = \sum_{i \in (I-P)} \sum_{j \in (J-Q)} \sum_{s=1}^{3} y_{ijs} \left(\sum_{g \in R(i,j)} E[g].t_{ij}.CPP_s \right) + \sum_{i \in P} \sum_{j \in Q} \sum_{s=1}^{3} y_{ijs}.FP_{ijs} \quad (10)$$

where all constraint sets will remain basically the same with the constraint sets of ARP. FP_{ijs}

denotes the fixed price of advertisement *j* telecasted in slot *s* of client *i*. The objective function (10) still aims to maximize the total expected revenue gained from telecasted advertisements of clients. The contribution of a client to the total expected revenue, if it is selected to telecast, can be the revenue obtained from (2) or from the fixed price logic that the client is willing to pay.

4.2. ARP-FP with relationships case (ARP-FPR)

It is not always the fact that TV networks decide according to the highest profit that they can gain from their clients. In some countries like Turkey, relationships have influence upon decisions. The relations between TV network and its clients that affect decisions may be interpersonal or intercompany. In fact, TV networks want to keep their clients which TV networks already have good relations with them and provide their continuity. In that case, networks sometimes consent less profit instead of not losing the client for future relations.

4.2.1. ARP-FPR with the ε-constraint method

TV networks usually determine their relationships with clients to give them priority in their operations. They define this relationship by a coefficient α_i which satisfies $0 \le \alpha_i \le 1$ for each client *i* named as *priority coefficient*. In real life, these parameters can be assigned by taking into account the gross sales performed with the client, the economic power of the client or the individual relationships between the network and the client. Furthermore, they have too many valuable clients and they must select the clients that have the highest priority coefficient to maximize also their expected revenues. Under this constraint the objective function is as in (11).

$$\max z = \sum_{i\in(I-P)} \sum_{j\in(J-Q)} \sum_{s=1}^{3} y_{ijs} \left(\sum_{g\in R(i,j)} E[g].t_{ij}.\alpha_i.CPP_s \right) + \sum_{i\in P} \sum_{j\in Q} \sum_{s=1}^{3} y_{ijs}.\alpha_i.FP_{ijs} \tag{11}$$

First part of the objective function (11) considers the revenue obtained from the advertisements $j \in (J-Q)$ of clients $i \in (I-P)$ and the second part considers the revenue obtained from advertisements $j \in Q$ of clients $i \in P$ when the following new constraint added to other constraints in the basic model is satisfied:

$$\sum_{i\in I} \sum_{j\in J} \sum_{s} y_{ijs}\alpha_i \ge \varepsilon_L \tag{12}$$

where ε_L denotes the lower level of the sum of relationship priority coefficients for all clients. It says that total relationships provided by all clients must be greater than the pre-specified level. This constraint must be satisfied to sustain the relationships with clients and maximize revenue.

5. Case Study

The proposed models are implemented in one of the biggest TV networks of Turkey. At first, we have defined a problem set with the staff of the network. As a real life implementation of the proposed models, an advertisement break of a program which will be telecasted on prime-time is specified. However TV Networks in Turkey do not want to share their real data with 3rd parties so that we do not have more data sets to test on.

To obtain ratings forecasts for all target groups, the audience that is predicted to watch an advertisement is modeled as ex-ante of random variable X with mean μ and standard deviation σ. Then, we have taken the expected values of ratings as we mentioned before that we assumed all ratings for all target groups are known deterministically for this problem.

We assume there are 21 clients. Some of them have only one advertisement to be telecasted and some of them have more than one advertisement. The duration of advertisements of the clients are given in Table 1. i denotes the client and j denotes the advertisements in the following tables. Table 2 gives the specific target groups that clients want to live up to. There are 92 different target audience groups in Turkey Broadcasting Market and the forecasted ratings of all these target groups that is expected to be gained in the specified advertisement break are known. For instance, Client 2 wants to reach the 25th target group for advertisement 1. This target group shows the audiences who are more than 20 years old and its expected rating point is $E[25^{th} \text{ target group}] = 1.059$. Moreover, client 18 wants to reach the 63rd target group which denotes the audience lives in Black Sea Region and its expected rating point is $E[.] = 0.979$.

Table 1. The duration of advertisements of the clients

i	*j* 1	2	3	4
1	30	20	-	-
2	20	-	-	-
3	20	-	-	-
4	20	15	20	-
5	20	-	-	-
6	25	-	-	-
7	20	10	10	20
8	5	5	10	-
9	5	-	-	-
10	30	15	15	-
11	12	-	-	-
12	16	32	8	32
13	20	-	-	-
14	10	-	-	-
15	48	10	10	-
16	16	-	-	-
17	20	-	-	-
18	30	10	10	-
19	21	10	10	-
20	22	10	-	-
21	15	-	-	-

Table 2. Target audience group requests of the clients

i	*j* 1	2	3	4
1	48	30	-	-
2	25	-	-	-
3	23	-	-	-
4	22	15	21	-
5	22	-	-	-
6	25	-	-	-
7	20	17	30	22
8	7	17	51	-
9	8	-	-	-
10	37	15	9	-
11	12	-	-	-
12	16	31	8	34
13	20	-	-	-
14	10	-	-	-
15	48	30	30	-
16	17	-	-	-
17	20	-	-	-
18	63	18	15	-
19	23	77	77	-
20	22	81	-	-
21	13	-	-	-

Cost per rating point (CPP) where the advertisement of a client is telecasted at order *s* is known deterministically and same for all clients and advertisements. They differ only according to the slot order. The CPP values are \$11, \$5 and \$10.5 for the first, second and third slot respectively.

In addition to this, upper and lower limit of total telecasting time for the advertisement break is specified 425 and 420 seconds, respectively. Finally, Client 4 and Client 12 are competitors and they do not want their advertisements to be telecasted in the same advertisement break, and TV network does not take into account the relationships with these clients to satisfy their constraints. We used the GAMS software package to find the optimal solution of proposed models.

5.1. Computational results for ARP model

The problem is solved firstly for the basic reservation model ARP. Table 3 gives the assignment of the advertisements to the slots. The total revenue gained from these assignments is 2336.794 USD. The total duration of the advertisement break after these assignments is 424 seconds. $(i.j)$ denotes the advertisement j of client i and s denotes the slot number in the following tables. As a result, advertisements of clients 3, 4, 16, 19 and 21 are not selected to be telecasted in any break. Moreover, advertisements of Client 4 and Client 12 are not assigned to the same advertisement break.

Table 3. Assignment of advertisements to the slots

(i,j)	*s* 1	2	3
1.1		1	
2.1		1	
5.1		1	
6.1		1	
7.1		1	
7.2		1	
7.3		1	
8.1		1	
8.3		1	
9.1		1	
10.1			1
10.2		1	
11.1		1	
12.2		1	
13.1		1	
14.1		1	
15.1	1		
17.1		1	
18.1		1	
18.2		1	
18.3		1	
20.1		1	

5.2. Computational results for ARP-FP model

Client 4 and Client 6 preferred to make their reservations with a fixed price. In that case, TV networks have to take into account these fixed amounts of revenues instead of CPP values. Table 4 gives the fixed prices that Client 4 and Client 6 offer to TV network to make their advertisements to 4 and $190 for the third slot.

Table 4. Fixed Prices Offered by Clients

(i,j)	*s*		
	1	**2**	**3**
4.1	200	170	190
4.2	140	90	100
4.3	180	140	150
6.1	250	100	200

Table 5 gives the assignment of the advertisements to the slots. For instance, 1st advertisement of the 1st client is assigned to the 2nd slot and the 1st advertisement of the 15th client is assigned to the 1st slot. The total revenue gained from this assignment is 2387.519 USD. The total duration of the advertisement break after these assignments decreases to 422 seconds but the revenue increases.

Table 5. Assignment of Advertisements to the Slots

(i,j)	*s*		
	1	**2**	**3**
1.1		1	
2.1		1	
4.1		1	
5.1		1	
6.1		1	
7.1		1	
7.3		1	
7.4		1	
8.1		1	
8.3		1	
10.1			1
10.2		1	
11.1		1	
13.1		1	
14.1		1	
15.1	1		
17.1		1	
18.1		1	
18.2		1	
18.3		1	
20.1		1	
21.1		1	

The advertisements of clients 3, 9, 12, 16 and 19 are not selected to be telecasted in any break. But, one of the advertisements of Client 4 is assigned to the advertisement break. This prevents Client 12 to be assigned to the advertisement break.

When we compared the assignments of ARP with ARP-FP, the effect of the clients which offer fixed prices to make their advertisements to be telecasted in the advertisement break is transpired. If the contribution of an advertisement to the objective function is higher than the contribution of one or more advertisements assigned to the advertisement break before, the new advertisement will take the place of these pre-existed advertisements. Without limiting the generality of the foregoing, all other constraints are satisfied after this new assignment. An advertisement of Client 4 and some new advertisements are assigned after the client gave a fixed price, where some of the advertisements of other clients are taken out of the assignment. The contributions of advertisements of the clients are given in Table 6.

With these contributions, 4.1 and 21.1 are assigned to the advertisement break where 9.1 and 12.2 are taken out of the assignment. The contribution of the first advertisement of Client 6 has also undergone a change because of the diverse strategies.

The difference between two groups, ARP and ARP-FP, $\left([170+100+27.825]-[15.475+180+51.625]\right)$ is \$50.725. It is the same with the difference of two objective values $(\$2387.519-\$2336.794)$.

5.3. Computational results for ARP-FPR

Table 7 represents the relationship parameters for all clients. Clients 3, 4, 15 and 16 have more than 90% priority coefficients and then clients 20 and 21 follow these clients. In this case, lower bound of total relationships is set to be 10. The optimal assignments of the advertisements of each client to the slots are shown in Table 8. Total revenue gained will be \$2124.979. It is important to take notice of the difference of assignments between two cases, say that ARP-FP and ARP-FPR. Advertisements 4.1, 6.1, 7.1, 7.4, 18.2, 18.3 are taken out of the assignment. It is also important to see that the relationship parameter of a client is not important itself only. For example, although the priority coefficient of client 4 (0.98) is higher than the priority level of client 12 (0.79), advertisement of client 12 is selected to be telecasted.

What is more, advertisements of clients like 16.1 is assigned to the advertisement break for the first time where it has a very low contribution to the objective function which is only $51.04, but have one of the highest relationship parameters 0.99.

Table 6. Contributions of advertisements to the objective function

(i.j)	**Contribution ($)**
4.1 (ARP-FP)	170
6.1 (ARP)	51.625
6.1 (ARP-FP)	100
9.1 (ARP)	15.475
12.2 (ARP)	180
21.1(ARP-FP)	27.825

Table 7. Relationship parameters for all clients

Client i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
α_i **(%)**	7	72	92	98	31	11	58	46	11	61	55	79	27	69	94	99	1	31	57	82	87

Table 8. Assignment of advertisements to the slots

(i,j)	***s***		
	1	**2**	**3**
1.1			1
2.1		1	
3.1		1	
5.1		1	
7.2		1	
7.3		1	
8.1		1	
8.3		1	
9.1		1	
10.1		1	
10.2		1	
11.1		1	
12.2		1	
13.1		1	
14.1		1	
15.1	1		
16.1		1	
17.1		1	
18.1		1	
19.1		1	
20.1		1	
21.1		1	

6. Conclusion and Future Directions

A key problem faced with the TV networks in Turkey is how to accept and televise the advertisements reserved by a client on a specified advertisement break which we have called "Advertisement Reservation Problem" is considered in this paper. But, the problem is complicated by many factors such as limited time inventory, different rating points for different target groups and the relationship between TV networks and clients. Many cases were developed to take into account all these factors and to solve this problem, mixed integer linear programming approach is proposed to solve these problems. This approach has been implemented to a case taken from one of the biggest TV networks of Turkey. Owing to the fact that TV Networks in Turkey do not want to share their real data with 3rd parties, we do not have the capacity to demonstrate the effectiveness and applicability of the model. However, we have successful examples from all around the World, especially from the USA, that these kinds of optimization models provide financial savings.

By using the results derived from the model, TV networks will have a profound knowledge on managing their operations. These models come to the conclusion to better use of advertisement inventory, and decrease the time needed to produce an advertisement assignment and the time needed to rework on plans created. TV networks also respond more quickly to their clients which enable them to have a stronger position against other networks. Although TV networks perform these analyses about one hour for the problem of 21 clients, 4 advertisements and 3 slots, it takes about 1 minute in GAMs model that we have developed. When the problem size increases, their computational time increases exponentially but ours spends approximately the same time with the previous one.

Results derived from the models used in this paper point up the effect of the relations between TV networks and their clients. Plainly, in such countries which have these kinds of special business environments, revenue management professionals take relations into consider. To better use revenue management techniques and to negative the relations in parties, new business mechanisms can be designed for advertisement reservation problems which will compromise arrival of the parties.

All the same, advertisement reservation problem is formulated from the networks' point of view and it can be reformulated from the client's point of view or an integrated approach. Furthermore, the problem has been modeled for one ad break only. Two or more advertising breaks case will have different strategies and it should give different assignments. Finally, since the audience ratings are not deterministic, the problem can be modeled using stochastic programming where uncertainty of audience levels will be included in such a model.

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