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RESEARCH ARTICLE

Analysing the market for digital payments in India using the predator-prey model

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ABSTRACT

Technology has revolutionized the way transactions are carried out in economies across the world. India too has witnessed the introduction of numerous modes of electronic payment in the past couple of decades, including e-banking services, National Electronic Fund Transfer (NEFT), Real Time Gross Settlement (RTGS) and most recently the Unified Payments Interface (UPI). While other payment mechanisms have witnessed a gradual and consistent increase in the volume of transactions, UPI has witnessed an exponential increase in usage and is almost on par with pre-existing technologies in the volume of transactions. This study aims to employ a modified Lotka-Volterra (LV) equations (also known as the Predator-Prey Model) to study the competition among different payment mechanisms. The market share of each platform is estimated using the LV equations and combined with the estimates of the total market size obtained using the Auto-Regressive Integrated Moving Average (ARIMA) technique. The result of the model predicts that UPI will eventually overtake the conventional digital payment mechanism in terms of market share as well as volume. Thus, the model indicates a scenario where both payment mechanisms would coexist with UPI being the dominant (or more preferred) mode of payment.

1. Introduction

The last couple of decades have witnessed technology penetrating our lives in unimaginable ways. One such area where technology has had a significant impact in is the financial sector. With the advent of technology, payment mechanisms are undergoing paradigm shifts. Electronic payment systems offer various advantages over physical currency, like speed, security, lower transaction costs for individuals, elimination of counterfeit currency, and enhanced regulation. For this reason, Central Banks are not only promoting and facilitating digital payment mechanisms, but some are also mooting the idea of completely shifting to electronic transactions by replacing physical cash with central bank digital currency (CBDC). Electronic payment mechanisms have been in vogue for a considerable period of time. By providing the aforementioned benefits to users, these mechanisms influence behaviour in very significant ways. Numerous modes of electronic payments have emerged in the past couple of decades including e-banking services, NEFT, RTGS and most recently the Unified Payments Interface or UPI.

UPI in particular, has witnessed phenomenal growth within a short span of its introduction. The UPI was launched by the National Payments Corporation of India (NPCI), a joint initiative of

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the RBI and leading banks, which has been a pioneer in developing efficient and accessible payment solutions in India. The UPI enables a set of standard application programming interface specifications to facilitate digital payments using the mobile phone [1]. It leveraged on the extensive mobile phone network and the increasing usage of smart phones, enhanced internet availability, and the growth of mobile-based payment applications in India. UPI allows for a range of financial and non-financial transactions by making mobile phones the primary payment device. The introduction of UPI coincided with two key events in the economic and business landscape, which contributed immensely to its popularity. The year 2016 saw the entry of new players like Jio, which propelled the data revolution in India, which drastically brought down the prices of internet data, thus increasing its coverage and usage. With data available at low cost, and increased availability of smart phones, UPI witnessed a consistent increase in the number of users, as well as the number of banks, live on the platform. The second factor that was significant in the initial increase in UPI usage was the demonetisation of high-value currency notes, which the Government of India announced in November 2016. This brought in noticeable changes in the perception of users regarding digital payment technologies. UPI is a significant improvement over its peers in numerous ways:

- The UPI allows for both "pull", i.e., payee initiated as well as "push", i.e., payer initiated transactions.
- UPI payments can be made using various platforms like apps, websites, etc.
- UPI eliminates the need to divulge multiple, sensitive details like bank account number, IFSC code, etc., by capturing all information in a single verifiable UPI ID.
- UPI payments are based on 2-factor authentication in which the customer only needs to enter a single MPIN, unlike other cashless payment modes where users need to enter multiple details like name, password, OTP, and others.
- UPI only requires the presence of a mobile phone and internet connection which reduces the infrastructure needed by a very large amount.
- UPI does not work in "silos" as the involved parties need not be on the same interface.

The above features have brought about an exponential increase in the usage of UPI. While this is of great importance to policymakers and lawmakers as it enhances the digitalisation of the financial sector, it is of higher significance for banks as it has a tremendous economic impact. The coexistence of UPI with similar payment technologies offers customers with a choice. When faced with a choice, the decision often depends on the opportunity cost of each alternative. As has been established, UPI outperforms its peers on important parameters like time taken to complete the transactions, cost incurred per transaction, and convenience, among others. Currently, the UPI allows for non-banking firms also to operate on the common infrastructure. This has given rise to a scenario where the market for UPI transactions is largely dominated by three technology companies, none of them being banks. If this trend were to continue, the dynamics would result in banks losing out the major portion of their revenue coming from transaction charges to these tech firms. Hence the need to study the competition between existing digital payment technologies and UPI, and whether they can coexist gains importance. While an empirical approach can be adopted to examine these questions, the predictive powers of such analyses are limited due to the fact that empirical research involves the use of past data in which variations are inherent. On the other hand, using suitable mathematical models to study the various scenarios arising out of competition can prove to be superior in describing and predicting the interaction among players in the market under study.

Mathematical models have played a central role in the understanding phenomena in various fields, including natural and applied sciences. One such model which has been widely studied is the Lotka-Volterra model or the Predator-Prey Model. Propounded to understand the dynamic nature of population growth of different species competing against each other, the model has been extended and modified extensively to mimic real life scenarios to a great extent. Though the model was initially confined to the study of evolutionary theories, it later found extensive application in economics. It was evident to researchers that competition in markets involving multiple players was not dissimilar to dynamics present among competing species. Thus the Lotka-Volterra model, and its extensions, were used in various contexts to study the different phenomena arising in economics. Some of the popular applications include the study of competition between different sectors like agriculture, industry and agriculture in a country; study of competition between different industries in the economy; competition between different technologies within an industry; competition between firms at different stages seeking investment; dynamics between websites competing for same user base; and competition between different companies within the same market to list a few.

The investigation about real-world problems is always a hot topic in the present context. The efficiency of the predictor-corrector method is effectively illustrated by researchers in [2] in order to examine the SIR model of COVID-19; in extension with this, the stability is derived in [3] for the numerical technique, which helps to solve predator-prey model, the predator-prey model associated with prey refuge was investigated in [4], the effect of a numerical method to solve the atmospheric ocean model is illustrated in [5]. In order to prove the essence and significance of mathematical modelling in connection with real-world problems, the authors in [6–8] investigated the omicron and its earlier version and presented some useful results. The current study can be extended by generalizing the integer order derivative with fractional order; for instance, the stability of the integro-differential systems within the frame of fractional order is connected by researchers in [9], the hyper-chaotic system is examined with the help of novel fractional operator in [10], the physical model with unstable cases is investigated in [11], the numerical method for higher order fractional system is proposed by researchers in [12], the chemical reaction model is investigated with the efficient numerical scheme in [13], the scholars in [14–16] investigated the fractional order models with numerical approaches. These above-cited studies can help the readers to extend the present work.

The purpose of this paper is to study one such application of the Lotka-Volterra model, i.e., in the context of the market for digital payments in India. While the estimates and forecasts of the aggregate transactions can be obtained by time series methods, the competition element among the platforms cannot be found using the same. Thus the paper uses a combination of ARIMA and LV model to analyse the dynamic between the competing platforms, i.e., Conventional Digital Payments (CDP) consisting of NEFT, RTGS and Internet Banking and the revolutionary technology UPI.

2. Literature review

2.1. Economic applications of the Lotka-Volterra model

The Lotka-Volterra model has been applied extensively to understand the competing relationships in various business ecosystems. Apedaille et al. [17] use the predator-prey mechanism to model the shares of agricultural, industrial and exospheric wealth in the open interacting economic systems. One of the earliest and most well-known applications of the predator-prey model was given by Maurer and Huberman [18] which developed a model to explain the domination of the internet by certain websites. Watanabe et al. [19] apply the Lotka-Volterra model to forecast the transition from analogue broadcasting to digital broadcasting in the context of Japan. Lee et al. [20] study the interaction between competing technologies in communication systems by inputting patent data to the Lotka-Volterra model. Tsenf et al. use the Lotka-Volterra model to analyse competition between smartphone operating systems and thus attempt to forecast sales volumes. Lee and Oh [21] use the Lotka-Volterra model to analyse the competition between two rival markets namely the Korean Stock Exchange and the Korean Securities Dealers Automated Quotation. Ren et al. [22] studies competition among websites by dividing consumers into 'users' and 'visitors' and formulating a two-competitor model to find a situation (represented by a stable solution) where the competing website can coexist. Brander and de Bettignies [23] use the predator-prey model to provide a contributing explanation for both high-venture capital concentration by industry and 'boom and bust' industry-level investment dynamics. Kreng and Wang [24] use the Lotka-Volterra equations to model the competition between LCD and Plasma Display televisions. Chiang & Wong [25] considered the LV-model to estimate market diffusion by considering the competition between desktops and notebook computers. A similar study in the Indian context by Pant and Bagai [26] looks at the coexistence of the organised and unorganised sectors in the retail industry where use a modified Lotka-Volterra model was used to describe the competition between the two sectors. Crookes and Blignaut [27] use the predator prey model to stimulate the intersectoral dynamics of the steel sector. Hung et al. [28] apply an enhanced Lotka-Volterra model to study the competition between convenience-oriented and budget-oriented retail stores in Taiwan by decomposing data into three components and hence obtaining more efficient estimates as a result. Nikolaieva and Bochko [29] have studied the behaviour of the market share of operating systems using the Lotka-Verra model and subsequently tried to predict the market share for Android and iOS operating systems using numerical integration. Evidently, the Lotka-Volterra model is a reliable forecasting method for two or more competing species.

2.2. UPI technology in India

Gochhwal [1] point out that penetration of telecommunication, increase in bank coverage, elimination of the need to share sensitive bank details, and reduction in time and cost compared to pre-existing electronic payment services are factors favouring enhanced usage of UPI. Mohapatra [30] emphasises the proliferation of smartphones, availability of an online individual identity, universal access to banking and the introduction of biometric sensors in smartphones as some trends which would aid in further developing cashless payment technologies. Kakade and Veshne [31] establish that among the reasons for the widespread use of UPI is its 24x7 availability and emphasise its role in enhancing transaction efficiency and making India a cashless economy. Vipin and Sumathy [32] found that habitual use of cash and complexity in using digital payments was the main barriers for trying digital payments cited by the users. Patil [33] analyses the adoption of UPI and studies the demographic factors affecting UPI perception among consumers using primary data. They found that while the age of consumers did not influence the perceptions regarding usefulness and cost, it did influence the perception regarding ease of use. There was no significant difference in perception among different educational groups and income categories. Philip [34] analysed the impact of UPI on customers' satisfaction using primary data and found that UPI had a significant positive impact on customers, and perceptions of UPI and traditional payment methods varied significantly among consumers. Kumar et al. [35] analysed the security dimension of UPI and other payment apps in India and discovered unreported multi-factor flaws in the authentication design, making the interface vulnerable to significant potential attacks.

3. Theoritical Framework

3.1. ARIMA

Auto-Regressive Integrated Moving Average is a forecasting technique used to analyse time series data. This model is applicable in cases where data displays non-stationary behaviour (i.e., non-constancy with respect to mean but not with respect to variance). Non-stationarity thus arising can be dealt with using differencing techniques, i.e., differencing the data with itself one or more times.

An ARIMA(p,d,q) implies

$$\begin{aligned} {y'}_t &= c + \varphi_1 {y'}_{t-1} + \varphi_2 {y'}_{t-2} \cdots \varphi_p {y'}_{t-p} \\ &+\varepsilon_t - \theta_1\varepsilon_{t-1} - \theta_2\varepsilon_{t-2} \cdots \theta_q\varepsilon_{t-q}, \end{aligned} \tag{1}$$

where ${y'}_t$ is the differenced series with order of differencing d; p is the order of the Auto-regressive part and q is the order of the Moving-Average part.

3.2. Lotka-Volterra equations

The simple Lotka-Volterra model or the predator-prey model is a system of non-linear ordinary differential equations that describe the trajectories of the population of two interacting species, namely predator and prey, over a time period. It is given by

$$\begin{aligned} \frac{dx}{dt} &= \alpha x - \beta xy, \\ \frac{dy}{dt} &= \delta xy - \gamma y, \end{aligned} \tag{2}$$

where

- x is the number of preys,
- y is the number of predators,
- $\frac{dx}{dt}$ and $\frac{dy}{dt}$ represent the instantaneous growth rates of the two populations,
- t represents time,
- α, β, γ, δ are positive real parameters describing the interaction of the two species.

While the equations in system 1 represent the dynamics where species preys on another, the same can be extended to represent the dynamic where both the species prey on each other. Thus the growth in one both species influences the population of the other species negatively. Such a system is given by

$$\begin{aligned} \frac{dx}{dt} &= \alpha x - \beta x^2 - \gamma xy, \\ \frac{dy}{dt} &= \varphi y - \psi y^2 - \mu xy, \end{aligned} \tag{3}$$

where

- x is the number of species x,
- y is the number of species y,
- $\frac{dx}{dt}$ and $\frac{dy}{dt}$ represent the instantaneous growth rates of the two populations,
- t represents time,
- α, β, γ, φ, ψ, and μ are positive real parameters describing the interaction of the two species.

In the above system (3), α and φ are the per-capita birth rates (we may also consider them as overall per-capita growth rates) of x and y respectively and incorporate deaths (independent of the other species) as well as births. Thus they are per-capita growth rates, or per-capita reproduction rates, while the parameters β and ψ are the self-interaction parameters, which denote the decline in x and y in the absence of the other species. The parameters γ and μ are the interaction parameters and describe the competition between the species.

While the ARIMA process provides parameters that fit the behaviour of time series data, the estimates and the subsequent forecasts obtained from the model are valid only when certain assumptions made regarding the error terms are satisfied. The ARIMA estimation technique makes two major assumptions regarding the errors:

i There is no serial correlation among the error terms.
ii The error terms are normally distributed with constant mean and finite variance, i.e., $a_t \sim N(\mu, \sigma^2)$

Upon fitting the model and obtaining the best fit parameter values, various tests can be performed to check if the assumptions are satisfied in order to validate the results of the model. The first assumption can be checked by Box-Pierce Test, Box-Ljung Test among others. With respect to testing normality, standard testing procedures like the Shapiro-Wilk test can be used.

The last couple of decades have witnessed technology penetrating our lives in unimaginable ways. One such area where technology has had a significant impact in is the financial sector. With the advent of technology, payment mechanisms are undergoing paradigm shifts. Electronic payment systems offer various advantages over physical currency, like speed, security, lower transaction costs for individuals, elimination of counterfeit currency, and enhanced regulation. For this reason, Central Banks are not only promoting and facilitating digital payment mechanisms, but some are also mooting the idea of completely shifting to electronic transactions by replacing physical cash with central bank digital currency (CBDC). Based on the literature review, it can be understood that in a market where two or more firms compete against each other, the growth in the market share of one firm affects the market share of the other. In such cases, it is not possible to draw a clear distinction as to which firm is the predator and which firm is the prey. Thus, using a model in which both populations compete against each other, as in the system, would be more appropriate to analyse such a market. This leads us to the proposed model to describe the dynamics in the market for digital payments in India. Consider the system of equations

$$\begin{aligned} \frac{dU}{dt} &= \alpha_1 U - \beta_1 U^2 - \gamma_1 UC, \\ \frac{dC}{dt} &= \alpha_2 C - \beta_2 C^2 - \gamma_2 UC, \end{aligned} \tag{4}$$

where

- U is the market share of the UPI platform,
- C is market share of the Conventional Digital Payment Mechanisms like NEFT, RTGS, and Internet Banking,
- $\frac{dU}{dt}$ and $\frac{dC}{dt}$ represent the instantaneous growth rates of the two competing platforms,
- t represents time,
- α_1, $\beta_1\, \gamma_1, \alpha_2$, β_2, and γ_2 are real parameters describing the interaction of the two technologies
- no new technologies are introduced in subsequent periods.

As in system (3), α_1 and α_2 are the growth rate of the market shares of UPI and CDP platforms simultaneously. While the per capita growth rate in a biological context signifies the reproductive capacity of the species, in the context of market competition, they signify the ability of the concerned player to attract new customers. In this case, α_1 represents the ability of the UPI as a platform to induce existing users to repeat transactions in the successive time period as well as attract new users to perform transactions using this mode. A similar explanation follows for α_2. Intuitively, α_1 and α_2 have a positive impact on U and C respectively.

In system (4), β_1 and β_2 give the respective death rates of the population. It is technically the internal interaction within the species. In this context, β_1 for example, describes the rate at which users of the UPI platform withdraw from using it. A similar explanation follows for β_2.

On the other hand, the interaction parameters. γ_1 and γ_2 capture the competition between U and C in a given time period. In particular, γ_1 specifies the rate at which the UPI platform loses its users to Conventional Digital Payments. Similarly, γ_2 is the rate at which Conventional Digital Payments lose users to the UPI platform. The model makes some generalising assumptions. They are as follows:

- We assume the total number of users to be sufficiently large so that random fluctuations can be ignored without consequence

- We assume that the two system model reflects the market sufficiently accurately
- We assume each population grows exponentially in the absence of the other competitor
- We assume that access to both the platforms, level of awareness regarding both platforms, access to the internet, etc. are uniform across geographies and time periods
- Literature suggests that users of UPI have a great experience using the platform, hence reducing the chance of customer withdrawal which brings us to the assumption
$$0 < \beta_1 < \beta_2 < 1.$$
- Given that UPI outperforms its competitors we expect it to behave more predatorily. Hence we assume that
$$0 < \gamma_1 < \gamma_2 < 1.$$
- Most importantly we assume that there is no limit on the growth (i.e., carrying capacity) on the number of transactions in a platform

3.3. Stability analysis

In order for the system to be at equilibrium, the rate of change with respect to time must be zero, i.e., $\frac{dU}{dt}$ and $\frac{dC}{dt}$ must be equal to zero. We obtain the solutions for these by equating the right hand side of the respective equations to zero. By solving these we get two points where the slopes are equal to zero $P_1(0,0)$ and $P_2(\frac{\alpha_1\beta_2-\alpha_2\gamma_1}{\beta_1\beta_2-\gamma_1\gamma_2}, \frac{\alpha_2\beta_1-\alpha_1\gamma_2}{\beta_1\beta_2-\gamma_1\gamma_2})$.

Clearly, P_1 is a trivial solution as it indicates a situation where both the platforms have zero transactions and hence is of no interest to us. On the other hand, P_2 describes a situation where both platforms have a positive number of transactions and are of special interest to us.

The stability of this fixed point can be analysed using the Jacobian matrix:

$$J = \begin{vmatrix} \alpha_1 - 2\beta_1 U - \gamma_1 C & -\gamma_1 U \\ -\gamma_2 C & \alpha_2 - 2\beta_2 C - \gamma_2 U \end{vmatrix},$$

$$J\ (P_2) = \begin{vmatrix} \beta_1\frac{\alpha_2\gamma_1-\alpha_1\beta_2}{\beta_1\beta_2-\gamma_1\gamma_2} & -\gamma_1\frac{\alpha_1\beta_2-\alpha_2\gamma_1}{\beta_1\beta_2-\gamma_1\gamma_2} \\ -\gamma_2\frac{\alpha_2\beta_1-\alpha_1\gamma_2}{\beta_1\beta_2-\gamma_1\gamma_2} & \beta_2\frac{\alpha_1\gamma_2-\alpha_2\beta_1}{\beta_1\beta_2-\gamma_1\gamma_2} \end{vmatrix}.$$

The eigenvalues of the above matrix are calculated to determine the stability of the system at the above point. Since one eigenvalue is positive and one eigenvalue is negative, we infer that the fixed point P_2 is a saddle point.

3.4. Existence and uniqueness of Ssolution

Let $\mathcal{G}(J)$ be the Banach space with the maximal norm given by $\|x\| = \max_{t\in\mathcal{J}} \mid x(t) \mid$ where $\mathcal{J} = [0, \mathcal{T}_1]$ and $\mathcal{T}_1 = \mathcal{G}(J) \times \mathcal{G}(J)$.
Let us consider

$$F_1(t,U) = \alpha_1 U - \beta_1 U^2 - \gamma_1 UC,$$
$$F_2(t,C) = \alpha_2 C - \beta_2 C^2 - \gamma_2 UC.$$

Theorem 1. *The kernel F_1 and F_2 admit the Lipschitz condition and contraction when $0 \le \Lambda_1, \Lambda_2 < 1$, where $\lambda_1 = \alpha_1 - \beta_1(\epsilon_1 + \kappa_1) - \gamma_1\epsilon_2$, $\lambda_2 = \alpha_2 - \beta_2(\epsilon_2 + \kappa_2) - \gamma_2\epsilon_1$.*

Proof. We assume that the solution of the system is bounded, such that $\|U\| \le \epsilon_1$ and $\|C\| \le \epsilon_2$.
Consider two functions U and U^*, such that

$$\begin{aligned}
&\|F_1(t,U) - F_1(t,U^*)\| \\
&= \|(\alpha_1 U - \beta_1 U^2 - \gamma_1 UC) \\
&- (\alpha_1 U^* - \beta_1 U^{2^*} - \gamma_1 CU^*)\| \\
&= \|(\alpha_1 - \gamma_1 C)(U - U^*) - \beta_1(U + U^*)(U - U^*)\| \\
&\le (\alpha_1 - \beta_1(\epsilon_1 + \kappa_1) - \gamma_1\epsilon_2)\|(U - U^*)\| \\
&\le \lambda_1\|U - U^*\|,
\end{aligned}$$

where $\|U^*\| = \kappa_1$ and $\|C^*\| = \kappa_2$.
Consider two functions C and C^*, such that

$$\begin{aligned}
&\|F_3(t,C) - F_3(t,C^*)\| \\
&= \|(\alpha_2 C - \beta_2 C^2 - \gamma_2 UC) \\
&- (\alpha_2 C^* - \beta_2 C^{2^*} - \gamma_2 UC^*)\| \\
&= \|(\alpha_2 - \gamma_2 U)(C - C^*) - \beta_2(C + C^*)(C - C^*)\| \\
&\le (\alpha_2 - \beta_2(\epsilon_2 + \kappa_2) - \gamma_2\epsilon_1)\|(C - C^*)\| \\
&\le \lambda_2\|C - C^*\|.
\end{aligned}$$

□

Theorem 2. *The solution of the model exists and is unique.*

Proof. Let, $\mathcal{K} = \max_{(U,C)\in\Lambda}\{\|F_1(U)\|, \|F_2(C)\|\}$.
The integral form of the system is given by

$$U(t) = U_0 + \int_0^t F_1(U(\tau))d\tau,$$
$$C(t) = C_0 + \int_0^t F_2(C(\tau))d\tau.$$

Using the successive approximations of the solution of the integral equations, we get

$$U_{n+1}(t) = U_0 + \int_0^t F_1(U_n(\tau))d\tau,$$
$$C_{n+1}(t) = C_0 + \int_0^t F_2(C_n(\tau))d\tau.$$

The solutions are continuous and satisfy

$$\begin{aligned}\|U_{n+1}(t) - T_0\| &= \|\int_0^t F_1(U_n(\tau))d\tau\| \\ &\leq \int_0^t \|F_1(U_n(\tau))\|d\tau \\ &\leq \mathcal{K}t.\end{aligned}$$

Let $\max\|U_1(t) - U_0\| \leq b$. We show that $\|U_{n+1}(t) - U_n(t)\| \leq (a_1 t)^{k-1} b$ using principal of mathematical induction. For $n = 1$ consider

$$\begin{aligned}&\|U_2(t) - U_1(t)\| \\ &= \|U_0 + \int_0^t F_1(U_1(\tau))d\tau - U_0 - \int_0^t F_1(U_0(\tau))d\tau\| \\ &= \|\int_0^t (F_1(U_1(\tau)) - F_1(U_0(\tau)))\, d\tau\| \\ &\leq \int_0^t \|F_1(U_1(\tau)) - F_1(U_0(\tau))\|d\tau \\ &\leq a_1 \int_0^t \|U_1(\tau) - U_0(\tau)\|d\tau \\ &\leq a_1 \max\|U_1(t) - U_0\|t \\ &\leq a_1 bt.\end{aligned}$$

Assume that the inequality holds for some $k \in \mathbb{N}$, i.e., $\|U_k(t) - U_{k-1}(t)\| \leq (a_1 t)^{k-1} b$.
Then for some integer $k \geq 2$, it follows that,

$$\begin{aligned}&\|U_{k+1}(t) - U_k(t)\| \\ &= \|U_0 + \int_0^t F_1(f_k(\tau))d\tau - U_0 - \int_0^t F_1(U_{k-1}(\tau))d\tau\| \\ &= \|\int_0^t F_1(U_k(\tau)) - F_1(U_{k-1}(\tau))d\tau\| \\ &\leq \int_0^t \|F_1(U_k(\tau)) - F_1(U_{k-1}(\tau))\|d\tau \\ &\leq a_1 \int_0^t \|U_k(\tau) - U_{k-1}(\tau)\|d\tau \\ &\leq (a_1 t)^k b.\end{aligned}$$

Let $at = \gamma$. For some $m, n \geq N$ we get,

$$\begin{aligned}\|U_m(t) - U_n(t)\| &\leq \sum_{k=n}^{m-1} \|U_{k+1}(t) - UU_k(t)\| \\ &\leq \sum_{k=N}^{\infty} \|U_{k+1}(t) - U_k(t)\| \\ &\leq \sum_{k=N}^{\infty} (a_1 t)^k b \\ &= \sum_{k=N}^{\infty} \gamma^k b \\ &= \frac{\gamma^N}{1-\gamma} b.\end{aligned}$$

This tends to 0 as $N \to \infty$. Therefore, for all $\epsilon > 0$ there exists N such that for $m, k \geq N$,

$$\|U_m(t) - U_n(t)\| \leq \epsilon,$$

i.e., $\{U_n\}$ is a Cauchy sequence in $\mathcal{G}(J)$ and therefore converges uniformly to a function U. Taking the limit as $n \to \infty$ on both sides of the definition of successive approximation we see that the function

$$U(t) = \lim_{n\to\infty} U_n(t),$$

admits

$$U(t) = U_0 + \int_0^t F_1(U(\tau))d\tau.$$

Since $f(t)$ is continuous, $F_1(U(t))$ is also continuous and using the Fundamental theorem of Integral Calculus, we get $U^{'}(t) = F_1(U(t))$. Similarly, we can show that C_n is a Cauchy sequence that converges uniformly $C(t)$, and we can obatine $C^{'}(t) = F_2(C(t))$. Furthermore, $U(0) = U_0$ and $C(0) = C_0$. Therefore $U(t), C(t)$ is a solution of the system.
Suppose $\bar{U}(t), \bar{C}(t)$ is another set of solution for the system. Now, consider

$$\begin{aligned}\|U - \bar{U}\| &= \|U_0 + \int_0^t F_1(U(x))dx - f_0 \\ &\quad - \int_0^t F_1(\bar{f}(\tau))d\tau\| \\ &= \|\int_0^t F_1(U(\tau)) - \int_0^t F_1(\bar{U}(\tau))d\tau\| \\ &\leq \int_0^t \|F_1(U(\tau)) - F_1(\bar{U}(\tau))\|d\tau \\ &\leq a_1 \int_0^t \|U(\tau) - \bar{U}(\tau)\|d\tau \\ &\leq a_1 t\|U - \bar{U}\|.\end{aligned}$$

Since $a_1 t < 1$, the inequality is satisfied only when $\|U - \bar{U}\| = 0$. Thus, $U(t) = \bar{U}(t)$. Similarly we can show $C(t) = \bar{C}(t)$. Therefore, the system has a unique solution. □

3.5. Boundedness

Theorem 3. *The solution of the model is uniformly bounded.*

Proof. Let $P(t) = U(t) + C(t)$. Taking the derivative along with the control parameter, we get

$$\begin{aligned}&\left(\frac{d}{dt} + \phi_1(t)\right)(P(t)) \\ &= \frac{d}{dt}[U(t) + C(t)] + \mu_1(t)[U(t) + C(t)] \\ &= \alpha_1 U - \beta_1 U^2 - \gamma_1 UC + \alpha_2 C - \beta_2 C^2\end{aligned}$$

$$- \gamma_2 UC + \phi_1(t)[U(t) + C(t)]$$
$$\leq \alpha_1 U + \alpha_2 C + \phi_1(t)[U(t) + C(t)].$$

The solution exists and is unique in

$$\Lambda = \{U, C) \in \mathbb{R}^2 : max(\mid U \mid, \mid C \mid) \leq \epsilon\}.$$

The previous inequality yields

$$\left(\frac{d}{dt} + \phi_1(t)\right)(P(t)) \leq \epsilon\left[\alpha_1 + \alpha_2 + 2\phi_1(t)\right].$$

Therefore, the solution of the system is bounded.

4. Methodology

4.1. Data

For the purpose of this study, real-time data regarding transactions facilitated by the different transforms are considered. In order to measure the activity happening on each platform, the total volume of transactions in each month is considered. Data was collected for two variables: Conventional Digital Payments (CDP) which is the sum of all transactions happening through NEFT, RTGS and Internet Banking platforms and UPI which is the volume of transactions happening through UPI platforms. The data for CDP was collected from the RBI, while data for UPI was sourced from the NCPI. The data was collected for a period of 62 months starting from January 2017 to February 2022.

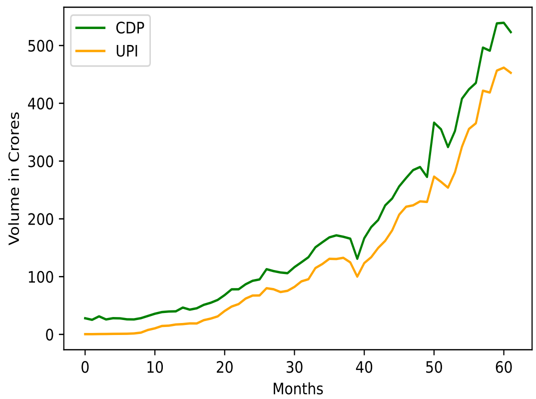

Figure 1. Volume of UPI and CDP Transactions 2017-22.

It is evident from Figure 1 that there is an explicitly increasing trend in the volume of transactions of both conventional digital payments as well as the UPI platforms. However, when the market share of each platforms is considered, there is a clear indication of competition among the two platforms. As seen in Figure 2, the UPI platform has witnessed a phenomenal increase in market share whereas the former has seen a consistent decline.

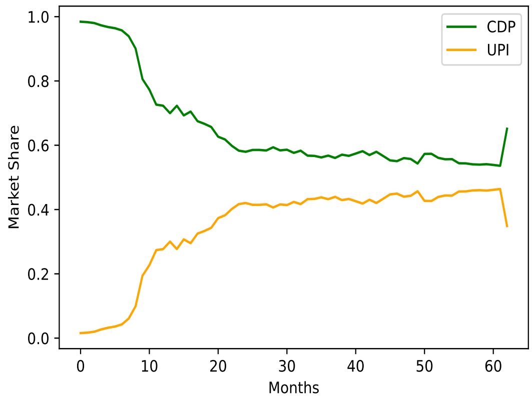

Figure 2. Market Share of UPI and CDP 2017-22.

4.2. Forecasting Method

The proposed methodology for estimating the market share of the respective platforms are captured in Figure 3. The estimation procedure consists of two modules. The first module involves estimating the total volume of transactions, $\hat{V}$, using time series techniques. The time series technique suitable for this purpose would be ARIMA as the volume of the transaction contains no significant seasonal or cyclical component. The second module is concerned with estimating the market share of each of the platforms i.e., U and C, which is determined by their respective competitive natures, using the Lotka-Volterra equations. The particular values of the parameters are obtained from real data and plugged into system 3. The above system of equations is of a non-linear kind and cannot be solved using known methods. Hence we need to use some numerical methods to obtain an approximate solution. For the purpose of estimating the market shares in different time periods, we propose to use the fourth order Runge-Kutta method. Once the estimates of the market share are obtained, it is combined with the ARIMA estimate to obtain the estimates of the volume of transactions in individual platforms, $\widehat{V_U}$ and $\widehat{V_C}$.

5. Results and Discussion

The fourth order Runge-Kutta method is employed to obtain a numerical solution of the market share. The iterative method is employed after using real data to estimate the value of the required parameters. Based on the collected data, the following values of the parameters in system 2 is chosen

$$\alpha_1 = 0.0664, \quad \beta_1 = 0.0005, \quad \gamma_1 = \ 0.02,$$

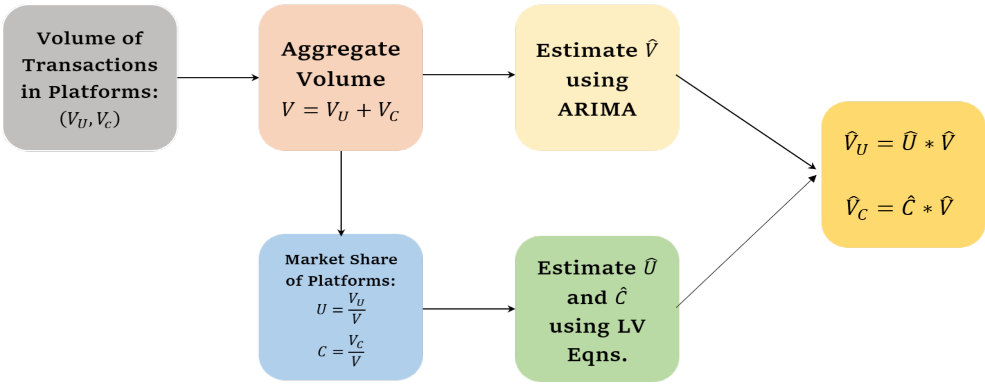

Figure 3. Proposed Forecasting Procedure

$$\alpha_2 = 0.0096, \quad \beta_2 = 0.0009, \quad \gamma_2 = 0.1,$$

and substituted in system 3. The results thus obtained are presented are in Figure 4. It can be observed that there is a progressive decline in the market share of CDP which is consistent with the trend established by the real data in Figure 2. Similarly, the market share of UPI is seen to witness continuously, which is again consistent with the trend established by real data. The results of the LV equations also establish that the growth in market share for UPI, and the decline in market share of CDP, reduces gradually. This is made evident by the plateauing and the stabilizing of the respective curves. The stability analysis of the same has been attached with the Appendix.

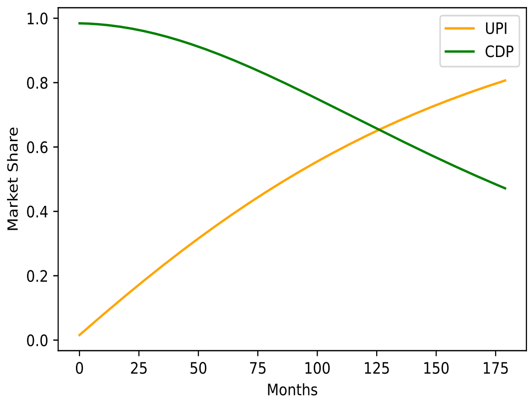

Figure 4. Estimated Market Share of UPI and CDP.

The ARIMA procedure requires that the errors of the fitted model are distributed normally. This assumption is not satisfied by the variable under consideration. Hence a log transformation is employed to ensure that error terms are normally distributed. The diagnostic tests of the new fitted model is added in the appendix. The new variable provides the best fit model to be ARIMA (0, 1, 0). The estimated equation is given as

$$\hat{V}_t = \ 0.0580 + \hat{V}_{t-1} + \ \varepsilon_t.$$

This signifies a positive association between the current values and the previously estimated terms. The values forecasted using the above equation are presented in Figure 5. The forecast is in line with the behaviour observed in the real data which shows a continuously increasing trend.

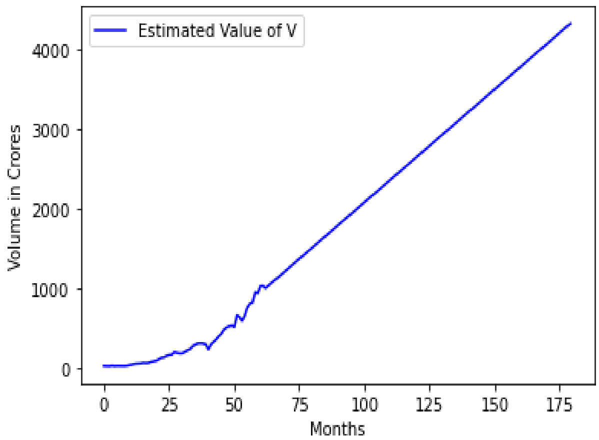

Figure 5. ARIMA estimate of aggregate volume.

The estimated value of the total number of transactions ($\hat{V}$) and the estimated market share of each platform (U and C) when combined, give the estimates of the volume of transactions in individual platforms, $\widehat{V_U}$ and $\widehat{V_C}$. The result of the same can be seen in Figure 6 which shows an almost exponential increase in the volume of transactions using the UPI platform. The volume of transactions on the CDP platforms on the other hand witnessed a steady increase before stabilizing after a given time period at a certain level. Just as in the case of the market share estimates, the volume of the UPI platform overtakes the volume of the CDP platform at a particular point in time.

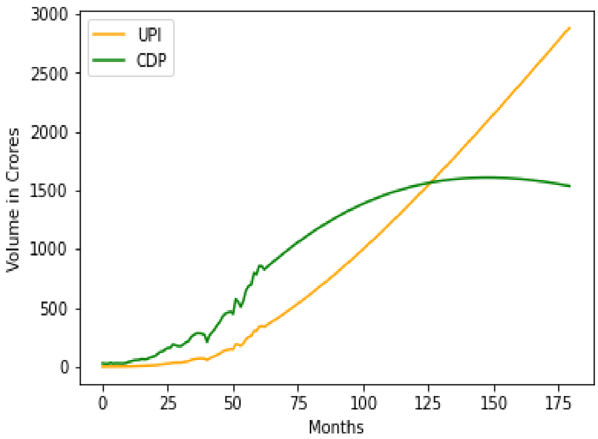

Figure 6. Estimated Volumes of UPI and CDP.

The above results establish certain phenomena explicitly. The first clear trend that emerges is that digital payment transactions in general witness a near exponential growth. The increase in the volume of digital payments is driven by an increase in both UPI and CDP platforms. This fits well with economic intuition and empirical evidence. As the Indian economy grows, and as the greater portion of it gets formalised, it would lead to greater adoption of digital payment mechanisms. This would in turn lead to an increase in the volume as well as the value of transactions being processed in each platform as suggested by the forecasts.

The second trend, and the one which is of more interest to us, is the coexistence of the two platforms in current as well as future time periods. This can be explained by some of the economic and policy related features of the market for digital payments. For example, there exists an upper limit on the value of the transaction that can be carried out using the UPI platform. This naturally shifts a finite portion of the market to conventional digital payment platforms which enable the transfer of money above a certain limit. Literature also suggests the existence of concerns among users regarding security, veracity and accessibility with regard to the UPI platform. Such concern may result in the CDP platforms retaining a certain market share despite the phenomenal growth of UPI. This seeming anomaly of decreasing market share but the increasing volume of the CDP platform can be understood in the perspective of the first trend. While the volume of total digital transactions increases, this causes an increase in the volume of transactions on the CDP platforms owing to the expansionary nature of the economy. The enormous increase in the volume of transactions on the UPI platform thus does not necessarily imply a shift in the user ship from one platform to another. While the rise in user ship of UPI up to a certain point (represented by the point where the two curves intersect) can be attributed to a shift from the CDP platforms, the volume of UPI transactions continues to rise beyond this point while the volume of transactions on CDP platforms stabilises. One possible explanation for this could be that while CDP platforms retain the high value transactions, UPI platforms gain popularity among low value transactions, replacing cash.

6. Concluding remarks

The purpose of this paper was to analyse the dynamics in the market for digital payments in India. The interaction between two competing platforms, conventional digital payments (which consist of NEFT, RTGS and Internet Banking) and the Unified Payments Interface, was examined using the Lotka-Volterra system of equations. The estimates of the competition element were combined with the estimate of the volume of transactions obtained using the ARIMA procedure to forecast the trends in the volume of transactions of the two platforms. The forecasts revealed that the volume of transactions in such platforms would increase manifold, thus highlighting the trend of digitalization of the economy. The results also suggest that the market share occupied by UPI would eventually overtake the market share of other platforms. However, the former would later exhibit a lack of growth and the latter a lack of decline, thus hinting at coexistence. While the results of the model do not indicate the extinction of services by banks, it asserts the supremacy of technological innovation by predicting that the technologically advanced UPI platform will dominate the market. This is a clarion call to banks and other financial institutions to explore, adopt and invest in new technologies if they seek to maintain their dominance over the financial sector.

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