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The system optimization perspective for multiproduct supply chain network

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Abstract. In this paper, multiproduct supply chain network model is developed with system optimization perspective. Each kind of products has an individual cost function and, at the same time, contributes to its own and other product's cost function in an individual way. The well-known equilibrium algorithm is extended to find system optimization pattern for such multiproduct supply chain network.

Keywords: System optimization, multiproduct supply chain, total cost minimization.
AMS Classification: (90c25)

1. Introduction

Today, supply chains are more extended and complex than ever before. At the same time, the current competitive economic environment requires that firms operate efficiently, which has spurred interest among researchers as well as practitioner to determine how to utilize supply chains more effectively and efficiently.

Furthermore, although there are numerous articles discussing multi-echelon supply chains, the majority deal with a homogeneous product, but, in application, deal is performed with multiproduct supply chain. Therefore, homogenous restriction is relaxed and worked with multiproduct in the same supply chain network. In this paper, system optimization perspective is utilized which minimizes the total cost in the system. So, we extend equilibrium algorithm to construct system-optimizing flow pattern [3].

Note that Min and Zhou [5] provided a synopsis of supply chain modeling and the importance of planning, designing, and controlling the supply chain as a whole. Nagurney, [6] subsequently, proved that supply chain network equilibrium problems, in which there is cooperation between tiers, but competition among decision-makers within a tier, can be reformulated and solved as transportation network equilibrium problems. Cheng and Wu [2] proposed a multiproduct, and multi criterion, supply-demand network equilibrium model. Davis and Wilson [4], in turn, studied differentiated product competition in an equilibrium framework.

2. Supply Chain Network Structure

Assume that multiproduct supply chain network involves a firm A, as depicted in figure 1. Let, $G = [N, L]$, denote the graph consisting of nodes $[N]$ and directed links $[L]$. Firm A is involved in the production, storage, and distribution of J products, denote typical product by superscript j. Assume that, firm A, has n_M manufacturing facilities, $M_1, \cdots, M_{n_M}$ and n_{D1} distribution

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centers that are denoted by $D_{1,1},\cdots,D_{n_D,1}$ without storage and n_{D2} distribution center with storages are denoted by $D_{1,2},\cdots,D_{n_D,2}$. A path consists of a sequence of links originating at a node A and denotes supply chain activities comprising manufacturing, storage, and distribution of the products to the retail nodes. Assume that, node A, to be the origin, R_k, $k=1,\ldots,n_R$ be the destinations and every origin-destination pair, O/D, be denoted by w. Let x_p^j, denote the nonnegative flow of product j, on path p, f_a^j, flow of product j on link a and let p_w, denote the set of paths connecting the origin/destination pair w. Also, let P, denote the set of all paths in the network and W, denote the set of all O/D pairs of nodes. The path flows group into the vector x and link flows into the vector f. In other words, the following notations is used:

$$x\equiv\{x_p:p\in P\},\quad x_p\equiv(x_p^1,\ldots,x_p^J)$$

$$f\equiv\{f_a:a\in L\},\quad f_a\equiv(f_a^1,\ldots,f_a^J)$$

Assume that, $d_{R_k}^j$, denote the demand for product j; $j=1,\ldots,J$ at retail outlet R_k; $k=1,\ldots,n_R$ associated with firm A; that is briefly denoted by d_w^j along with

$$D\equiv\{d_w:w\in W\},\quad d_w\equiv(d_w^1,\ldots,d_w^J).$$

The links from the top-tiered A, to the manufacturing nodes $M_1,\ldots,M_{n_M}$, in figure 1, represent the manufacturing links. The links from the manufacturing nodes, in turn, to the distribution center nodes, correspond to the shipment links. The links joining first distribution center nodes and second distribution center nodes correspond to the storage links for the products. Finally, the links joining second distribution center nodes to retails correspond to the storage links for the products.

In this paper, supply chain model is constructed to include several products and flow of each product effects other products so that each product has an individual cost function and, at the same time, contributes to its own and other product's cost function in a particular way. An extended equilibrium algorithm is developed to construct an optimal flow pattern in which the total cost in network is minimized. Modified equilibrium algorithm is defined as composition of operators for several products in the same network in comparison with prior methods that first, the multiproduct supply chain network was converted into single-product network. Therefore, this method needs a few number of iteration for converging in comparison with other methods as discussed in [8].

Let, $c_a^j(f_a^1,\ldots,f_a^J)$, denote the cost of one unit shipment of product j, $j=1,\ldots,J$ on link a, which is a function of other product's flows on same link. That is:

$$c_a^j=c_a^j(f_a^1,\ldots,f_a^J),\quad j=1,\ldots,J,\quad \forall a\in L. \tag{1}$$

In other word,

$$C\equiv\{c_a:a\in L\},\quad c_a\equiv(c_a^1,\ldots,c_a^J)$$

The total cost shipment f_a^j unit for j product on link a that are denoted by $\hat{c}_a^j(f_a^1,\ldots,f_a^J)$, define as follows:

$$\hat{c}_a^j(f_a^1,\ldots,f_a^J)=c_a^j(f_a^1,\ldots,f_a^J)\times f_a^j. \tag{2}$$

That is, the total cost on a link a, is equal to the link cost on the link times the flow on the link. The total cost on a path p, $\hat{C}_p$, is given by the sum of the product costs on the links that comprise the path, that is:

$$\hat{C}_p^j=\sum_{a\in L}\hat{c}_a^j(f_a^1,\ldots,f_a^J)\delta_{ap},\quad \forall p\in P, \tag{3}$$

where $\delta_{ap}=1$, if link a is contained in path P and $\delta_{ap}=0$, otherwise.

In the system optimized problem, the total cost in network is minimized, where the total cost in network is given by:

$$\sum_{a\in L}\sum_{j=1}^{J}\hat{c}_a^j(f_a^1,\ldots,f_a^J). \tag{4}$$

Perhaps, the simplest nontrivial example of a cost function is provided by linear model:

$$c_a^j(f_a^1,\ldots,f_a^J)=\sum_{l=1}^{J}g_a^{jl}f_a^l+h_a^j,\forall a\in L,j=1,\ldots,J \tag{5}$$

And accordingly, the total cost functions are nonlinear (quadratic), given by:

$$\hat{c}_a^j(f_a^1,\ldots,f_a^J)=\sum_{l=1}^{J}g_a^{jl}f_a^jf_a^l+h_a^jf_a^j, \tag{6}$$

for all $a\in L$ and $j=1,\ldots,J$, where g_a^{jl} , h_a^j are given constants.

The following conservation of flow equations must hold for firm A in which, each product j, and each retail outlet R_k:

$$\sum_{p\in P_{R_k}}x_p^j=d_{R_k}^j,\quad j=1,\ldots,J,\quad k=1,\ldots,n_R, \tag{7}$$

That is, the demand for each product must satisfy each retail outlet.

A flow pattern x, with the demand D, is called feasible, if equation (7) satisfies for every $w\in W$.

Every flow pattern x, generates a load pattern f and x is called compatible with f.

A flow pattern f, is called feasible, if there exists at least one feasible flow pattern x, compatible with f and following flow equation holds:

$$f_a^j = \sum_{p \in P} x_p^j \delta_{ap}, \quad j = 1, \ldots, J, \quad \forall a \in L, \quad (8)$$

where $\delta_{ap} = 1$, if link a is contained in path P and $\delta_{ap} = 0$, otherwise.

Expression (8) states that the flow on a link a is equal to the sum of all the path flows on path p containing link a.

The path flows must be nonnegative, that is:

$$x_p^j \geq 0, \quad j = 1, \ldots, J, \quad \forall p \in P. \quad (9)$$

Assume that, the total cost function for each product on each link is convex, continuously differentiable, and it has bounded third order partial derivatives. The multiproduct supply chain cost minimization problem can be formulated jointly as follows:

$$\text{Minimize} \quad \sum_{a \in L} \sum_{j=1}^{J} \hat{c}_a^j(f_a^1, \ldots, f_a^J) \quad (10)$$

Subject to :

$$\sum_{p \in P_{R_k}} x_p^j = d_{R_k}^j, j = 1, \ldots, J, k = 1, \ldots, n_R$$

$$\sum_{p \in P} x_p^j \delta_{ap} = f_a^j, \quad j = 1, \ldots, J, \quad \forall a \in L$$

$$x_p^j \geq 0, \quad j = 1, \ldots, J, \quad \forall p \in P$$

Observe that, this problem is a system optimization problem.

The triple $T = (G, D, C)$ will be called a supply chain network, where G is a directed network, D is a demand vector and C is a cost vector as defined before.

Definition 1. *For a given multiproduct supply chain network $T = (G, D, C)$, a feasible flow pattern x that minimizes the total cost function*

$$TC = \sum_{a \in L} \sum_{j=1}^{J} \hat{c}_a^j(f_a^1, \ldots, f_a^J), \quad (11)$$

is called a system optimization flow pattern.

Theorem 1. *For a given multiproduct supply chain network $T = (G, D, C)$, suppose that in (10), $TC(f)$, is strictly convex and its feasible set is convex. Then, there is a unique system-optimizing flow pattern, such that $TC(f)$ is the minimum of TC.*

Proof. Straight forward. □

Theorem 1, concludes that, every feasible flow pattern x, compatible with f is a system-optimizing flow pattern. Thus, a system-optimizing flow pattern always exists and, in particular, is unique, if and only if there exists a unique feasible flow pattern x compatible with f.

In addition to the above existence and uniqueness theorem, the assumption of convexity of $TC(f)$ implies that the system-optimizing flow pattern satisfies the (K.K.T.)[1] conditions [1].

Theorem 2. *For a given $T = (G, D, C)$, x, is a system-optimizing flow pattern if and only if it enjoys the following property. Let $w \in W$, be connected by the paths $p_1, \ldots, p_m$, then these paths can be numbered as:*

$$\begin{aligned} \hat{C}'_{p_1}(f) &= \cdots = \hat{C}'_{p_s}(f) \quad (12) \\ &= \lambda_w \leq \hat{C}'_{p_{s+1}}(f) \leq \cdots \leq \hat{C}'_{p_m}(f) \\ & x_{p_r} > 0 \quad r = 1, \ldots, s \\ & x_{p_r} = 0 \quad r = s+1, \ldots, m, \end{aligned}$$

where the total marginal cost is denoted by

$$\hat{C}'_p(f) \equiv \sum_{j=1}^{J} \sum_{l=1}^{J} \sum_{a \in L} \frac{\partial \hat{c}_a^l(f_a^1, \ldots, f_a^J)}{\partial f_a^j} \delta_{ap}, \quad (13)$$

Proof. Straight forward. □

Therefore, all used paths have equal and minimal total marginal costs and unused paths have higher (or equal) total marginal costs than those of the used paths.

In fact, a system-optimizing solution corresponds to Wardrop's second principle [7] and is one that minimizes the total cost and all utilized paths connecting each O/D pair have equal and minimal marginal total costs.

3. Algorithm

In this section, an extended variant of well-known equilibrium algorithm is developed to find the system optimization pattern for a multiproduct supply chain network. The algorithm constructs a system-optimizing flow pattern by iteration, i.e., starting from an arbitrary initial feasible flow pattern [2] x^0, it generates a sequence $\{x^n\}$ of feasible flow patterns converging to the set of system-optimizing flow patterns. The passage from x^{n-1} to x^n is attained by applying an operator E, i.e., $x^n = Ex^{n-1}$. Once E has been defined, the description of the algorithm is complete.

[1] Karush-Kuhn-Tucker

[2] Such a flow pattern can be obtained with all-nothing assignment method.

Let $Z[T]$ stand for the set of all feasible flow patterns of the supply chain network T. An operator

$$E : Z[T] \to Z[T] \tag{14}$$

is defined as the composition

$$E = E_{w(m)} \circ \ldots \circ E_{w(1)} \tag{15}$$

of operators

$$E_{w(l)} : Z[T] \to Z[T], \quad l = 1, \ldots, m, \tag{16}$$

where $w(1), \ldots, w(m)$ is an arbitrary ordering of the set W. In turn, $E_{w(l)}$ will be defined as the composition

$$E_{w(l)} = E^J_{w(l)} \circ \cdots \circ E^1_{w(l)} \tag{17}$$

of operators

$$E^j_{w(l)} : Z[T] \to Z[T], \quad j = 1, \ldots, J, \tag{18}$$

where E^j_w sends a feasible flow pattern x to another feasible flow pattern $\hat{x} = E^j_w x$, which is constructed by the following procedure.

Among the elements of P_w, determine the paths q and r requiring with minimum and maximum total marginal cost

$$\begin{aligned} \hat{C}'^j_q(f) &= \min_{p \in P_{R_k}} \{\hat{C}'^j_p(f)\} \\ \hat{C}'^j_r(f) &= \max_{p \in P_{R_k}, x^j_p > 0} \{\hat{C}'^j_p(f)\} \end{aligned} \tag{19}$$

where $\hat{C}'^j_p(f)$ denotes the total marginal cost on path p for product j, given by

$$\hat{C}'^j_p(f) \equiv \sum_{l=1}^{J} \sum_{a \in L} \frac{\partial \hat{c}^l_a(f^1_a, \ldots, f^J_a)}{\partial f^j_a} \delta_{ap}, \tag{20}$$

and then set

$$\begin{aligned} \hat{x}^l_p &= x^l_p \quad l \neq j, \quad p \in P \\ \hat{x}^j_p &= x^j_p \quad p \neq q, \quad p \neq r \\ \hat{x}^j_q &= x^j_q + \delta \\ \hat{x}^j_r &= x^j_r - \delta, \end{aligned} \tag{21}$$

where δ is selected so that the total cost $TC(\hat{f})$ is minimized over the class of admissible load patterns $\hat{f}$ that are induced by the class of flow patterns $\hat{x}$ given by (8). In the case of the quadratic model, δ can be determined explicitly through the formula

$$\delta = \min\{x^j_r, \frac{\hat{C}^{j\prime}_r(f) - \hat{C}^{j\prime}_q(f)}{2\sum_{a \in L} g^{jj}_a (\delta_{aq} - \delta_{ar})^2}\}. \tag{22}$$

In fact, the equilibrium algorithm conveys the flow from paths with positive and maximum flow to paths with minimum flow till the flow in network is equilibrated.

Theorem 3. *Let f be the (unique) system optimizing flow pattern. For any $x^0 \in Z$, let $x^n \equiv E^n x^0$ where f^n denotes the flow pattern induce by x^n. Then*

$$f^n \to f \quad as \quad n \to \infty.$$

Proof. The proof is similar to the proof of Theorem 3.1 in Dafermos and Sparrow [3]. □

4. Numerical Example

In this section, a numerical example is presented to demonstrate the algorithm for a simple 2-typical-product supply chain network so that the reader can become familiar with the realization of the operators E^j_w , E_w, and E.

In practice it is important to decide for which $n.x^n$ is sufficiently close to a system-optimizing flow pattern in order to stop the algorithm.

Characteristics of the Network

Set of nodes: $N = \{A, M_1, M_2, D_{1,1}, D_{1,2}, R_1, R_2\}$
Set of links: $L = \{1, 2, 3, 4, 5, 6, 7\}$ as defined in Table 1
Set of admissible paths: $P = \{p_1, p_2, p_3, p_4\}$, $p_1 = (1, 3, 5, 6), p_2 = (1, 3, 5, 7), p_3 = (2, 4, 5, 6)$, $p_4 = (2, 4, 5, 7)$.
Set of connected pair of nodes: $W = \{w_1, w_2\}$ with $w_1 = (A, R_1),\ w_2 = (A, R_2)$
Set of paths which connect $w_1 : P_{w_1} = \{p_1, p_3\}$
Set of paths which connect $w_2 : P_{w_2} = \{p_2, p_4\}$
Cost structure: Suppose that the quadratic model is applied, i.e., the total shipment cost for products of typical j on a link a of the network is of the form

$$\hat{c}^j_a = \sum_{l=1}^{2} g^{jl}_a f^j_a f^l_a + h^j_a f^j_a, \quad \forall a \in L, \quad j = 1, 2,$$

where, g^{jl}_a , h^j_a are given constants.

Also, the set of demands are given by $D = \{d_{w_1}, d_{w_2}\}$, $d_{w_1} = \{d^1_{w_1}, d^2_{w_1}\}$, $d_{w_2} = \{d^1_{w_2}, d^2_{w_2}\}$ with $d^1_{w_1} = d^2_{w_1} = d^1_{w_2} = d^2_{w_2} = 5$. The marginal costs correspond to the products of each typical product along the paths of the network is evaluated by using equation (20) and then application of the algorithm can be performed. First, an initial feasible flow pattern x^0 is selected, so that, it equally distributes the demands among the available paths, i.e.,

$$x^1_1 = x^2_1 = x^1_2 = x^2_2 = x^1_3 = x^2_3 = x^1_4 = x^2_4 = 2.5,$$

where x^j_i is an abbreviation of $x^j_{p_i}, i = 1, 2, 3, 4$. The resulting feasible flow pattern is

$$\begin{aligned} &f^1_1 = 5,\ f^1_2 = 5,\ f^1_3 = 5,\ f^1_4 = 5, \\ &f^1_5 = 10,\ f^1_6 = 5,\ f^1_7 = 5, \\ &f^2_1 = 5,\ f^2_2 = 5,\ f^2_3 = 5,\ f^2_4 = 5, \\ &f^2_5 = 10,\ f^2_6 = 5.\ f^2_7 = 5. \end{aligned}$$

Now by applying operators $E^1_{w_1}, E^2_{w_1}, E^1_{w_2}, E^2_{w_2}$, the feasible flow pattern can be improved. Until, a system-optimizing flow pattern is obtained.

The paths with minimum and maximum total marginal costs in network and the total marginal costs for these paths and δ values in each iteration was reported in Table 2.

Table 3 and 4 contain the link and path flow patterns obtained when $E^1_{w_1}$ through $E^2_{w_2}$ are applied, so that the resulting optimal flow patterns in Table 5 are the system-optimizing flow patterns for this network.

Therefore, the minimum total cost in supply chain network for optimal flow pattern is obtained as following:

$$TC(f) = \sum_{a\in L}\sum_{j=1}^{J}\sum_{l=1}^{J} g_a^{jl} f_a^j f_a^l + h_a^j f_a^j$$

5. Conclusion

In this paper, multiproduct supply chain network model is constructed utilizing a system-optimization perspective and the total cost is minimized in the network. Therefore, system-optimizing pattern is necessary in order to optimize the network. Since the deal is with multiproduct, modified equilibrium algorithm is defined as composition of operators. In fact, system-optimizing flow pattern can be determined with the extended equilibrium algorithm, so that total cost for sending products in supply chain network is minimized.

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Table 1. Definition of links and associated personal link cost functions

$Link a$	$From Node$	$To Node$	$\hat{c}_a^1(f_a^1, f_a^2)$	$\hat{c}_a^2(f_a^1, f_a^2)$
1	A	M_1	$1(f_1^1)^2 + 2f_1^2 f_1^1 + 11f_1^1$	$2(f_1^2)^2 + 2f_1^1 f_1^2 + 8f_1^2$
2	A	M_2	$2f_2^1)^2 + 2f_2^2 f_2^1 + 8f_2^1$	$1(f_2^2)^2 + 2f_2^1 f_2^2 + 6f_2^2$
3	M_1	$D_{1,1}$	$3(f_3^1)^2 + 2.5f_3^2 f_3^1 + 7f_3^1$	$4(f_3^2)^2 + 2.5f_3^1 f_3^2 + 7f_3^2$
4	M_2	$D_{1,1}$	$4(f_4^1)^2 + 1.5f_4^2 f_4^1 + 3f_4^1$	$3(f_4^2)^2 + 1.5f_4^1 f_4^2 + 11f_4^2$
5	$D_{1,1}$	$D_{1,2}$	$1(f_5^1)^2 + f_5^2 f_5^1 + 6f_5^1$	$4(f_5^2)^2 + f_5^1 f_5^2 + 11f_5^2$
6	$D_{1,2}$	R_1	$3(f_6^1)^2 + 1.5f_6^2 f_6^1 + 4f_6^1$	$4(f_6^2)^2 + 1.5f_6^1 f_6^2 + 10f_6^2$
7	$D_{1,2}$	R_2	$4(f_7^1)^2 + f_7^2 f_7^1 + 7f_7^1$	$2(f_7^2)^2 + 2f_7^1 f_7^2 + 8f_7^2$

Table 2. The paths with minimum and maximum total marginal costs in network and δ values in each iteration by applying $E_{w_1}^1$ through $E_{w_2}^2$

$Apply$	$E_{w_1}^1$	$E_{w_1}^2$	$E_{w_2}^1$	$E_{w_2}^2$
q	p_1	p_3	p_2	p_4
r	p_3	p_1	p_4	p_2
$\hat{C}_q'^j(f)$	198	297.45	208.40	252.5475
$\hat{C}_r'^j(f)$	201	264.45	228.55	267.6675
δ	0.15	1.65	1.0075	0.756

Table 3. The resulting path flow patterns by applying $E_{w_1}^1$ Through $E_{w_2}^2$

$Path\ P$	$E_{w_1}^1.\ x_p^1$	$E_{w_1}^1.\ x_p^2$	$E_{w_1}^2.\ x_p^1$	$E_{w_1}^2.\ x_p^2$	$E_{w_2}^1.\ x_p^1$	$E_{w_2}^1.\ x_p^2$	$E_{w_2}^2.\ x_p^1$	$E_{w_2}^2.\ x_p^2$
p_1	2.65	2.5	2.65	0.85	2.65	0.85	2.65	0.85
p_2	2.35	2.5	2.35	2.5	3.3575	2.5	3.3575	1.744
p_3	2.5	2.5	2.5	4.15	2.5	4.15	2.5	4.15
p_4	2.5	2.5	2.5	0.85	1.4925	2.5	1.4925	3.256

Table 4. The resulting links flow patterns by applying $E_{w_1}^1$ Through $E_{w_2}^2$

$Link\ a$	$E_{w_1}^1.\ f_a^1$	$E_{w_1}^1.\ f_a^2$	$E_{w_1}^2.\ f_a^1$	$E_{w_1}^2.\ f_a^2$	$E_{w_2}^1.\ f_a^1$	$E_{w_2}^1.\ f_a^2$	$E_{w_2}^2.\ f_a^1$	$E_{w_2}^2.\ f_a^2$
1	5	5	5	5	5	3.35	6.0075	3.35
2	5	5	5	5	5	6.65	3.9925	6.65
3	5	5	5	5	5	5	6.0075	3.35
4	5	5	5	5	5	6.65	3.9925	6.65
5	10	10	10	10	10	10	10	10
6	5.15	5	5.15	5	5.15	5	5.15	5
7	4.85	5	4.85	5	4.85	5	4.85	5

Table 5. The resulting optimal flow patterns

$Link\ a$	$From Node$	$To Node$	f_a^{1*}	f_a^{2*}
1	A	M_1	5.4219	0.8000
2	A	M_2	4.5781	9.2000
3	M_1	$D_{1,1}$	5.4219	0.8000
4	M_2	$D_{1,1}$	4.5781	9.2000
5	$D_{1,1}$	$D_{1,2}$	10.0000	10.0000
6	$D_{1,2}$	R_1	5.0000	5.0000
7	$D_{1,2}$	R_2	5.0000	5.0000

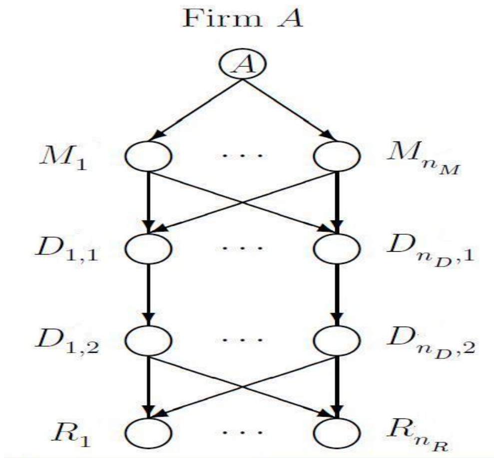

Figure 1. Supply chain network of firm A

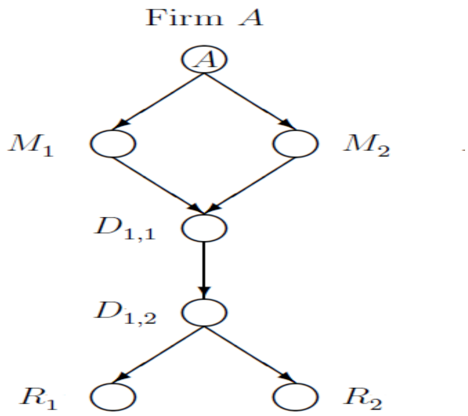

Figure 2. Supply chain network topology for the numerical example