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RESEARCH ARTICLE

Novel approach for nonlinear time-fractional Sharma-Tasso-Olever equation using Elzaki transform

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ABSTRACT

In this article, we demonstrated the study of the time-fractional nonlinear Sharma-Tasso-Olever (STO) equation with different initial conditions. The novel technique, which is the mixture of the *q*-homotopy analysis method and the new integral transform known as Elzaki transform called, *q*-homotopy analysis Elzaki transform method (*q*-HAETM) implemented to find the adequate approximated solution of the considered problems. The wave solutions of the STO equation play a vital role in the nonlinear wave model for coastal and harbor designs. The demonstration of the considered scheme is done by carrying out some examples of time-fractional STO equations with different initial approximations. *q*-HAETM offers us to modulate the range of convergence of the series solution using $\hbar$, called the auxiliary parameter or convergence control parameter. By performing appropriate numerical simulations, the effectiveness and reliability of the considered technique are validated. The implementation of the new integral transform called the Elzaki transform along with the reliable analytical technique called the *q*-homotopy analysis method to examine the time-fractional nonlinear STO equation displays the novelty of the presented work. The obtained findings show that the proposed method is very gratifying and examines the complex nonlinear challenges that arise in science and innovation.

1. Introduction

Fractional calculus (FC) is an incipient tool in the field of mathematics with strong execution in the diverse areas of science and engineering. FC is defined as the generalization of the classical calculus where we study the integral and differential operators of fractional order, even can be lengthened to a complex set. In the past few decades, many mathematical minds have strengthened this concept and designed various fractional differential and integral operators [1, 2]. The progressive functioning of the demonstration of the classical derivatives is done using the nonlocality of the fractional operators. Fractional operators are undeniably used to define sophisticated memory and a range of objects that may be studied using normal mathematical methods such as classical differential calculus. Latterly, fractional operators with nonlocality have been demonstrated and foreseen in the absence of a singular kernel. However, we are still at the initial stage of implementing the concept of FC in various areas of research. Nowadays, FC is a very promising tool due to its larger applications in the dynamics of complex nonlinear phenomena.

The idea of fractional calculus has its origin in the correspondence between *L'hospital* and *Leibniz*. Additionally, it was shown that FC is much more suitable to handle most complex real-world issues than classical calculus. Fractional calculus's richness in applied research has grown over time. Several studies have now proved its potential to deal with a variety of issues., particularly in the fields of science domains like robotics [3], viscoelasticity [4], image processing [5],

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biological population models [6], and several more [7-27]. Compare to the integer-order differential equations, fractional counterparts are much more reserved to get adequate exact solutions for highly nonlinear problems. For this purpose, many numerical and analytical techniques are developed to solve this category of problems.

Along with the development of the classical theory in physics, the concept of fractional calculus and its operators has dragged much attention due to its importance in applied physics such as plasma physics, chemical kinematics, fluid mechanics, optical fibres, probability, statistics, etc. Although it has a long history, in recent decades, scientists have been attracted to fractional differential equations (FDE) due to its extensive applications in wide areas of science and engineering upon which few systems which are inherently nonlinear in nature are much studied by physicists, mathematicians, engineers, meteorologists, etc.

Nonlinear fractional differential equations (NLFDEs) which describe the change in the variables over time was difficult to solve and unpredictable and are most commonly approximated by linear equations. The basic common approach to solve NLFDEs is either to change the variables so that, the solution for the equation will become simpler like the linear equation or transform the problem that can result in a linear equation. Sometimes, the problem will be converted into one or more ordinary differential equation(s) which may or may not be solvable further. For example, weather forecasting is one of the non-linear behaviour systems in which, some parameters are complete of random behaviour, where simple changes in one part of the system produce complex results throughout the system. Resulting in difficulty with accurate long-term weather forecasts even with current advanced technology. Therefore, the investigation of the exact solutions for NLFDEs plays an important role in the study of a nonlinear system of equations such as Navier–Stokes equations of fluid dynamics, Nonlinear optics, Nonlinear Schrödinger equation, Boltzmann equation, General relativity, Van der Pol oscillator, etc.

The inquisition of soliton results of complex nonlinear evolution equations has great significance in the examination of the nonlinear field. These solutions are very informative towards the essential nonlinear science aspects. In this article, we are investigating the nonlinear time-fractional STO equation [28] given as follows:

$$D_t^{\alpha}u(x,t) + 3au_x^2 + 3au^2u_x + 3auu_{xx} + au_{xxx} = 0,\ t > 0,\ 0 < \alpha \leq 1, \tag{1}$$

where a is the random real constant, u is the dependent variable, t and x are the temporal and spatial variables respectively. The STO equation is similar to the KdV equation which can describe evolutionary physics phenomena and interaction with nonlinear waves, like continuum mechanics, fluid dynamics, solitons and turbulence, aerodynamics, etc. The STO equation incorporates the double nonlinear term and linear dispersive term. The solution of the STO equation has been acquired by numerous methods. The Backlund transformation and Hirota's direct method have been implemented to get the fusion and fission of the solitary wave solutions. It's been revealed that the fission of solutions is obtained for $a < 0$ and when $a > 0$ waves depict only the fusion of solutions [29]. The potential symmetries and the generalized symmetries of the STO equation are studied in [30, 31]. Furthermore, to examine the soliton solutions of nonlinear PDEs are analyzed by numerous effective methods so far, like Hirota's method [32], Scattering transformation [33], the First integral method [34, 35], Kudryashov method [36], Extended homoclinic test function method [37, 38], Functional variable method [39], Ansatz method and simplest equation approach [40-42], and others. Various researchers across the globe have given many methods and approaches to solve the nonlinear differential equations among which, Sharma–Tasso–Olver equation which is popularly known as the STO equation has not been much investigated. With this motivation, this work highlights the new generalized novel approach for the nonlinear time-fractional Sharma-Tasso-Olever equation using the Elzaki transform.

To solve linear and nonlinear problems, a semi-analytical tool, known as the homotopy analysis method (HAM) is a very efficient scheme recommended and demonstrated by *Liao* [43-45]. Further, for solving nonlinear problems, the q-homotopy analysis method (q-HAM) as a furnished concept of HAM was introduced by El-Tavil and Hussain [46, 47]. Latterly, the combination of the semi-analytical schemes with the Laplace transform is hired to scrutinize nonlinear equations such as Abel integral equation [48], nonlinear fractional shock wave equation [49], nonlinear boundary value problem on the semi-infinite domain [50], two-dimensional Burger's equation [51], class of nonlinear differential equations [52], nonlinear fractional Zakharov-Kuznetsov equation [53], fractional Klein-Gordon-Schrödinger equations [54], fractional coupled Burger's equations [55], and so on.

The study of the nonlinear STO equation using various numerical and analytical techniques is covered in a large body of literature. The innovative aspect of the current study is the investigation of the nonlinear time-fractional Sharma-Tasso-Olever equation utilizing a powerful analytical tool known as the q-homotopy analysis Elzaki transform method. The primary goal of this work is to use the new integral transform known as the Elzaki transform to investigate the fractional behaviour of the problem under consideration. The presented work has not been performed before using the considered algorithm.

In the present work, we investigate the reliability and effectiveness of the q-homotopy analysis Elzaki transform method (q-HAETM) [56] for solving the time-fractional nonlinear STO equation. The considered technique is the amalgamation of the Elzaki transform (ET) scheme and the q-homotopy analysis method (q-HAM). The Elzaki transform is the new integral transform obtained by the classical Fourier integral, which was presented by Tarig Elzaki [57] to alleviate the procedure of addressing the solutions for ordinary and partial differential equations. The combination of an Elzaki transform with the decomposition algorithm is applied to solve the numerous nonlinear partial differential equations [58], ADM Elzaki and VIM Elzaki [59], homotopy perturbation Elzaki transform method [60], the nonlinear regularized long-wave models are studied with the help of Elzaki transform in [61], and so on. The benefits of the q-HAETM include not requiring discretization, linearization, perturbations, or any rigid assumptions, significantly reducing the complexity of complex computations, promising a wide convergence region, offering a non-local effect, and not requiring complex polynomials, integrations, or physical parameter calculations. To limit the convergence zone and frequent convergence of the obtained solution to a minimum tolerable region, the studied approach is also natured by auxiliary and homotopy parameters. It produces more digestible outcomes for the identical grid point and series solution sequence. Additionally, the technology under consideration preserves greater accuracy despite requiring less time, making it incredibly efficient and trustworthy. The feasibility and optimism of the considered strategy are demonstrated by its capacity to provide highly precise precision, a large convergence range, and a straightforward solution technique.

The rest of the work is organized as follows: Section 2 covers prefaces of the fractional integral in Reimann-Liouville sense, ET, and Caputo fractional derivative. The fundamental notion of the investigated methodology is explained in Section 3, and the results for the time-fractional STO equation are discussed in Section 4. Plots are used to explain the responsiveness and pattern of the acquired fractional-order findings. The numerical simulations of the results obtained using q-HAETM are cited in comparison with ADM, HPM, and OHAM. The final section contains comments on the findings obtained.

2. Preliminaries

Here we present some basic notions of Fractional operators and the Elzaki transform:

Definition 1. The fractional Riemann-Liouville integral of a function $f(t) \in C_\mu (\mu \geq -1)$, is presented [1] by

$$J^\alpha f(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-\vartheta)^{\alpha-1} f(\vartheta) d\vartheta, \tag{2}$$

$$J^0 f(t) = f(t). \tag{3}$$

Definition 2. The derivative with fractional order α of $f \in C_{-1}^n$ in the Caputo sense [1] is:

$$D_t^\alpha f(t) = \begin{cases} \frac{d^n f(t)}{dt^n}, & \alpha = n \in \mathbb{N}, \\ \frac{1}{\Gamma(n-\alpha)} \int_0^t (t-\vartheta)^{n-\alpha-1} f^{(n)}(\vartheta) d\vartheta, & \alpha \in (n-1, n), n \in \mathbb{N}. \end{cases} \tag{4}$$

Definition 3. The Elzaki transform (ET) of a function $f(t)$ is demarcated as follows [57]:

$$E\{f(t)\} = \tilde{f}(s) = s \int_0^\infty e^{-\frac{t}{s}} f(t) dt.$$

The ET of some basic functions are given below [57]

$E\{t^n\} = n!\, s^{n+2}$, where $n = 0,1,2,3, \ldots$

$E\{e^{at}\} = \frac{s^2}{1-as}$,

$E\{\sin(at)\} = \frac{as^3}{1+a^2s^2}$,

$E\{\cos(at)\} = \frac{as^2}{1+a^2s^2}$,

$E\{\sinh(at)\} = \frac{as^3}{1-a^2s^2}$,

$E\{\cosh(at)\} = \frac{as^2}{1-a^2s^2}$.

Definition 4. The ET of a derivative in Eq. (4) is presented as [60]

$$E[D_t^\alpha f(t)] = \frac{\tilde{f}(s)}{s^\alpha} - \sum_{r=0}^{n-1} s^{2-\alpha+r} f^{(r)}(0), \quad (n-1 < \alpha \leq n), \tag{5}$$

where $\tilde{f}(s)$ denote the ET of the function $f(t)$.

3. The basic concept of the q-homotopy analysis Elzaki transform method (q-HAETM)

Consider the following nonlinear fractional PDE involving linear (N) and nonlinear (R) operators to illustrate the basic principle of the considered method:

$$D_t^\alpha\, \mathcal{U}(x,t) + R\, \mathcal{U}(x,t) + N\, \mathcal{U}(x,t) = f(x,t), \quad 0 < \alpha \leq 1, \tag{6}$$

where $D_t^\alpha \mathcal{U}(x,t)$is the Liouville-Caputo fractional derivative of $\mathcal{U}(x,t), f(x,t)$ is the source term. Currently, hiring the ET on Eq. (6) leads to

$$\frac{1}{s^\alpha} E[\mathcal{U}(x,t)] - \sum_{k=0}^{n-1} s^{2-\alpha+k} \mathcal{U}^{(k)}(x,0) + E[R\mathcal{U}(x,t)] + E[N\mathcal{U}(x,t)] = E[f(x,t)], \tag{7}$$

By reducing Eq. (7), we get

$$E[\mathcal{U}(x,t)] - s^\alpha \sum_{k=0}^{n-1} s^{2-\alpha+k} \mathcal{U}^k(x,0) + s^\alpha \{E[R\mathcal{U}(x,t)] + E[N\mathcal{U}(x,t)] - E[f(x,t)]\} = 0. \tag{8}$$

The nonlinear operator N is defined under the homotopy analysis approach as follows

$$N[\varphi(x,t;q)] = E[\varphi(x,t;q)] - s^{\alpha}\sum_{k=0}^{n-1} s^{\alpha-k-1}\varphi^{(k)}(x,t;q)(0^{+}) + s^{\alpha}\{E[R\varphi(x,t;q)] + E[N\varphi(x,t;q)] - E[f(x,t)]\}, \tag{9}$$

where E is the Elzaki transform and $\varphi(x,t;q)$ is a real function of $x,t,$ and q(embedding parameter) $\in \left[0,\frac{1}{n}\right]$ $(n \geq 1)$.

The homotopy is defined as:

$$(1-nq)E[\varphi(x,t;q) - \mathcal{U}_0(x,t)] = \hbar q H(x,t) N[\varphi(x,t;q)], \tag{10}$$

where $\mathcal{U}_0(x,t)$ is an initial guess of $\mathcal{U}(x,t)$,$\hbar \neq 0$ is an auxiliary parameter. For $q=0$ and $q=1/n$, respectively we have:

$$\varphi(x,t;0) = \mathcal{U}_0(x,t), \qquad \varphi\left(x,t;\frac{1}{n}\right) = \mathcal{U}(x,t). \tag{11}$$

As a result, by changing q from 0 to $\frac{1}{n}$, the solution $\varphi(x,t;q)$ converges from $\mathcal{U}_0(x,t)$ to $\mathcal{U}(x,t)$. The function $\varphi(x,t;q)$ can then be enlarged with the utilization of the Taylor theorem across q.

$$\varphi(x,t;q) = \mathcal{U}_0(x,t) + \sum_{m=1}^{\infty}\mathcal{U}_m(x,t)q^m, \tag{12}$$

with

$$\mathcal{U}_m(x,t) = \frac{1}{m!}\frac{\partial^m \varphi(x,t;q)}{\partial q^m}\Big|_{q=0}. \tag{13}$$

The series (12) joins at $q=\frac{1}{n}$, resulting in the fundamental nonlinear equation, and it's one of the solutions of the type, by selecting the$n$ and $\hbar$ (auxiliary parameter) the initial guess $\mathcal{U}_0(x,t)$ and $H(x,t)$ properly.

$$\mathcal{U}(x,t) = \mathcal{U}_0(x,t) + \sum_{m=1}^{\infty}\mathcal{U}_m(x,t)\left(\frac{1}{n}\right)^m. \tag{14}$$

Then divide by $m!$ by differentiating Eq. (10) m times with respect to q. Finally, we derive the deformation equation of order m as follows for q=0.

$$E[\mathcal{U}_m(x,t) - K_m\mathcal{U}_{m-1}(x,t)] = \hbar H(x,t)\Re_m(\vec{\mathcal{U}}_{m-1}). \tag{15}$$

and the vectors considered in the form as

$$\vec{\mathcal{U}}_m = \{\mathcal{U}_0(x,t), \mathcal{U}_1(x,t), \dots, \mathcal{U}_m(x,t)\}. \tag{16}$$

Eq. (15) is the recursive equation that may be represented by the effect of the inverse Elzaki transform

$$\mathcal{U}_m(x,t) = K_m\mathcal{U}_{m-1}(x,t) + \hbar E^{-1}\left[H(x,t)\Re_m(\vec{\mathcal{U}}_{m-1})\right], \tag{17}$$

where

$$\Re_m(\vec{\mathcal{U}}_{m-1}) = \frac{1}{(m-1)!}\frac{\partial^{m-1}N[\varphi(x,t;q)]}{\partial q^{m-1}}\Big|_{q=0}. \tag{18}$$

and

$$K_m = \begin{cases} 0, & m \leq 1, \\ n, & m > 1. \end{cases} \tag{19}$$

Finally, we find the component-wise q-HAETM series solution using Eq. (17).

4. Solution for nonlinear Sharma-Tasso-Olever equation of fractional order

The investigation of the following examples witnesses the efficacy and resolution of the contemplated scheme.

4.1. Example 1

The Sharma-Tasso-Olever equation

$$D_t^{\alpha}u(x,t) + 3au_x^2 + 3au^2u_x + 3auu_{xx} + au_{xxx} = 0. \tag{20}$$

with the starting solution

$$u(x,0) = \frac{2k(\tanh(kx)+w)}{w\tanh(kx)+1}. \tag{21}$$

Introduce ET on Eq. (20) along with the starting solution in (21), which leads to

$$E[u(x,t)] - s^2\left\{\frac{2k(\tanh(kx)+w)}{w\tanh(kx)+1}\right\} + s^{\alpha}E\{3au_x^2 + 3au^2u_x + 3auu_{xx} + au_{xxx}\} = 0. \tag{22}$$

The nonlinear operator N is defined as

$$N[\varphi(x,t;q)] = E[\varphi(x,t;q)] - s^2\left\{\frac{2k(\tanh(kx)+w)}{w\tanh(kx)+1}\right\} + s^{\alpha}E\left\{3a\frac{\partial\varphi^2(x,t;q)}{\partial x} + 3a\varphi^2(x,t;q)\frac{\partial\varphi(x,t;q)}{\partial x} + 3a\varphi(x,t;q)\frac{\partial^2\varphi(x,t;q)}{\partial x^2} + a\frac{\partial^3\varphi(x,t;q)}{\partial x^3}\right\}. \tag{23}$$

The m[th] order deformation equation is

$$E[u_m(x,t) - K_m u_{m-1}(x,t)] = \hbar\Re_m[\vec{u}_{m-1}], \tag{24}$$

where

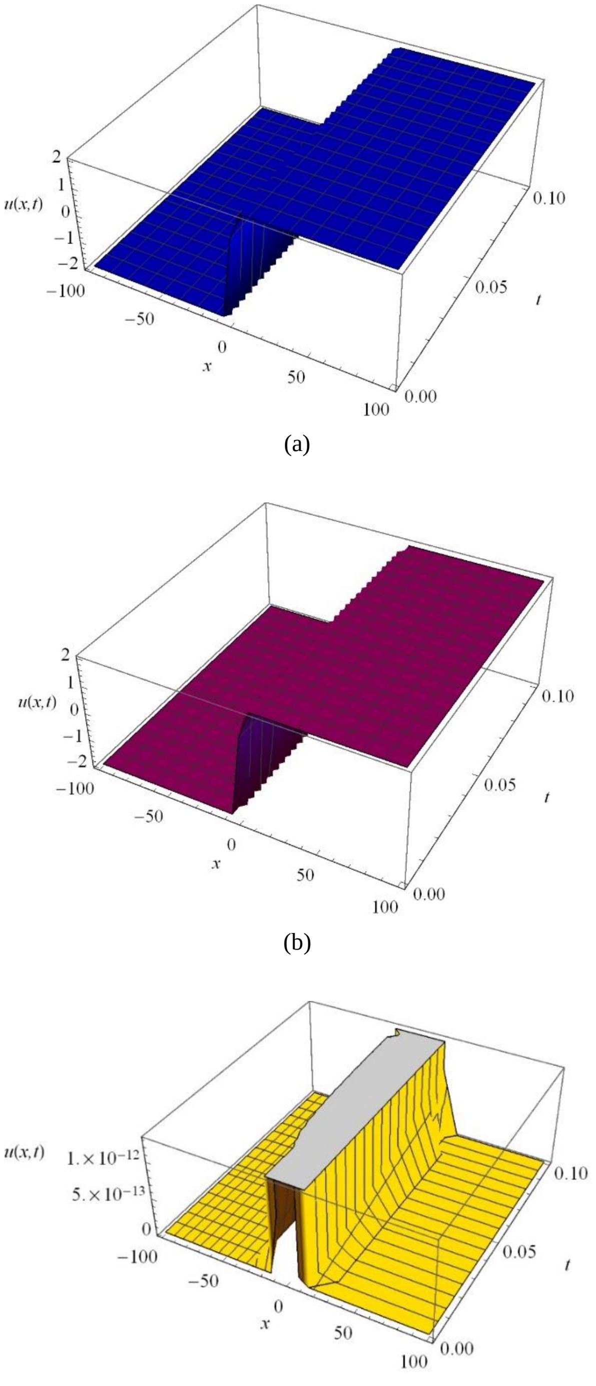


Figure 1. 3D plots of (a) q-HAETM solution (b) Exact solution (c) Absolute error= $|u_{Exact} - u_{App.}|$ at $\alpha = 1, n = 1, a = 1, k = 1, w = 0.5$, and $\hbar = -1$.

$$\Re_m[\vec{u}_{m-1}] = E[u(x,t)] - \left(1 - \frac{K_m}{n}\right) s^2 \left\{\frac{2k(\tanh(kx)+w)}{w\tanh(kx)+1}\right\} + s^\alpha E\left\{3a\sum_{i=0}^{m-1} \frac{\partial u_i}{\partial x}\frac{\partial u_{m-i-1}}{\partial x} + 3a\sum_{i=0}^{m-1}\sum_{j=0}^{i} u_i u_{i-j}\frac{\partial u_{m-i-1}}{\partial x} + 3a\sum_{i=0}^{m-1} u_i \frac{\partial^2 u_{m-i-1}}{\partial x^2} + a\frac{\partial^3 u_{m-1}}{\partial x^3}\right\}. \quad (25)$$

Apply inverse ET on Eq. (24), we obtain

$$u_m(x,t) = K_m u_{m-1}(x,t) + \hbar E^{-1}\{\Re_m[\vec{u}_{m-1}]\}, \quad (26)$$

From Eq. (26), we arrive at:

$$u_0(x,t) = \frac{2k(\tanh(kx)+w)}{w\tanh(kx)+1},$$

$$u_1(x,t) = \frac{8ak^4(w^2-1)\hbar t^\alpha}{\Gamma(\alpha+1)(w\sinh(kx)+\cosh(kx))^2},$$

$$u_2(x,t) = \frac{8ak^4(w^2-1)\hbar(n+\hbar)t^\alpha}{\Gamma(\alpha+1)(w\sinh(kx)+\cosh(kx))^2} + \frac{8a^2k^7(w^2-1)\hbar^2 t^{2\alpha}\operatorname{sech}^6(kx)(4(w^3+w)\cosh(4kx))}{\Gamma(\alpha+1)\Gamma(2\alpha+1)(w\tanh(kx)+1)^6} - \frac{8a^2k^7(w^2-1)\hbar^2 t^{2\alpha}\operatorname{sech}^6(kx)16w(w^2-1)\cosh(2kx)}{\Gamma(\alpha+1)\Gamma(2\alpha+1)(w\tanh(kx)+1)^6} - \frac{2\sinh(2kx)((w^4+6w^2+1)\cosh(2kx)-4w^4+4)}{\Gamma(\alpha+1)\Gamma(2\alpha+1)(w\tanh(kx)+1)^6},$$

$\vdots$

Finally, after getting further iterative terms, the essential series solution of Eq. (20) is presented by

$$u(x,t) = u_0(x,t) + \sum_{m=1}^{\infty} u_m(x,t)\left(\frac{1}{n}\right)^m. \quad (27)$$

By taking $n = 1$, $\alpha = 1$, and $\hbar = -1$ then the attained solution $\sum_{m=1}^{N} u_m(x,t)\left(\frac{1}{n}\right)^m$, will end up with the exact solution $u(x,t) = \frac{2k(\tanh(k(x-4ak^2t))+w)}{w\tanh(k(x-4ak^2t))+1}$ which is of the Sharma-Tasso-Olever equation as $N \to \infty$.

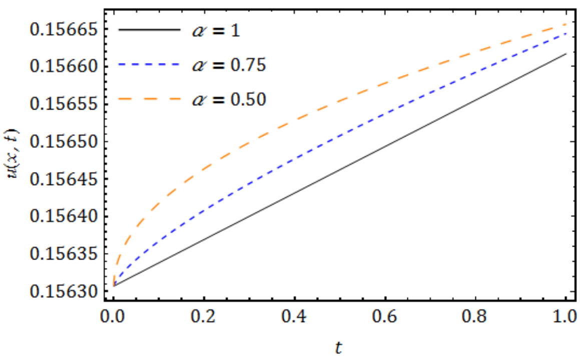
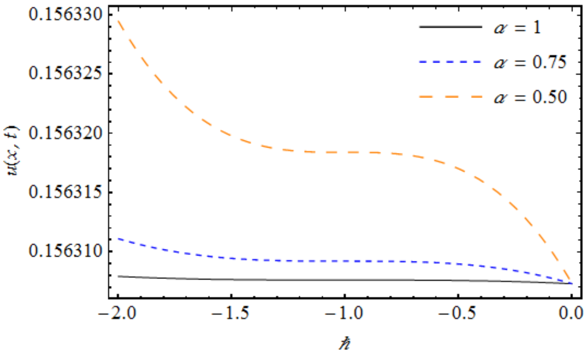


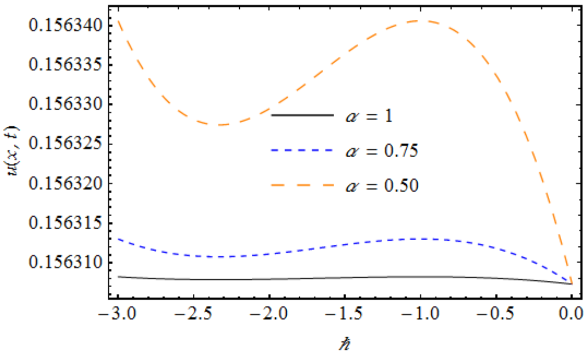
Figure 2. $u(x,t)$ versus t for contemplated, Ex. 1. when $\hbar = -1$, $x = 5, a = 1, k = 0.1, w = 0.5$, and $n = 1$ for distinct α.

Table 1. Absolute errors of ADM, HPM, OHAM [63], and the q-HAETM for Ex. 1 at $\boldsymbol{\alpha = 1, n = 1, k = 1, a = 1, \hbar = -1, w = 0.5}$ and $\boldsymbol{t = 0.01}$.

x	ADM	HPM	OHAM	q-HAETM
2	5.3799×10^{-3}	5.3799×10^{-3}	4.6088×10^{-3}	3.8991×10^{-3}
3	2.4002×10^{-3}	2.4002×10^{-3}	6.4466×10^{-4}	5.3356×10^{-4}
4	9.4208×10^{-4}	9.4208×10^{-4}	8.7636×10^{-5}	7.2319×10^{-5}
5	3.5464×10^{-4}	3.5464×10^{-4}	1.1867×10^{-5}	9.7893×10^{-6}
6	1.3156×10^{-4}	1.3156×10^{-4}	1.6062×10^{-6}	1.3248×10^{-6}
7	4.8547×10^{-5}	4.8547×10^{-5}	2.1737×10^{-7}	1.7930×10^{-7}
8	1.7879×10^{-5}	1.7879×10^{-5}	2.9419×10^{-8}	2.4266×10^{-8}
9	6.5802×10^{-6}	6.5802×10^{-6}	3.9814×10^{-9}	3.2840×10^{-9}
10	2.4211×10^{-6}	2.4211×10^{-6}	5.3883×10^{-10}	4.4445×10^{-10}



(i)



(ii)

Figure 3. $\hbar$-curve for the acquired solution $y(x,t)$ versus $\hbar$ for considered Ex. 1 when (i) $n = 1$ and (ii) $n = 2$ when $a = 1, k = 0.1, w = 0.5, t = 0.001, x = 5$ for distinct α.

Table 2. Absolute errors of ADM, HPM, OHAM [63], and the *q*-HAETM for Ex. 1 at $\boldsymbol{\alpha = 1, n = 1, k = 1, a = 1, \hbar = -1, w = 0.5}$ and $\boldsymbol{t = 0.001}$.

x	ADM	HPM	OHAM	*q*-HAETM
2	7.2096×10^{-4}	7.2096×10^{-4}	4.6795×10^{-4}	3.8602×10^{-4}
3	5.3361×10^{-4}	5.3361×10^{-4}	6.5443×10^{-5}	5.2797×10^{-5}
4	2.3942×10^{-4}	2.3942×10^{-4}	8.8962×10^{-6}	7.1556×10^{-6}
5	9.4126×10^{-5}	9.4126×10^{-5}	1.2046×10^{-6}	9.6860×10^{-7}
6	3.5453×10^{-5}	3.5453×10^{-5}	1.6305×10^{-7}	1.3109×10^{-7}
7	1.3154×10^{-5}	1.3154×10^{-5}	2.2066×10^{-8}	1.7741×10^{-8}
8	4.8545×10^{-6}	4.8545×10^{-6}	2.9864×10^{-9}	2.4010×10^{-9}
9	1.7879×10^{-6}	1.7879×10^{-6}	4.0416×10^{-10}	3.2494×10^{-10}
10	6.5802×10^{-7}	6.5802×10^{-7}	5.4698×10^{-11}	4.3975×10^{-11}

4.2. Example 2

The Sharma-Tasso-Olever equation

$$D_t^{\alpha} u(x,t) + 3au_x^2 + 3au^2 u_x + 3auu_{xx} + au_{xxx} = 0, \tag{28}$$

with initial conditions

$$u(x,0) = -\sqrt{2}\sqrt{B_0}\tan\left(\frac{\sqrt{B_0}x}{\sqrt{2}}\right). \tag{29}$$

Introduce ET on Eq. (28) along with the starting solution in (29), which leads to

$$E[u(x,t)] + s^2\left\{\sqrt{2}\sqrt{B_0}\tan\left(\frac{\sqrt{B_0}x}{\sqrt{2}}\right)\right\} + s^{\alpha}E\{3au_x^2 + 3au^2u_x + 3auu_{xx} + au_{xxx}\} = 0\}. \tag{30}$$

The nonlinear operator N is defined as

$$N[\varphi(x,t;q)] = E[\varphi(x,t;q)] + s^2\left\{\sqrt{2}\sqrt{B_0}\tan\left(\frac{\sqrt{B_0}x}{\sqrt{2}}\right)\right\} + s^{\alpha}E\left\{3a\frac{\partial\varphi^2(x,t;q)}{\partial x} + 3a\varphi^2(x,t;q)\frac{\partial\varphi(x,t;q)}{\partial x} + 3a\varphi(x,t;q)\frac{\partial^2\varphi(x,t;q)}{\partial x^2} + a\frac{\partial^3\varphi(x,t;q)}{\partial x^3}\right\}. \tag{31}$$

The m^th^ order deformation equation is

$$E[u_m(x,t) - K_m u_{m-1}(x,t)] = \hbar\Re_m[\vec{u}_{m-1}], \tag{32}$$

where

$$\Re_m[\vec{u}_{m-1}] = E[u(x,t)] + \left(1 - \frac{K_m}{n}\right)s^2\left\{\sqrt{2}\sqrt{B_0}\tan\left(\frac{\sqrt{B_0}x}{\sqrt{2}}\right)\right\} + s^{\alpha}E\left\{3a\sum_{i=0}^{m-1}\frac{\partial u_i}{\partial x}\frac{\partial u_{m-i-1}}{\partial x} + 3a\sum_{i=0}^{m-1}\sum_{j=0}^{i}u_i u_{i-j}\frac{\partial u_{m-i-1}}{\partial x} + 3a\sum_{i=0}^{m-1}u_i\frac{\partial^2 u_{m-i-1}}{\partial x^2} + a\frac{\partial^3 u_{m-1}}{\partial x^3}\right\}. \tag{33}$$

Apply inverse ET on Eq. (32), we obtain

$$u_m(x,t) = K_m u(x,t) + \hbar E^{-1}\{\Re_m[\vec{u}_{m-1}]\}. \tag{34}$$

From Eq. (34), we arrive at:

$$u_0(x,t) = -\sqrt{2}\sqrt{B_0}\tan\left(\frac{\sqrt{B_0}x}{\sqrt{2}}\right),$$

$$u_1(x,t) = -\frac{2aB_0{}^2\hbar t^{\alpha}\sec^2\left(\frac{\sqrt{B_0}x}{\sqrt{2}}\right)}{\Gamma(\alpha+1)},$$

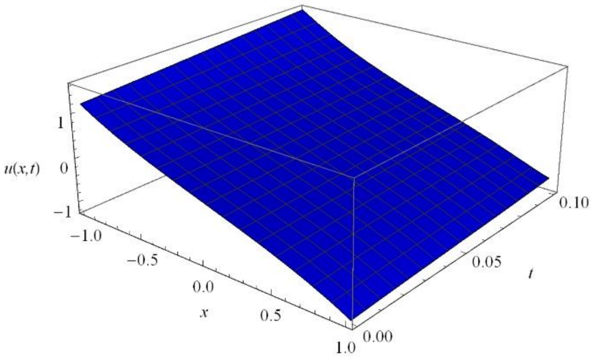
$$u_2(x,t) = -\frac{2aB_0{}^2\hbar(n+\hbar)t^{\alpha}\sec^2\left(\frac{\sqrt{B_0}x}{\sqrt{2}}\right)}{\Gamma(\alpha+1)} - \frac{a^2B_0{}^{7/2}\hbar^2t^{2\alpha}\sec^6\left(\frac{\sqrt{B_0}x}{\sqrt{2}}\right)\left(\sqrt{2}\,\Gamma(\alpha+1)(8\sin(\sqrt{2}\sqrt{B_0}x) + \sin(2\sqrt{2}x))\right)}{2\Gamma(2\alpha+1)} + \frac{a^2B_0{}^{7/2}\hbar^2t^{2\alpha}\sec^6\left(\frac{\sqrt{B_0}x}{\sqrt{2}}\right)\sqrt{2}\,\Gamma(\alpha+1)24aB_0{}^{3/2}\hbar t^{\alpha}}{2\Gamma(2\alpha+1)},$$

$\vdots$

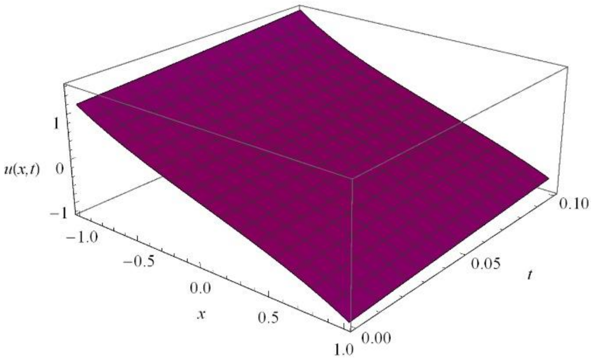
Finally, after getting further iterative terms, the essential series solution of Eq. (28) is presented by

$$u(x,t) = u_0(x,t) + \sum_{m=1}^{\infty} u_m(x,t)\left(\frac{1}{n}\right)^m. \tag{35}$$

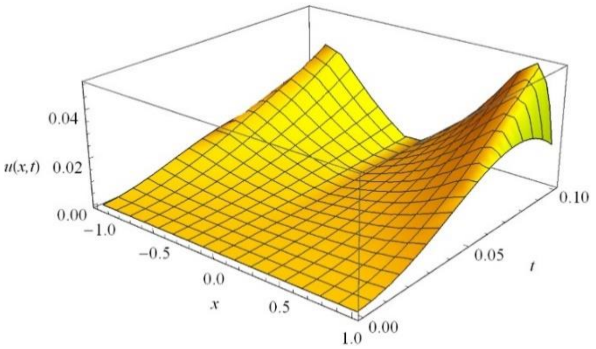
If we set $n = 1$, $\alpha = 1$, and $\hbar = -1$ then the secure solution $\sum_{m=1}^{N} u_m(x,t)\left(\frac{1}{n}\right)^m$, converges to exact solution $u(x,t) = -\sqrt{2B_0}\tan\left(\frac{1}{2}\sqrt{2B_0}\left(x - \frac{\lambda t^{\alpha}}{\Gamma(\alpha+1)}\right)\right)$ of the integer-order Sharma-Tasso-Olever equation as $N \to \infty$.



(a)



(b)



(c)

Figure 4. Surfaces of **(**a) q-HAETM solution (b) Exact solution (c) Absolute error= $|u_{Exact} - u_{App.}|$ at $\hbar = -1, B_0 = 1, \lambda = 2, a = 1, n = 1$, and $\alpha = 1$.

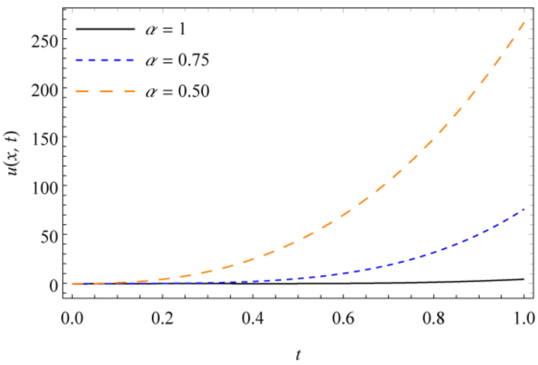
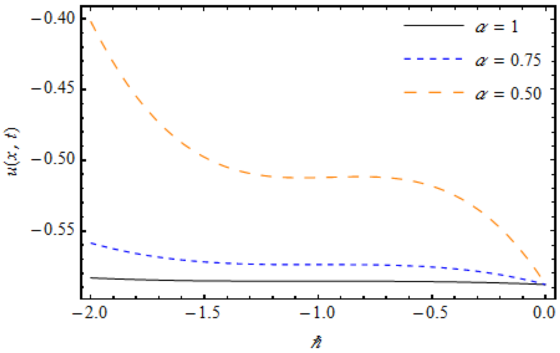
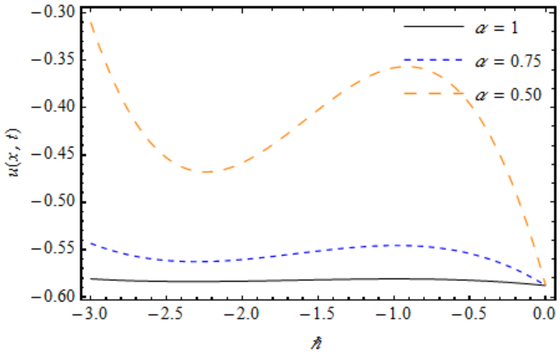


Figure 5. $u(x,t)$ versus t for Ex. 2 at $\hbar = -1, x = 5, \mathrm{B}_0 = 1, \lambda = 2, a = 1$, and $n = 1$ for distinct α.



(i)



(ii)

Figure 6. A plot of approximate solution $u(x,t)$ with respect to $\hbar$ for Ex. 2 when (i) $n = 1$ and (ii) $n = 2$ when $x = 5, \mathrm{B}_0 = 1, \lambda = 2, a = 1$, and $t = 0.001$ for distinct α.

Table 3. Numerical simulations for Ex. 2 at $n = 1, \alpha = 1, \hbar = -1, \mathrm{B}_0 = 1, a = 1, \lambda = 2$ for various values of x and at $t = 0.001, t = 0.01$.

t	x	$\alpha = 1$	$\alpha = 0.75$	$\alpha = 0.5$
0.001	0.1	3.0758×10^{-7}	1.5406×10^{-5}	7.9499×10^{-4}
	0.2	6.3401×10^{-7}	3.1368×10^{-5}	1.5953×10^{-3}
	0.3	1.0070×10^{-6}	4.9479×10^{-5}	2.4768×10^{-3}
	0.4	1.4581×10^{-6}	7.1155×10^{-5}	3.4833×10^{-3}
	0.5	2.0307×10^{-6}	9.8300×10^{-5}	4.6578×10^{-3}
0.01	0.1	3.4407×10^{-5}	6.1324×10^{-4}	1.0980×10^{-2}
	0.2	6.8106×10^{-5}	1.1773×10^{-3}	2.1147×10^{-2}
	0.3	1.0657×10^{-4}	1.8089×10^{-3}	3.1485×10^{-2}
	0.4	1.5292×10^{-4}	2.5468×10^{-3}	4.1785×10^{-2}
	0.5	2.1137×10^{-4}	3.4366×10^{-3}	5.1021×10^{-2}

4.3. Example 3

The Sharma-Tasso-Olever equation

$$D_t^{\alpha} u(x,t) + a{u^3}_x + \frac{3}{2} a{u^2}_{xx} + a u_{xxx} = 0, \tag{36}$$

with initial conditions

$$u(x,0) = \sqrt{\frac{1}{a}} \tanh\left(\sqrt{\frac{1}{a}} x\right). \tag{37}$$

Introduce ET on Eq. (36) along with the starting solution in (37), which leads to

$$E[u(x,t)] - s^2\left\{\sqrt{\frac{1}{a}}\tanh\left(\sqrt{\frac{1}{a}}x\right)\right\} + s^\alpha E\left\{a{u^3}_x + \frac{3}{2}a{u^2}_{xx} + au_{xxx}\right\} = 0\}. \tag{38}$$

The nonlinear operator N is defined as

$$N[\varphi(x,t;q)] = E[\varphi(x,t;q)] - s^2\left\{\sqrt{\frac{1}{a}}\tanh\left(\sqrt{\frac{1}{a}}x\right)\right\} + s^\alpha E\left\{a\frac{\partial\varphi^3(x,t;q)}{\partial x} + \frac{3}{2}a\frac{\partial^2\varphi^2(x,t;q)}{\partial x^2} + a\frac{\partial^3\varphi(x,t;q)}{\partial x^3}\right\}. \tag{39}$$

The m[th] order deformation equation is

$$E[u_m(x,t) - K_m u_{m-1}(x,t)] = \hbar\Re_m[\vec{u}_{m-1}]. \tag{40}$$

where

$$\Re_m[\vec{u}_{m-1}] = E[u(x,t)] + \left(1 - \frac{K_m}{n}\right)s^2\left\{\sqrt{\frac{1}{a}}\tanh\left(\sqrt{\frac{1}{a}}x\right)\right\} + s^\alpha E\left\{a\sum_{i=0}^{m-1}\sum_{j=0}^{i}\frac{\partial u_i}{\partial x}\frac{\partial u_{i-j}}{\partial x}\frac{\partial u_{m-i-1}}{\partial x} + \frac{3}{2}a\sum_{i=0}^{m-1}\frac{\partial^2 u_i}{\partial x^2}\frac{\partial^2 u_{m-i-1}}{\partial x^2} + a\frac{\partial^3 u_{m-1}}{\partial x^3}\right\}. \tag{41}$$

Apply inverse ET on Eq. (40), we obtain

$$u_m(x,t) = K_m u(x,t) + \hbar E^{-1}\{\Re_m[\vec{u}_{m-1}]\}. \tag{42}$$

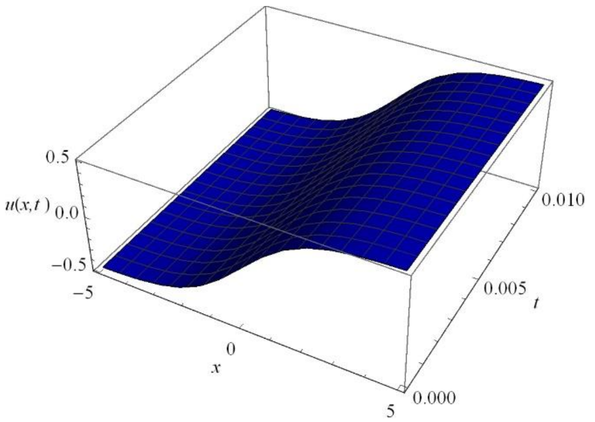
From Eq. (42), we arrive at:

$$u_0(x,t) = \sqrt{\frac{1}{a}}\tanh\left(\sqrt{\frac{1}{a}}x\right),$$

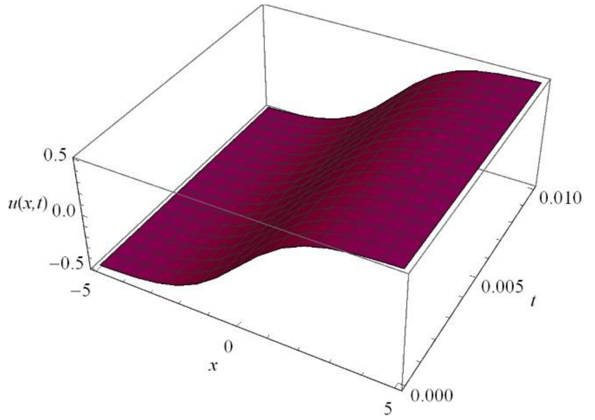
$$u_2(x,t) = \frac{\hbar t^\alpha\left(a\cosh\left(4\sqrt{\frac{1}{a}}x\right) - 2(a-3)\cosh\left(2\sqrt{\frac{1}{a}}x\right) - 3a - 4\right)\operatorname{sech}^6\left(\sqrt{\frac{1}{a}}x\right)}{2a^2\Gamma(\alpha+1)},$$

$$u_1(x,t) = \frac{\hbar(n+\hbar)t^\alpha(a\cosh(4\sqrt{\frac{1}{a}}x) - 2(a-3)\cosh(2\sqrt{\frac{1}{a}}x) - 3a - 4)\operatorname{sech}^6(\sqrt{\frac{1}{a}}x)}{2a^2\Gamma(\alpha+1)} - \frac{1}{4\Gamma(\alpha+1)\Gamma(2\alpha+1)}\left(\Gamma(\alpha+1)\cosh^5\left(\sqrt{\frac{1}{a}}x\right)\left(-52a^2\cosh\left(6\sqrt{\frac{1}{a}}x\right) + a^2\cosh\left(8\sqrt{\frac{1}{a}}x\right) + 4(149a^2 + 149a - 588)\cosh\left(2\sqrt{\frac{1}{a}}x\right) + 4(7a^2 - 223a + 72)\cosh\left(4\sqrt{\frac{1}{a}}x\right) + 515a^2 + 60a\cosh\left(6\sqrt{\frac{1}{a}}x\right) + 1548a + 2160\right)\right)\left(\left(\frac{1}{a}\right)^{7/2}\hbar^2 t^{2\alpha}\operatorname{sech}^{15}\left(\sqrt{\frac{1}{a}}x\right)\right) - \frac{4\left(\frac{1}{a}\right)^5\hbar^3 t^{3\alpha}\sinh\left(\sqrt{\frac{1}{a}}x\right)\tanh\left(\sqrt{\frac{1}{a}}x\right)\operatorname{sech}^{15}\left(\sqrt{\frac{1}{a}}x\right)}{4\Gamma(\alpha+1)^2\Gamma(3\alpha+1)} \frac{\left((12-8a)\cosh\left(2\sqrt{\frac{1}{a}}x\right) + a\cosh\left(4\sqrt{\frac{1}{a}}x\right) - 9(a+2)\right)^2}{4\Gamma(\alpha+1)^2\Gamma(3\alpha+1)},$$

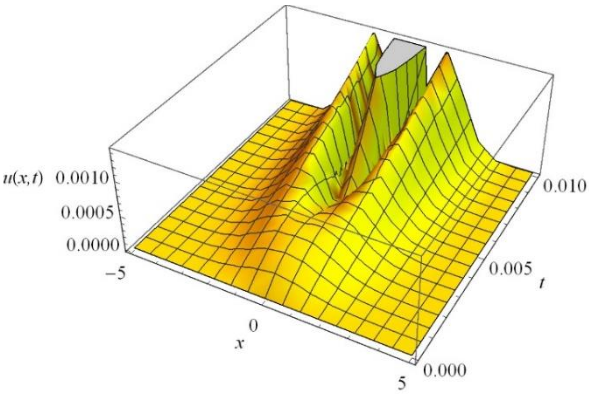
$\vdots$



(a)



(b)



(c)

Figure 7. (a) 3D plot for q-HAETM solution (b) surface of exact solution (c) approximated solution surface at $\hbar = -1.858$, $a = 4, n = 1$ and $\alpha = 1$.

Finally, after getting further iterative terms, the essential series solution of Eq. (36) is presented by

$$u(x,t) = u_0(x,t) + \sum_{m=1}^{\infty}u_m(x,t)\left(\frac{1}{n}\right)^m. \tag{43}$$

If we set $n = 1$, $\alpha = 1$, and $\hbar = -1$ then the secure solution $\sum_{m=1}^{N} u_m(x,t)\left(\frac{1}{n}\right)^m$, converges to exact solution $u(x,t) = \sqrt{\frac{1}{a}}\tanh\left(\sqrt{\frac{1}{a}}(x-t)\right)$ of the integer-order Sharma-Tasso-Olever equation as $N \to \infty$.

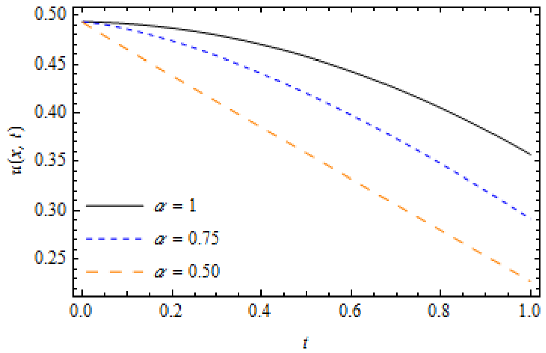
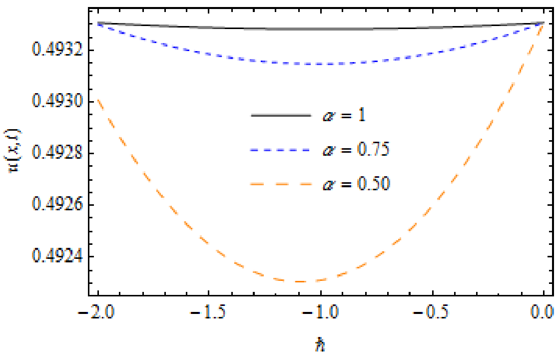
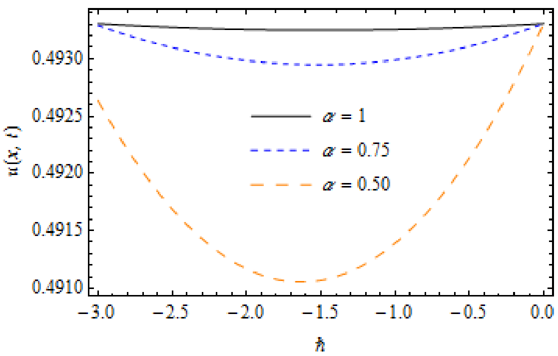


Figure 8. $u(x,t)$ versus t for the contemplated Ex. 3 at $\hbar = -1.858, x = 5, a = 4$, and $n = 1$ for distinct of α.



(i)



(ii)

Figure. 9. $\hbar$-curve for acquired solution $u(x,t)$ for Ex. 3 when (i) $n = 1$ and (ii) $n = 2$ when $x = 5, a = 4$, and $t = 0.001$ for distinct α.

Table 4. Numerical simulations for Ex. 3 at $n = 1, \alpha = 1, \hbar = -1, B_0 = 1, a = 1, \lambda = 2$ for various values of x and at $t = 0.001, t = 0.002$.

t	x	$\alpha = 1$	$\alpha = 0.75$	$\alpha = 0.5$
	5	2.8246×10^{-7}	4.1132×10^{-5}	4.9465×10^{-4}
	4	4.2905×10^{-7}	9.2494×10^{-5}	8.1866×10^{-4}
0.001	3	8.7092×10^{-6}	1.3618×10^{-4}	7.2756×10^{-4}
	2	5.7987×10^{-5}	3.6881×10^{-5}	8.8362×10^{-3}
	1	1.9745×10^{-4}	4.3659×10^{-4}	2.9978×10^{-2}
	5	8.1686×10^{-7}	7.4104×10^{-5}	8.4707×10^{-4}
	4	6.5000×10^{-7}	1.5605×10^{-4}	1.2792×10^{-3}
0.002	3	1.9479×10^{-5}	1.5950×10^{-4}	2.2392×10^{-3}
	2	1.2665×10^{-4}	3.8463×10^{-4}	1.8766×10^{-2}
	1	3.6421×10^{-4}	1.5111×10^{-3}	6.0264×10^{-2}

5. Numerical results and discussion

The numerical research for non-integer order STO equations using the q-HAETM is implemented in the current part. In terms of absolute error, Figure 1 depicts the resemblance of the solution obtained using the discussed method to the precise solution for Eq. (20). We can see that the obtained solution and the exact solution are the best matches with each other. The recommended technique's conclusion for Eq. (20) is plotted against time in figure 2. The solution increases with an increase in time for considering various fractional orders. The performance of n with $\hbar$ in an accomplished outcome of the provided method is shown in figure 3. The optimal region for the convergence of the obtained series solution in terms of the auxiliary parameter $\hbar$ can be depicted in figure 3. Table 1 and Table 2 cite the accurateness of the considered method in comparison with various methods namely, ADM, HPM, and OHAM through absolute errors at $t = 0.001$ and $t = 0.01$ respectively. The link between the results obtained by the proposed method in terms of absolute error and the precise answer for Ex. 2 is depicted in figure 4. We can compare both obtained solution and the exact solution to check the accuracy of the projected algorithm. The deed of the safe results of Ex. 2 with the change in time t is depicted in figure 5. As we can see, the solution increases with an increase in time t. The effectiveness of n in the produced solution by the proposed algorithm is shown in figure 6. Also, the solution is affected by various fractional orders with the time t. However, we attained a better accuracy rate with the consideration of fractional order differential operators too. This shows that the presented scheme is highly suitable to deal with nonlinear fractional differential equations. The approximated error results acquired for various values of α with the help of the considered scheme are cited in Table 3. Figure 7 depicts the relationship between the results acquired by the q-HAETM concerning absolute error and the exact solution for Ex. 3. Figure 8 is decorated with the variation of attained solution with time t. According to the considered initial approximation, the solution gradually decreases with an increase in time t. The performance of the embedding parameter ($\hbar$) for distinct values of n in the secured solution by the proposed strategy is shown in figure 9. Table 4 cites the accuracy of the obtained solutions in terms of absolute error.

6. Conclusion

In this paper, we have demonstrated how to solve the nonlinear time-fractional STO equation using the effective q-HAM with the Elzaki transform. We have examined three examples with distinct starting solutions to prove the significance as well as the effectiveness of the considered scheme. Moreover, we can compare the obtained results with the exact solutions to witness the same. The rate of convergence of the obtained series solution to the exact solution is accelerated with the help of optimal values of convergence control parameter $\hbar$. Presented numerical simulations guarantee results with higher accuracy. The numerical simulations are executed by using the considered technique in comparison with the other schemes like ADM, HPM, and OHAM in terms of approximated errors. The secure outputs indicate that a considered methodology was used to generate a standardized analytical solution. In this study, the detailed analysis of the fractional behaviour of the nonlinear STO equation and its solution is achieved by considering different initial approximations. The process of finding the solution for the considered problem using the Elzaki transform was effortless. The proposed approach is effective in delivering a simple solution, a critical convergence zone, and a non-local influence. Finally, we claim that our proposed technique is incredibly dependable and can be applied to large study classifications relating to fractional-order nonlinear scientific methods, which aid us in better understanding the nonlinear compound phenomena in linked domains of innovation and science.

References

[1] Kilbas, A. A., Srivastava, H. M., Trujillo, J. J. (2006). Theory and Applications of Fractional Differential Equations. *North-Holland Mathematics Studies.*

[2] Podlubny, I. (1999). Fractional Differential Equations. *Academic Press, San Diego, CA.*

[3] Xin, Z., Jing, Z., Wenru, L., Wenbo, X. (2021). Research on fractional sliding mode synchronous control of robotic arms under uncertain disturbance. *Atomatic Control and Computer Sciences, 55*(1), 26-37.

[4] Wang, X., Petru, M., Xia, L., (2021). Modelling the dynamical behaviour of the flax fibre rainforced composite after water using a modified Huet-Sayegh viscoelastic model with fractional derivatives. *Construction and Building Materials, 290*, 122879.

[5] Ghamisi, P., Couceiro, J. A., Benediktsson, J. A., Ferreira, N. M. (2012). An efficient method for segementation of images based on fractional calculus and natural selection. *Expert Systems with Applications, 39*(16), 12407-12417.

[6] Rashid, S., Kubra, T., Ullah, S. (2021). Fractional spatial diffusion of a biological population model via a new integral transform in the setting of power and Mittag-Leffler nonsingular kernel. *Physica Scripta, 96*(11), 114003.

[7] Veeresha, P., Prakasha, D. G., Baskonus, H. M. (2019). New numerical surfaces to the mathematical model of cancer chemotherapy effect in Caputo fractional derivatives. *Chaos, 29*, 013119.

[8] El Mfadel, A., Melliani, S., & Elomari, M. H. (2021). A note on the stability analysis of fuzzy nonlinear fractional differential equations involving the Caputo fractional derivative. *International Journal of Mathematics and Mathematical Sciences, 2021*, 1-6.

[9] Veeresha, P., W, Gao., Prakasha, D. G., Malagi, N. S., Ilhan, E., Baskonus, H. M. (2021). New dynamical behaviour of the coronavirus (2019-nCoV) infection system with nonlocal operator from reservoirs to people. *Information Sciences Letters, 10*(2), 205-212.

[10] Hammouch, Z., Yavuz, M., & Özdemir, N. (2021). Numerical solutions and synchronization of a variable-order fractional chaotic system. *Mathematical Modelling and Numerical Simulation with Applications, 1*(1), 11-23.

[11] Sunitha, M., Fehmi, G., Amal, A., Malagi, N. S., Sandeep, S., Rekha, J. G., Punith Gowda R. J. (2023). An efficient analytical approach with novel integral transform to study the two-dimensional solute transport problem. *Ain Shams Engineering Journal, 14*(3), 101878.

[12] Logeswari, K., Ravichandran, C., & Nisar, K. S. (2020). Mathematical model for spreading of COVID-19 virus with the Mittag–Leffler kernel. *Numerical Methods for Partial Differential Equations*, 2020, 1-16.

[13] Yavuz, M., (2020). Europian option pricing models described by fractional operators with classical and generalized Mittag-Leffler kernels. *Numerical Methods for Partial Differential Equations*, 1-23.

[14] Özköse, F., & Yavuz, M. (2022). Investigation of interactions between COVID-19 and diabetes with hereditary traits using real data: A case study in Turkey. *Computers in biology and medicine, 141*, 105044.

[15] Veeresha, P., Yavuz, M., & Baishya, C. (2021). A computational approach for shallow water forced Korteweg–De Vries equation on critical flow over a hole with three fractional operators. *An International Journal of Optimization and Control: Theories & Applications (IJOCTA), 11*(3), 52-67.

[16] Chalishajar, D., Ravichandran, C., Dhanalakshmi, S., & Murugesu, R. (2019). Existence of fractional impulsive functional integro-differential equations in Banach spaces. *Applied System Innovation, 2*(2),

18.

[17] Jothimani, K., Kaliraj, K., Panda, S. K., Nisar, K. S., & Ravichandran, C. (2021). Results on controllability of non-densely characterized neutral fractional delay differential system. *Evolution Equations & Control Theory, 10*(3), 619.

[18] Nisar, K. S., Jothimani, K., Kaliraj, K., & Ravichandran, C. (2021). An analysis of controllability results for nonlinear Hilfer neutral fractional derivatives with non-dense domain. *Chaos, Solitons & Fractals, 146*, 110915.

[19] Prakasha, D. G., Malagi, N. S., & Veeresha, P. (2020). New approach for fractional Schrödinger-Boussinesq equations with Mittag-Leffler kernel. *Mathematical Methods in the Applied Sciences*, *43*(17), 9654-9670.

[20] Prakasha, D. G., Malagi, N. S., Veeresha, P., & Prasannakumara, B. C. (2021). An efficient computational technique for time-fractional Kaup-Kupershmidt equation. *Numerical Methods for Partial Differential Equations*, *37*(2), 1299-1316.

[21] Özköse, F., Yavuz, M., Şenel, M. T., & Habbireeh, R. (2022). Fractional order modelling of omicron SARS-CoV-2 variant containing heart attack effect using real data from the United Kingdom. *Chaos, Solitons & Fractals*, *157*, 111954.

[22] Veeresha, P. (2021). A numerical approach to the coupled atmospheric ocean model using a fractional operator. *Mathematical Modelling and Numerical Simulation with Applications, 1*(1), 1-10.

[23] Yavuz, M., & Sene, N. (2020). Approximate solutions of the model describing fluid flow using generalized ρ-Laplace transform method and heat balance integral method. *Axioms*, *9*(4), 123.

[24] Valliammal, N., & Ravichandran, C. (2018). Results on fractional neutral integro-differential systems with state-dependent delay in Banach spaces. *Nonlinear Studies*, *25*(1), 159171.

[25] Evirgen, F., & Yavuz, M. (2018). An alternative approach for nonlinear optimization problem with Caputo-Fabrizio derivative. *In ITM Web of Conferences* (Vol. 22, p. 01009), EDP Sciences.

[26] Malagi, N. S., Veeresha, P., Prasannakumra, B. C., Prasanna, G, D., Prakasha, D. G. (2020). A new computational technique for the analytic treatment of time-fractional Emden-Fowler equations. *Mathematics and Computers in Simulation*, *190*, 362-376.

[27] Evirgen, F., Uçar, S., Özdemir, N., & Hammouch, Z. (2021). System response of an alcoholism model under the effect of immigration via non-singular kernel derivative. *Discrete & Continuous Dynamical Systems-S, 14*(7), 2199.

[28] Wang, G. W., & Xu, T. Z. (2014). Invariant analysis and exact solutions of nonlinear time fractional Sharma–Tasso–Olver equation by Lie group analysis. *Nonlinear Dynamics*, *76*(1), 571-580.

[29] Chen, A. (2010). Multi-kink solutions and soliton fission and fusion of Sharma–Tasso–Olver equation. *Physics Letters A*, *374*(23), 2340-2345.

[30] Wang, G., Kara, A. H., & Fakhar, K. (2016). Nonlocal symmetry analysis and conservation laws to an third-order Burgers equation. *Nonlinear Dynamics*, *83*(4), 2281-2292.

[31] Liu, H. (2015). Painlevé test, generalized symmetries, Bäcklund transformations and exact solutions to the third-order Burgers' equations. *Journal of Statistical Physics*, *158*(2), 433-446.

[32] Hirota, R. (1971). Exact solution of the Korteweg—de Vries equation for multiple collisions of solitons. *Physical Review Letters*, *27*(18), 1192.

[33] Ablowitz, M. J., Ablowitz, M. A., Clarkson, P. A., & Clarkson, P. A. (1991). *Solitons, nonlinear evolution equations and inverse scattering* (Vol. 149), Cambridge University Press.

[34] Eslami, M., Fathi Vajargah, B., Mirzazadeh, M., & Biswas, A. (2014). Application of first integral method to fractional partial differential equations. *Indian Journal of Physics*, *88*(2), 177-184.

[35] Eslami, M., Mirzazadeh, M., Vajargah, B. F., & Biswas, A. (2014). Optical solitons for the resonant nonlinear Schrödinger's equation with time-dependent coefficients by the first integral method. *Optik*, *125*(13), 3107-3116.

[36] Sanchez, P., Ebadi, G., Mojaver, A., Mirzazadeh, M., Eslami, M., & Biswas, A. (2015). Solitons and other solutions to perturbed Rosenau-KdV-RLW equation with power law nonlinearity. *Acta Physica Polonica A*, *127*(6), 1577-1586.

[37] Dai, Z., Liu, J., & Liu, Z. (2010). Exact periodic kink-wave and degenerative soliton solutions for potential Kadomtsev–Petviashvili equation. *Communications in Nonlinear Science and Numerical Simulation*, *15*(9), 2331-2336.

[38] Xu, Z., Chen, H., & Dai, Z. (2014). Rogue wave for the (2+ 1)-dimensional Kadomtsev–Petviashvili equation. *Applied Mathematics Letters*, *37*, 34-38.

[39] Mirzazadeh, M., & Biswas, A. (2014). Optical solitons with spatio-temporal dispersion by first integral approach and functional variable method. *Optik*, *125*(19), 5467-5475.

[40] Biswas, A., Mirzazadeh, M., Savescu, M., Milovic, D., Khan, K. R., Mahmood, M. F., & Belic, M. (2014). Singular solitons in optical metamaterials by ansatz method and simplest equation approach. *Journal of Modern Optics*, *61*(19), 1550-1555.

[41] Wang, C. (2016). Spatiotemporal deformation of lump solution to (2+ 1)-dimensional KdV equation. *Nonlinear Dynamics*, *84*(2), 697-702.

[42] Khalid, M., Sultana, M., Zaidi, F., & Arshad, U.

(2015). Application of Elzaki transform method on some fractional differential equations. *Mathematical Theory and Modeling*, *5*(1), 89-96.

[43] Liao, S. (2003). *Beyond perturbation: introduction to the homotopy analysis method*. Chapman and Hall/CRC.

[44] Liao, S. J. (1995). An approximate solution technique not depending on small parameters: a special example. *International Journal of Non-Linear Mechanics*, *30*(3), 371-380.

[45] Liao, S. (2012). *Homotopy analysis method in nonlinear differential equations* (pp. 153-165). Beijing: Higher education press.

[46] El-Tawil, M. A., & Huseen, S. N. (2012). The q-homotopy analysis method (q-HAM). *International Journal of Applied Mathematics and Mechanics*, *8*(15), 51-75.

[47] El-Tawil, M. A., & Huseen, S. N. (2013). On convergence of the q-homotopy analysis method. *International Journal of Contemporary Mathematical Sciences*, *8*(10), 481-497.

[48] Kumar, S., Kumar, A., Kumar, D., Singh, J., & Singh, A. (2015). Analytical solution of Abel integral equation arising in astrophysics via Laplace transform. *Journal of the Egyptian Mathematical Society*, *23*(1), 102-107.

[49] Kumar, D., Singh, J., Kumar, S., & Singh, B. P. (2015). Numerical computation of nonlinear shock wave equation of fractional order. *Ain Shams Engineering Journal*, *6*(2), 605-611.

[50] Khan, M., Gondal, M. A., Hussain, I., & Vanani, S. K. (2012). A new comparative study between homotopy analysis transform method and homotopy perturbation transform method on a semi infinite domain. *Mathematical and Computer Modelling*, *55*(3-4), 1143-1150.

[51] Khan, M. (2014). A novel solution technique for two dimensional Burger's equation. *Alexandria Engineering Journal*, *53*(2), 485-490.

[52] Khuri, S. A. (2001). A Laplace decomposition algorithm applied to a class of nonlinear differential equations. *Journal of Applied Mathematics*, *1*(4), 141-155.

[53] Kumar, D., Singh, J., & Kumar, S. (2014). Numerical computation of nonlinear fractional Zakharov–Kuznetsov equation arising in ion-acoustic waves. *Journal of the Egyptian Mathematical Society*, *22*(3), 373-378.

[54] Veeresha, P., Prakasha, D. G., Singh, J., Kumar, D., & Baleanu, D. (2020). Fractional Klein-Gordon-Schrödinger equations with Mittag-Leffler memory. *Chinese Journal of Physics*, *68*, 65-78.

[55] Singh, J., Kumar, D., & Swroop, R. (2016). Numerical solution of time-and space-fractional coupled Burgers' equations via homotopy algorithm. *Alexandria Engineering Journal*, *55*(2), 1753-1763.

[56] Singh, J., Kumar, D., Purohit, S. D., Mishra, A. M., & Bohra, M. (2021). An efficient numerical approach for fractional multidimensional diffusion equations with exponential memory. *Numerical Methods for Partial Differential Equations*, *37*(2), 1631-1651.

[57] Elzaki, T. M. (2011). The new integral transform Elzaki transform. *Global Journal of pure and applied mathematics*, *7*(1), 57-64.

[58] Khalid, M., Sultana, M., Zaidi, F., & Arshad, U. (2015). An Elzaki transform decomposition algorithm applied to a class of non-linear differential equations, *Journal of Natural Sciences Research*, *5*, 48-56.

[59] Manafian, J., & Zamanpour, I. (2014). Application of the ADM Elzaki and VIM Elzaki transform for solving the nonlinear partial differential equations, Sci. *Road Journal*, *2*(4), 37-50.

[60] Rashid, S., Hammouch, Z., Aydi, H., Ahmad, A.G. & Alsharif, A.M. (2021). Novel computations of the time-fractional Fisher's model via generalized fractional integral operators by means of the Elzaki transform, *Fractal and Fractional*, *5*(3), 94.

[61] Yavuz, M. (2020). Nonlinear regularized long-wave models with a new integral transformation applied to the fractional derivatives with power and Mittag-Leffler kernel. *Advances in Difference Equations. 2020*(1), 1-18.

[62] Abdeljawad, T. (2011). On Riemann and Caputo fractional differences. *Computers & Mathematics with Applications*, 62(3), 1602-1611.

[63] Nawaz, R., & Zada, L. (2018). Solving time fractional Sharma-Tasso-Olever equation by optimal homotopy asymptotic method. *In AIP Conference Proceedings*, *1978*(1), 310002.

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