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RESEARCH ARTICLE

Optimal matchday schedule for Turkish professional soccer league using nonlinear binary integer programming

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ABSTRACT

Sports scheduling problems are interesting optimization problems that require the decision of who play with whom, where and when to play. In this work, we study the sports scheduling problem faced by the Turkish Football Federation. Given the schedule of games for each round of the season, the problem is to determine the match days with the goal of having a fair schedule for each team. The criteria we employ to establish this fairness are achieving an equal distribution of match days between the teams throughout the season and the ideal assignment of games to different days in each round of the tournament. The problem is formulated as a nonlinear binary integer program and is solved optimally for each week. Our results indicate that significant improvements over the existing schedule can be achieved if the optimal solution is implemented.

1. Introduction

Due to increasing popularity of sports on the globe as well as demands of stakeholders such as teams, players, TV broadcasters, their sponsors and fans, scheduling sport tournaments has become quite important. In the presence of high economic stakes, the tournament organizers are hard-pressed to design a fair schedule that strikes a reasonable balance in addressing the needs of all the stakeholders.This is indeed an enormous task because the objectives of multiple stakeholders are often in conflict and the fixtures have to be determined subject to a wide variety of constraints. Therefore, over the years, sports scheduling has turned into a challenging decision problem that can best be handled by a rigorous application of operations research methods.

The main focus of this paper is the scheduling of the Turkish Super League which is a task undertaken by the Turkish Football Federation (TFF). Each year, the TFF officials first finalize the seasonal fixture of the league which regularly includes 18 teams (although the Federation recently announced the cancellation of relegation for the 2019-20 season due to the pandemic, which implies that there are 21 participants in the 2020-21 season). To be more specific, they determine who will play with whom each week. Then, the actual schedule is determined taking into account other tournaments organized by UEFA and FIFA (governing bodies of soccer in Europe and the world, respectively) as well as the domestic Turkish Cup. Naturally, each season, number of games played by any team in the Super League is known with certainty. On the other hand, UEFA competitions and the Turkish Cup are single-elimination tournaments, so there is uncertainty about the number of rounds each team will play without getting knocked out. Therefore, determination of when league games are played is a dynamic process that has to take the most recent status of other tournaments into account. With 18 teams in the league,

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the fixture is divided into 34 weeks of play. Unless there are exceptional circumstances, games of any week are scheduled from Friday to Monday; i.e., four days of the week.

The critical aspect of this procedure is the fixture determination for its supposed influence on each team's overall performance. To illustrate, if a team is scheduled to play two consecutive games with relatively stronger teams, additional fatigue may impair the endurance of the players and cause a significant drop in performance in the latter match. This negative influence is called the carryover effect, and its minimization is one of the many issues that should be addressed in fixture determination. However, in recent years, club administrators as well as the technical staff in Turkey are also increasingly vocal in their criticism on which day of the week their next game in the fixture is scheduled. It is a widespread belief in the soccer community that not all days of play are equally beneficial. Their criticisms are centered around two main arguments: first economic, second performance related. Both arguments are parallel in claiming that scheduled matches on Fridays and Mondays are less favorable. Those that raise the economic concerns complain about the loss of revenues for the reduced attendance of spectators on the weekdays (see for instance [1], [2], [3], [4]) whereas the performance concerns are largely centered around unusual loss of points on weekdays due to various reasons. News on Beşiktaş´s unexpectedly low performance on Fridays in the season $2017-18$ after two consecutive years of league championship (see [5]) and Trabzonspor's loss of points on Fridays in the first half of $2007 - 2008$ (see [6]) are examples of reports that support this view on reduced performance. Other examples may be found in [7] and [8].

Given these concerns, the problem is to determine the day of each game with the objective of having a fair distribution among all the participating clubs. Since the future of other tournaments affecting the Turkish Super League is uncertain, any proposed method should be dynamic in nature. The metric of day distribution is also quite important because it will be the main element of the method that maintains fairness in the resulting schedule. Schedules of other related tournaments, and the rest period requirements add constraints to this challenging problem that we formulate as a nonlinear binary integer programming problem.

In the sections below, we first give an overview of the related literature. The third section is where we present the main problem formulation that we also solve optimally for each round of the season on the fly. In particular, using the data of the 2018-2019 season, we compare the performance of the optimal schedule generated by our program against that obtained manually by the TFF officials. An extended discussion of the results is offered in the fourth section. Section 5 concludes.

2. Literature

Constructing schedules for sports competitions has attracted the attention of academicians in the last five decades ([9]). In particular, the literature for round robin tournaments where every team has to play every other team a certain number of times is rich. We provide a short overview of the literature here, however interested reader can examine [10], [11] and [12] for extensive surveys on round robin scheduling. Various objectives are utilized for obtaining optimal/near optimal schedules for those tournaments. Examples of those objectives include (but are not limited to) minimizing rest mismatches ([13], [14]), minimizing breaks ([15], [16], [17], [18]), minimizing travel distances ([19]), and minimizing carry-over effects ([20], [21], [22]). Bulk of the literature concentrates on addressing a number of these and other objectives (such as maximizing gate revenue, attendance etc.) simultaneously subject to constraints that are specific to a particular tournament ([23], [24], [25], [26], [27], [28]).

Not surprisingly, scheduling of round robin tournaments have been handled both through various types of integer programming formulations and from the perspective of graph theory. Where theoretical formulations fail against the complexity of the problem, heuristics are employed. First, we highlight some of the previous applications of integer programming on fixture determination. [29] proposed an integer programming formulation for round robin tournaments taking into account constraints such as not allowing forbidden games and fairness constraints related to breaks as well as carryover effects. [30] studied the problem of scheduling Argentina's professional basketball leagues, considering the measure of total distance traveled by the teams. The authors formulated the problem as a variation of the Traveling Tournament Problem. In [9], the current schedule of the Korea baseball league was analyzed in the presence of measures such as the total travel distance of teams and match fairness. The author proposed an integer programming model for solving the problem, and developed a heuristic algorithm for handling large-sized problems. [17] proposed an integer programming formulation for scheduling the professional soccer league in Ecuador. The authors also developed a heuristic approach for solving the problem in a reasonable amount of time. [16] addressed the problem of scheduling the South American qualifiers to the 2018 FIFA World Cup, proposing an integer programming formulation for solving it. [20] studied the problem of minimizing weighted carry over effects formulating the underlying problem using integer programming and proposed an efficient heuristic that yields high-quality solutions. In a recent work, [22] focused on a round robin scheduling problem with the goal of minimizing carry over effects in Turkish Super League while keeping the number of breaks per team below a certain threshold, and proposed a better schedule for this league.

Applications of graph theory in round robin tournament scheduling are also widely found. In some of these applications, we observe the use of the "circle method" which ensures that each team has one game in each round and there is no unassigned game by the end of the tournament ([31], [32]). Graph theoretic studies also include articles that discuss break minimization for solving round robin scheduling problems ([33], [34]). [33] transformed the sports scheduling problem into a maximum cut problem. The authors demonstrated that their method outperforms previous methods that are based on integer and constraint programming. [34] provided theoretical results for round robin scheduling problems where the number of teams is even, making sure that the schedule is constructed with minimum number of breaks. [35] addressed football league scheduling problems in the presence of uncertainty. In their work, the authors considered several quality measures such as breaks and canceled matches. They developed a variety of proactive and reactive approaches for solving the underlying problem.

Among the objectives that have been used for a better round robin schedule, minimizing rest mismatches distinguishes itself for its natural influence on determination of matchdays given a fixture. For example, [13] introduced a "Rest Mismatch Problem" under this objective. The authors tackled the problem using integer and constraint programming models that aim to find a schedule with a minimum number of rest mismatches between the opponents, and proposed a heuristic algorithm that yields a zero-mismatch schedule in certain cases. In a more recent work, same authors focused on the problem of determining matchdays with the objective of minimizing the total rest difference (which is a slightly different objective than the one in the previous paper) given that the games in each round are known ([14]). The authors showed that the problem can be solved by solving each round separately. We should note that these two papers also have two common features with this study; first is the round by round treatment of the entire season and second is the focus on determination of the matchday.

As indicated in the introduction, an important motivation of our paper is the need to determine the matchdays in the Turkish Super League in the presence of criticisms regarding the unfair distribution of weekdays throughout the season among the fixtures of participating teams. One reason behind this concern is reduced revenues generated from games scheduled certain days of the week. The determinants of fan attendance to stadiums has received most attraction in the sports economics literature. Among the determinants that have been most often reported in empirical studies are competitive balance, variables that measure the performance of home team and the away team, various economic factors, TV broadcasting (see [36], [37], [38], [39], [40]), and the matchdays. Regarding the matchdays, [41] examined data from top European soccer leagues in Germany, Spain, and France. Their analysis revealed that all of the examined leagues have a lower attendance in games that are held on "non-frequently played" days as compared to frequently played days. In addition, the home advantage that weak teams have on non-frequent days is significantly lower than that of stronger teams. The conventional wisdom that weekday matches attract less attendance than weekend matches is also supported for the Spanish league in [42]. [43], in a comparative study across various soccer league divisions in England, reported that midweek scheduling has more negative impact on attendance in lower divisions. As far as the impact of TV broadcasting is concerned, [44] published a study on English soccer leagues as well that presents evidence for a higher degree of negative influence of satellite coverage on match attendance during weekdays rather than weekends.

2.1. Contribution

We study the matchday schedule problem for the Turkish professional soccer league that has unique features due to the needs of different stakeholders. The problem is formulated as a nonlinear binary integer programming through the use of unique fairness criteria. Particularly, an equal distribution of match days between the teams throughout the season and the ideal assignment of games to different days in each round of the tournament are taken into account in our mathematical model. We solve the underlying integer program optimally for each week and demonstrate that the optimal solution significantly outperforms the existing schedule generated manually by TFF officials.

Further, our work differs from the related work (described in [13] and [14]) in the following aspects. First, in both references, authors assign schedules to matches in the league with the objective of minimizing rest differences, whereas our model does not deal with rest difference. Second, unlike the solutions of the formulations of those work, our solution is highly dependent on the matchday schedules in all the previous rounds. Finally, there is a strong relation between the decomposed IP models of such work and the quadratic assignment problem (QAP), whereas the nature of the objective function of our model does not lend itself to the application of QAP.

We are now in a position to describe our problem and its mathematical formulation.

3. Problem description and mathematical formulation

We first provide brief information about the Turkish professional soccer league.

3.1. Turkish Professional Soccer League

The Turkish Super League (TSL) is the top tier professional soccer league of Turkey. The league regularly contains 18 teams that play with their opponents in both parts of the season, one being played at their home venue and the other at an opponent's venue. Such a tournament is called Double Round Robin Tournament. Further, teams earn three points for a win and one point for a draw, whereas no points are given for a loss.

Teams to take part in international cups are determined by the following rules. Teams ranked first and second at the end of the season qualify for the UEFA Champions League. The champion of the Turkish Cup along with teams ranked third and fourth qualify for the UEFA Europa League. These rules may be subject to change because each season the number of teams allowed to participate is redetermined by UEFA based on the overall performance of Turkish teams in five successive years before the season.

Due to the concerns indicated in the introduction, the administrators of the TSL have been facing the issue of making sure that games are fairly distributed. Since it is a quite uncommon practice of the federation in Turkey to schedule TSL games to the middle of the week, we will proceed according to the assumption that teams play Friday to Monday. There are a total of nine games in each round of play. The distribution of matches played Friday to Monday is mathematically represented as $e-f-g-h$ where e, f, g, h represent the number of games played on Friday, Saturday, Sunday and Monday, respectively. Surely, $e+f+g+h=9$. The days and hourly times of all the games during a season is scheduled by the officials of the TSL manually. On top of that, the season contains a number of rounds for which a matchday schedule has to be constructed by taking into account the games of teams attending the Champions League and the Europa League as well as the games held for the Turkish Cup from Tuesdays to Thursdays. Accordingly, a minimum of two consecutive days without any competitive encounter are inserted in between back-to-back matches (possibly in different tournaments).

Table 1. Rankings of the favorite teams in different seasons.

Season	GS	FB	BJK	MB
2014-2015	1	2	3	4
2015-2016	6	2	1	4
2016-2017	4	3	1	2
2017-2018	1	2	4	3
2018-2019	1	6	3	2

Table 2. Total number of games the favorite teams played on different days between 2014 and 2019.

Day	GS	FB	BJK	MB
Friday	20	13	18	17
Saturday	72	49	48	60
Sunday	55	76	67	68
Monday	19	29	33	22

Tables 1 and 2 display the rankings and matchday distributions of the four most successful teams between 2014 and 2019 ([45]). These teams are Galatasaray (GS), Fenerbahçe (FB), Beşiktaş (BJK), and Medipol Başakşehir (MB). Apparently, the number of games each team plays on certain days vary significantly (see Table 2). Our mathematical analysis aims to reduce this variability for these and other teams in the league. The mathematical formulation of our problem is given next.

3.2. Mathematical formulation

Our mathematical formulation is inspired by the Monden heuristic, which is used for mixed model assembly line scheduling to minimize the variation of the component parts for manufacturing end-items. This method determines the order of end items to be assembled which will make the actual usage rate of components most closely match their average usage rates ([46]). In order to choose which end item to schedule in the next assembly, all possible options are evaluated by summing the squared deviations of the actual use rates of the components. The minimum of the summed values is selected for the next assembly. The related equation is given by:

$$D_{ki} = \sqrt{\sum_{j=1}^{\beta} (\frac{K \times N_j}{Q} - X_{j,k-1} - B_{ij})^2}, \qquad (1)$$

where: D_{ki} is the deviation for end item i and order number k; β is the number of variable components; K is the number of the sequence for scheduling; N_j is the sum of components j required for the final assembly sequence; Q is the sum of the end items in the final assembly; $X_{j,k-1}$ is the cumulative number of component j used until sequence $k-1$; B_{ij} is the number of component j required to make end item i.

This method ensures that the actual usage for all components tracks closely to their average usages. We utilized Eqn. 1 in constructing the mathematical formulation of our problem. In particular, we formulate the scheduling problem faced by the TSL administrators for each round of the season as a nonlinear binary integer program, to achieve an equal matchday distribution. Monden heuristic is used for problems in a completely different domain, but there is one useful analogy which we briefly explain here. The concept of Just-in-time (JIT) manufacturing advocates a schedule in which multiple products are produced in a mixed sequence that plans for the same rate of production in any reasonable unit of time. This rate is adjusted based on incoming rate of demand. For example, consider a production environment with only three items (i.e., Item 1, Item 2, and Item 3) to produce. If in a period of 8 hours, 16 units of Item 1, 24 units of Item 2, and 32 units of Item 3 should be produced, then we schedule production of 2, 3, and 4 units per hour of Items 1, 2, and 3, respectively. In our problem, each team is analogous to a different product and each round is analogous to a different period of time where the rate of games to be played by each team on any day throughout the season is the same for establishing fairness. The details of how this works and the resulting mathematical model are provided below.

3.2.1. Parameters and decision variables

As mentioned earlier, our model is dynamic in nature. When it is used to find an equitable distribution of matchdays among the participating teams in a given round of play, it seeks to minimize the differences between the total number of times that each team has made an appearance until that round on any given day. Roughly speaking, when determining the game to be played on a Friday in any round that includes nine games, the model first compares the total number of Friday games that each team has played up to that point in the season. Then it assigns the teams with a minimal number of Friday appearances until that round to the Friday slot as long as other constraints are satisfied. In doing so, the model assumes an ideal distribution of matchdays for each team throughout season that includes 34 games in total, which is denoted by (s^*, t^*, u^*, v^*) where numbers s^* to v^* represent the total number of games played from Friday to Monday respectively. Since there are 34 weeks in any seasonal fixture, $s^* + t^* + u^* + v^* = 34$. For fairness, the ideal distribution is essentially the same for each team. Since weekend games are more preferable from the perspective of all stakeholders, ideal distribution over 34 rounds should indicate that a greater portion of games for each team to be played over the weekend. The presence of the ideal distribution establishes an objective ground for the assignment of matchdays as equally as possible in the entire season.

To emphasize, the comparative analysis indicated in the above paragraph draws an analogy with the Monden heuristic in the following sense: like we align the usage of items in production closely with their average usages, we make sure that the total number of appearances on any day of the week by the end of a specific round is as close as possible to an ideal distribution of matchdays associated with that round. Accordingly, we calculate a metric for each team to measure a matchday count deviation from an ideal and fair distribution in each round. This deviation metric, D_{dr}, $d \in DS = \{Fri, Sat, Sun, Mon\}$, $1 \leq r \leq 34$ is given below:

$$D_{dr} = \sqrt{\sum_{i=1}^{I} (x_{ird} + \sum_{k=1}^{r-1} x_{ikd} - r \times a_d)^2}, \qquad (2)$$

where

$$x_{ird} = \begin{cases} 1, & \text{if team } i \text{ has a game in round } r \text{ on day } d, \\ 0 & \text{otherwise.} \end{cases}$$

Here $i = 1, \ldots, I$, $r = 1, \ldots, R$, $d \in CD = \{Wed, Thu, Fri, Sat, Sun, Mon, Tue\}$, $I = 18$ and $R = 34$, as there are 18 teams. It will later become clear that x_{ird}'s are the decision variables of our model. Furthermore, a_d is the ideal matchday distribution per team in one round. It is calculated by dividing the ideal number of games played from Friday to Monday to the number of rounds as follows:

$$a_d = \begin{cases} \frac{s^*}{34}, & \text{if } d = \text{Fri.} \\ \frac{t^*}{34}, & \text{if } d = \text{Sat.} \\ \frac{u^*}{34}, & \text{if } d = \text{Sun.} \\ \frac{v^*}{34}, & \text{if } d = \text{Mon.} \end{cases}$$

Let us now exemplify the use of this metric. Consider a season with 34 rounds in which the ideal number of games on Saturday is 10. This ideal number is for the entire season, however, it is also an anchor to calculate the round-specific ideal for any round r. For instance, if $r = 7$, the ideal count of Saturday games (which is calculated by the term $r \times a_d$ in D_{dr}) is $7 \times 10/34 = 2.0588$. Then the value of the matchday decision variable in round $r = 7$ for any team i, $x_{i7Sat.}$, is set such that the total number of Saturday games played by team i *by the end of* round $r = 7$ (which is represented by the term $x_{ird} + \sum_{k=1}^{r-1} x_{ikd}$) is as close to 2.0588 as possible. This is repeated for any $i = 1, \ldots, 18$, and weighted against the round-specific deviation metric that we introduce in the next paragraph. The main motivation is to keep balance not only by the end of the season, but rather at each step of the way throughout the season. This is crucial because the presence of imbalance may challenge the perception of fairness when it occurs in any round of the season, not just the end.

In addition to the seasonal matchday count deviation metric, D_{dr}, there is another form of deviation that the model calculates: deviation from an ideal distribution of nine games from Friday to Monday in any round. This metric is related to the aforementioned distribution of matches that are mathematically represented as $e - f - g - h$ where e denotes the number of games played in one round on Friday, f on Saturday, g on Sunday, and h on Monday. Let $e^* - f^* - g^* - h^*$ be an ideal distribution from which our next metric measures the deviation for each round of play. This deviation metric is denoted by V_r and is computed using the below formula:

$$V_r = \sqrt{\sum_{d \in DS} \Big(\frac{\sum_{i=1}^{I} x_{ird}}{2} - b_d \Big)^2}, \qquad (3)$$

where b_d represents the ideal number of games on day d, which should be determined by the federation based on team and broadcasting company preferences. It should therefore be equal to:

$$b_d = \begin{cases} e^*, & \text{if } d = \text{Fri.} \\ f^*, & \text{if } d = \text{Sat.} \\ g^*, & \text{if } d = \text{Sun.} \\ h^*, & \text{if } d = \text{Mon.} \end{cases}$$

In (3), $\frac{\sum_{i=1}^{I} x_{ird}}{2}$ represents the total number of games on day d in round r. It is customary to schedule one match on Friday, and one on Monday (i.e., after setting $e^* = h^* = 1$) leaving the rest of the meetings to take place on Saturdays and Sundays. However the model that we propose in this paper can accommodate any pattern of play in each round based on relative importance of multiple deviations from ideal distribution in the objective function. In comparison to the seasonal matchday count deviation metric in (2), V_r has a simpler functional form. Nevertheless, V_r is essentially similar to many other deviation or distance metrics widely available in the literature; it is the square root of the sum of squared deviations from the ideal number of games in each day of the round. The main role of this metric is not to establish fairness, though. Instead, it is formulated to assign more games to weekends in each round as preferred by all the stakeholders for various reasons.

There are three parameters that represent the relative importance weights of deviations defined in this section:

w_d: weight assigned to deviation D_{dr} for each day $d \in DS = \{Fri, Sat, Sun, Mon\}$ in the objective function, $1 \leq r \leq 34$,

c_1: weight for the weighted sum of the seasonal matchday deviations, i.e., $\sum_{d \in DS} w_d \times D_{dr}$, in round $r \in \{1, ..., 34\}$,

c_2: weight assigned to deviation V_r.

One paper that we should highlight here for the presence of a methodological similarity is the study by [47]. As we mentioned in the previous section, inspired from the JIT's mixed production scheduling algorithm, our model generates a dynamically evolving matchday assignment for each team keeping a close track of the fair distribution of the available days for this purpose. A similar fairness concern is also central in the study by [47]. Their aim is to accomplish a fair assignment of referees by following criteria such as keeping a relatively balanced load of officiating throughout the season and setting up each referee to serve in an equal number of games played by each participating team. However, they do not use similar deviation metrics in their formulation, whereas the deviation functions D_{dr} $d \in DS = \{Fri, Sat, Sun, Mon\}$, $1 \leq r \leq 34$ and V_r are critical to the objective function in this paper.

The idea of establishing fairness by setting up each team to play an equal number of times on a given matchday (from Friday to Monday) as the season progresses is somewhat similar to the balancedness concept introduced in [48] as well. This concept requires for each team that the number of home and away games differ by at most one. Later, it was extended by [49] to ensure that in any round the difference between the number of home games played by any two teams up to that point in the schedule does not exceed a fixed number g. Such a schedule is called g-ranking-balanced. However, this concept is not directly applicable to our study for two reasons. First and foremost, they are not suitable for formulating the fairness criteria which are also the main motivations of our paper. Second, they are not practical for use in a multi-criteria analysis.

We state the entire problem formulation in the next subsection.

3.2.2. Objective function and constraints

Using the above parameters and decision variables, we solve a nonlinear binary integer program for $r = 1, \ldots, 34$. Owing to Champions League, European League, and Turkish Cup games before/after certain rounds, the constraints of the program differ from one round to another. We therefore provide the entire set of constraints for one specific round for illustrative purposes (i.e., $r = 7$). The fixture for this round is provided in Table 3. The games in this fixture were to be scheduled between September 28 and October 1, Friday to Monday. Without loss of generality, each team is assigned an arbitrary index 1 to 18. Specific constraints that should be considered are as follows: team 1 has a Champions League game the following Wednesday (October 3), teams 8, 11, and 13 have Turkish cup games on the Wednesday (September 26) before round 7, and team 7 has a Turkish Cup game on Thursday (September 27) which is also right before round 7.

The resulting nonlinear binary integer program for round 7 is expressed as follows:

$$\min c_1 \times \left(\sum_{d \in DS} w_d \times \sqrt{\sum_{i=1}^{I} (x_{ird} + \sum_{k=1}^{r-1} x_{ikd} - r \times a_d)^2} \right) + c_2 \times \sqrt{\sum_{d \in DS} \left(\frac{\sum_{i=1}^{I} x_{ird}}{2} - b_d \right)^2} \tag{4}$$

subject to:

Feasibility constraints:

$$x_{i,7,Fri} + x_{i,7,Sat} + x_{i,7,Sun} + x_{i,7,Mon} = 1, \quad i = 1, \ldots, I \tag{5}$$

$$x_{i,7,Thu} + x_{i,7,Fri} + x_{i,7,Sat} \leq 1, \quad i = 1, \ldots, I \tag{6}$$

$$x_{i,7,Sun} + x_{i,7,Mon} + x_{i,7,Tue} \leq 1, \quad i = 1, \ldots, I \tag{7}$$

$$x_{i,7,Wed} + x_{i,7,Thu} + x_{i,7,Fri} \leq 1, \quad i = 1, \ldots, I, i \neq 1 \tag{8}$$

$$x_{i,7,Mon} + x_{i,7,Tue} + x_{i,7,Wed} \leq 1, \quad i = 1, \ldots, I, i \neq 8, 11, 13 \tag{9}$$

Assignment constraints:

Table 3. Games scheduled for round 7.

Game	Home Team	Away Team	Home Team index	Away Team index
1	Galatasaray	Erzurum	1	17
2	Trabzon	Kasımpaşa	4	14
3	Alanya	Akhisar	9	18
4	Beşiktaş	Kayseri	3	10
5	Sivas	Bursa	12	16
6	Göztepe	Konya	15	8
7	Çaykur Rize	Fenerbahçe	11	6
8	Ankaragücü	Antalya	13	7
9	Başakşehir	Malatya	2	5

$$x_{1,7,d} - x_{17,7,d} = 0, \quad d \in DS \tag{10}$$
$$x_{4,7,d} - x_{14,7,d} = 0, \quad d \in DS \tag{11}$$
$$x_{9,7,d} - x_{18,7,d} = 0, \quad d \in DS \tag{12}$$
$$x_{3,7,d} - x_{10,7,d} = 0, \quad d \in DS \tag{13}$$
$$x_{12,7,d} - x_{16,7,d} = 0, \quad d \in DS \tag{14}$$
$$x_{15,7,d} - x_{8,7,d} = 0, \quad d \in DS \tag{15}$$
$$x_{11,7,d} - x_{6,7,d} = 0, \quad d \in DS \tag{16}$$
$$x_{13,7,d} - x_{7,7,d} = 0, \quad d \in DS \tag{17}$$
$$x_{2,7,d} - x_{5,7,d} = 0, \quad d \in DS \tag{18}$$

Additional constraints:

$$x_{1,7,Wed} = 1 \tag{19}$$
$$x_{7,7,Thu} = 1 \tag{20}$$
$$x_{11,7,Wed} = 1 \tag{21}$$
$$x_{13,7,Wed} = 1 \tag{22}$$
$$x_{8,7,Wed} = 1 \tag{23}$$
$$\sum_{d \in CD} x_{i,7,d} = 1, \quad i = 1, \ldots, I, i \neq \{1, 7, 8, 11, 13\} \tag{24}$$

Note that the term $\sum_{k=1}^{r-1} x_{ikd}$ appearing in D_{dr} in (4) uses the data coming from the previous rounds. To be more specific, this term constitutes the optimal solutions of the previous rounds. Similarly, the optimal solution of the current round will be added to the data that will be input into the mathematical models of the remaining rounds. In this sense, the mathematical model of each round is connected to each other through this term.

Constraints 5 to 8 are all formulated for a similar purpose: that each team will play only one game between the days indicated by the decision variables. Constraint 5 imposes this restriction from Friday to Monday because each team has one game to play in one round. Constraint 6 does the same from Thursday to Saturday, Constraint 7 serves for a similar purpose from Sunday to Tuesday, and finally Constraint 8 from Wednesday to Friday all because of the minimum rest requirement between games. Constraint 8 excludes an index value of 1 although $x_{1,7,Wed} = 1$ because team 1 is to play a Wednesday game after round 7 whereas 8 is imposed for teams that are scheduled to play a Wednesday game before round 7. Constraint 9 ensures that each team except teams 8, 11, and 13 can have at most one game from Monday to Wednesday. Exclusion of teams 8, 11, and 13 in constraint 9 is for a similar reason why team 1 index is excluded in constraint 8. In sum, feasibility constraints should be amended in each round to reflect the implications of the two-day rest period between games.

Constraints 10 through 18 are assignment constraints created for each scheduled game (see Table 3). For instance, Constraint 10 ensures that for each day of Friday through Monday, if team 1 has a game, team 17 must also have a game on the same day; similarly if team 1 does not have a game on any of those days, team 17 must not have a game on the same day, either.

Constraints 19 through 23 state that team 1 has a game on Wednesday, team 7 has a game on Thursday, and teams 8, 11, and 13 have games on Wednesday. Note that these constraints are required to make sure that information on the Champions League and Turkish Cup fixture of the associated teams are reflected to the model. Finally, Constraint 24 ensures that each team except teams 1, 7, 8, 11, and 13 will have only one game from Wednesday to Tuesday.

As stated earlier, we solve the aforementioned mathematical model for each round by slightly revising its constraints, depending on what teams have additional games right before/after the respective round. The underlying nonlinear binary integer program is solved by AMPL via its solver, BARON. It takes less than a minute to solve our model for a given round.

Our computational results are discussed in the next section.

Table 4. Optimal and manual matchday schedules for round 7.

Game	Optimal Schedule	Manual Schedule
1	Sunday	Friday
2	Sunday	Saturday
3	Saturday	Saturday
4	Friday	Saturday
5	Sunday	Sunday
6	Saturday	Sunday
7	Sunday	Sunday
8	Monday	Monday
9	Saturday	Monday

4. Results

We compare the optimal matchday schedule obtained by AMPL, BARON with the manual schedule constructed by the TSL officials for the 2018-2019 soccer season of Turkey ([45]). The parameters of our models are set to the following values: $c_1 = c_2 = 1$; $w_{Fri} = 0.11, w_{Sat} = 0.33, w_{Sun} = 0.44, w_{Mon} = 0.11$. Weights c_1 and c_2 are the same indicating that both deviation

Table 5. Seasonal matchday distribution of games for each team obtained by the manual schedule.

Team index	Friday	Saturday	Sunday	Monday
1	7	11	13	3
2	4	10	13	7
3	5	10	13	6
4	6	13	11	4
5	4	10	16	4
6	4	11	11	8
7	3	13	13	5
8	4	11	14	5
9	4	11	12	7
10	1	17	13	3
11	3	14	12	5
12	5	13	13	3
13	4	12	14	4
14	5	11	10	8
15	5	14	12	3
16	7	10	15	2
17	4	11	15	4
18	1	11	15	7

Table 6. Seasonal matchday distribution of games for each team obtained by the optimal schedule.

Team index	Friday	Saturday	Sunday	Monday
1	5	12	15	2
2	4	11	16	3
3	3	12	15	4
4	5	12	14	3
5	4	11	16	3
6	3	12	15	4
7	3	12	15	4
8	4	12	14	4
9	4	12	15	3
10	5	12	15	2
11	3	12	16	3
12	3	12	16	3
13	3	12	16	3
14	3	12	15	4
15	3	12	16	3
16	4	12	15	3
17	4	12	15	3
18	3	12	15	4

components of objective function are equally important. As indicated in the introduction, economic and other concerns seem to support the view that weekend games are relatively more preferable and maybe more important. This is why we choose our baseline values for seasonal deviation metric weights for weekends (i.e., w_{Sat}, w_{Sun}) to be greater than weights for the weekdays (i.e., w_{Fri}, w_{Mon}).

Regarding the ideal distribution of matches, we set $e^* - f^* - g^* - h^* = 1 - 3 - 4 - 1$, and (s^*, t^*, u^*, v^*) = (5, 12, 12, 5) for illustrative purposes and obtain our results accordingly. Like in the case of seasonal deviation metric weights, the relative preferability of the weekend games by the stakeholders is reflected to these ideal distributions.

We begin with the comparison of the optimal matchday schedule with the manual schedule for round 7. As observed in Table 4, the optimal schedule ensures the 1 - 3 - 4 - 1 pattern, whereas the manual schedule does not do so, yielding the pattern 1 - 3 - 3 - 2.

Table 7. Summary statistics for the manual schedule in the entire season.

Matchday	Std. Dev.	Min.	Max.
Friday	1.62	1	7
Saturday	1.85	10	17
Sunday	1.59	10	16
Monday	1.87	2	8

Table 8. Summary statistics for the optimal schedule in the entire season.

Matchday	Std. Dev.	Min.	Max.
Friday	0.77	3	5
Saturday	0.32	11	12
Sunday	0.65	14	16
Monday	0.65	2	4

Tables 5 and 7 show the day distribution of games obtained by the manual schedule for the entire season and the resulting summary statistics for each day from Friday to Monday, respectively. Similar statistics for the optimal solution obtained by solving the underlying nonlinear binary integer programs successively for the entire season are presented in Tables 6 and 8. Results indicate that the optimal solution significantly outperforms the manual schedule in terms of the standard deviation of the matchday schedule for Friday through Monday (see Tables 7 and 8). As an example, while standard deviation obtained by the manual schedule for Friday is 1.62, with minimum and maximum values being 1 and 7, standard deviation obtained by the optimal schedule for the same day is 0.77, with minimum and maximum values being 3 and 5. Finally, average reduction in standard deviation over the manual schedule over those four days is 65%.

Further, the optimal solution for each round reveals that the 1 - 3 - 4 - 1 pattern is ensured for the entire season except a few rounds. The TFF officials implement the 1 - 4 - 4 - 0 pattern in those few rounds due to the need to provide enough rest days for the teams with players who play in national games held right after those rounds. In contrast, the manual schedule deviates from this pattern in nearly half of the 34 rounds.

The seasonwide solution that we summarize in this section can be potentially improved upon by employing multi-round solutions based on gradual information flow from other tournaments, or comparing alternative solutions in earlier rounds that could bring better solutions in future rounds of the season. However, the dynamic approach we present here is a good start in exploring schedules that could mitigate the problems summarized in the introduction.

Table 9. Summary statistics for the optimal schedule ($c_2 = 0.2$).

Matchday	Std. Dev.	Min.	Max.
Friday	0.51	4	5
Saturday	0.46	12	13
Sunday	0.58	12	14
Monday	0.65	3	6

4.1. Sensitivity analysis

In order to observe the impact of weights, c_1 and c_2, on the performance of the optimal solution, we performed sensitivity analysis. As stated earlier, the base-case values for c_1 and c_2 are set to 1 (note that c_1 corresponds to seasonal deviation, whereas c_2 corresponds to round specific daily deviation). To explore the sensitivity of our results to these weights, we adjusted their relative values keeping the value of c_1 at 1. In particular, we generated different cases by gradually decrementing the value of c_2. It turned out that the results do not change when c_2 is greater than 0.2 (i.e., the seasonal matchday distribution of games for each team remains the same). This implies that the round specific daily deviation term of the objective function has more impact on the optimal solution. In other words, the model provides robust results even when we assign a significantly larger weight to striking a reasonable seasonwide balance up to a point in which seasonal deviation is at least five times more important than the round specific deviation. One may attribute this behavior to the partial compatibility of two deviation metrics in the sense of taking ideal matchday distributions that assign a higher number of games to weekends. As for the $c_2 = 0.2$ case, the optimal solution differs only slightly from the baseline case optimal solution in terms of the matchday distribution and the summary statistics for each matchday (Fri. to Mon.; see Tables 9 and 10).

Furthermore, Table 11 presents optimal seasonal and round specific daily deviations for each round for the baseline case and the $c_2 = 0.2$ case. Unsurprisingly, the baseline case optimal solution yields better results in terms of round specific deviation, whereas the $c_2 = 0.2$ case is superior in terms of

Table 10. The matchday distribution of games obtained by the optimal schedule ($c_2 = 0.2$).

Team index	Friday	Saturday	Sunday	Monday
1	5	12	13	4
2	4	12	13	5
3	4	12	14	4
4	5	12	13	4
5	5	13	12	4
6	5	12	13	4
7	4	12	13	5
8	5	12	13	4
9	5	13	12	4
10	4	12	14	4
11	5	13	12	4
12	5	12	13	4
13	5	13	13	3
14	4	12	13	5
15	4	13	13	4
16	5	12	13	4
17	5	12	13	4
18	4	12	12	6

seasonal deviation. The sensitivity analyses also revealed that decrementing c_2 even further leads to implausible results that cannot be implemented in practice due to the concerns of TFF.

5. Conclusions

In this work, we discuss the sports scheduling problem faced by the administrators of the Turkish Super League (TSL). Given the scheduled games for each round, the problem is to determine a fair matchday schedule for each round in terms of day distribution. We formulate our problem as a nonlinear binary integer program and solve the underlying problem optimally for each round. Our results reveal that the implementation of the optimal schedule can significantly improve the manual schedule constructed by the TSL officials. We observe that the results are largely robust in the relative changes between the weights of two major deviation metrics. They also illustrate the significance of the round specific matchday distribution concerns against the need for seasonal fairness.

Future research may consider measures such as rest mismatch or breaks in addition to seasonal or round specific day distribution. In this regard, uncertain and dynamic natures of these problems can be addressed using mathematical models from the field of dynamic stochastic optimization (see [50] and [51]). Additionally, it is worthwhile to investigate cases with multi-round solutions obtained as other tournaments progress or to incorporate uncertainty into such problems owing to the fact that many games are rescheduled or canceled due to bad weather conditions in popular soccer leagues.

References

[1] Sözcü (2016). *Fenerbahçe TFF'ye başvurdu (Fenerbahçe consulted TFF)* [online]. Available from: `https://skor.sozcu.com.tr/2016/12/30/fenerbahce-tffye-basvurdu-maclara-cikmayiz-tepkisi-572320/amp` [Accessed 1 September 2021].

[2] Ajansspor (2017). *Hafta içi tribünler dolmuyor (Tribunes are not full on weekdays)* [online]. Available from: `https://ajansspor.com/haber/hafta-ici-tribunler-dolmuyor-118912` [Accessed 1 September 2021].

[3] Habertürk (2011). *Tribünler dolmuyor (Tribunes are not full)* [online]. Available from: `https://www.haberturk.com/spor/futbol/haber/686439-tribunler-dolmuyor` [Accessed 1 September 2021].

[4] Yeniçağ (2009). *TFF'ye büyük tepki (A huge reaction against TFF)* [online]. Available from: `https://www.yenicaggazetesi.com.tr/tffye-buyuk-tepki-21521h.htm` [Accessed 1 September 2021].

[5] Sözcü (2017). *Beşiktaş'a cumalar yaramıyor (Beşiktaş does not benefit from Fridays)* [online]. Available from:

Table 11. The objective function values obtained by the optimal schedules (for the baseline case and the $c_2 = 0.2$ case, indexed as 1 and 2, respectively).

Round	Seas. Dev.$_1$	Daily Dev.$_1$	Seas. Dev.$_2$	Daily Dev.$_2$
1	1.91	0	1.91	0
2	1.99	0	1.99	0
3	2.3	0	2.3	0
4	2.24	0	2.24	0
5	2.27	0	1.97	1.4
6	2.38	0	2.18	0
7	2.87	0	2.49	0
8	3.4	0	3.08	0
9	3.09	0	3.04	0
10	3.65	0	2.74	3.45
11	3.79	0	2.93	0
12	4.26	0	3.5	0
13	4.33	0	3.56	0
14	4.89	0	3.83	0
15	5.34	0	3.99	0
16	6.02	0	3.86	3.15
17	6.25	0	4.32	0
18	6.42	0	4.57	0
19	6.71	0	4.07	2.8
20	6.51	0	4.16	0
21	6.67	0	4.58	0
22	6.65	0	4.48	0
23	6.53	0	3.85	3.75
24	6.88	0	4.16	0
25	6.8	0	4.23	0
26	6.86	0	4.42	0
27	7.08	0	4.42	0
28	7.17	0	4.05	2.45
29	7.43	0	4.31	0
30	7.73	0	3.62	3.7
31	7.8	0	3.34	1.4
32	7.9	0	2.94	3.7
33	8.3	0	3.18	0
34	8.27	0	3.5	0
average	5.37	0	3.46	0.76

https://www.goal.com/tr/amp/haber/besiktasa-cumalar-yaramiyor/1wtdm3xat35i41tzisu6ezqjv9 [Accessed 1 September 2021].

[6] Sondakika (2017). *Trabzonspor'a cuma günü oynadığı maçlar yaramıyor (Trabzonspor does not benefit from Friday matches)* [online]. Available from: `https://www.sondakika.com/haber/haber-trabzonspor-a-cuma-gunu-oynadigi-maclar-yaramiyor` [Accessed 1 September 2021].

[7] Yeniçağ (2010). *Kartala cuma yaramıyor (Kartal does not benefit from Fridays)* [online]. Available from: `https://www.fanatik.com.tr/amp/kartala-cuma-yaramiyor-175352` [Accessed 1 September 2021].

[8] Fotomaç (2018). *Başakşehire pazartesi maçları yaramıyor (Başakşehir does not benefit from Monday matches)* [online]. Available from: `https://m.fotomac.com.tr/medipol-basaksehir/2018/02/27/basaksehire-pazartesi-maclari-yaramiyor/amp` [Accessed 1 September 2021].

[9] Kim, T. (2019). Optimal approach to game scheduling of multiple round-robin tournament: Korea professional baseball league in focus. *Computers and Industrial Engineering*, 136, 95–105.

[10] Rasmussen, R.V. & Trick, M.A. (2008). Round robin scheduling—a survey. *European Journal of Operational Research*, 188(3), 617–636.

[11] Ribeiro, C.C. (2012). Sports scheduling: Problems and applications. *International Transactions in Operational Research*, 19(1-2), 201–226.

[12] Kendall, G., Knust, S., Ribeiro, C.C., & Urrutia, S. (2010). Scheduling in sports: An annotated bibliography. *Computers and Operations Research*, 37(1), 1–19.

[13] Atan, T., & Çavdaroğlu, B. (2018). Minimization of rest mismatches in round robin tournaments. *Computers and Operations Research*, 99, 78–89.

[14] Çavdaroğlu, B., & Atan, T. (2020). Determining matchdays in sports league schedule to minimize rest differences. *Operations Research Letters*, 48(3), 209–216.

[15] Rasmussen, R.V., & Trick, M.A. (2007). A Benders approach for the constrained minimum break problem. *European Journal of Operational Research*, 177(1), 198–213.

[16] Duran, G., Guajardo, M., & Saure, D. (2017). Scheduling the South American Qualifiers to the 2018 FIFA World Cup by integer programming. *European Journal of Operational Research*, 262(3), 1109–1115.

[17] Recalde, D., Torres, R., & Vaca, P. (2013). Scheduling the professional Ecuadorian football league by integer programming. *Computers and Operations Research*, 40(10), 2478–2484.

[18] Bulck, D.V., & Goossens, D. (2020). Handling fairness issues in time-relaxed tournaments with availability constraints. *Computers and Operations Research*, 115, 1048–1056.

[19] Kendall, G., & Westphal, S. (2013). Sports Scheduling: Minimizing Travel for English Football Supporters. In: A. Uyar, E. Ozcan, N. Urquhart eds. *Automated Scheduling and Planning. Studies in Computational Intelligence.* Springer, Berlin, Heidelberg, 61–90.

[20] Guedes, A.C.B., & Ribeiro, C.C. (2011). A heuristic for minimizing weighted carry-over effects in round robin tournaments. *Journal of Scheduling*, 14(6), 655–667.

[21] Januario, T., & Urrutia, S. (2016). A new neighborhood structure for round robin scheduling problems. *Computers and Operations Research*, 70, 127–139.

[22] Günneç, D., & Demir, E. (2019). Fair-fixture: Minimizing carry-over effects in football leagues. *Journal of Industrial and Management Optimization*, 15(4), 1565–1577.

[23] Durán, G., Guajardo, M. & Wolf-Yadlin, R. (2012). Operations research techniques for scheduling Chile's second division soccer league. *Interfaces*, 42(3), 273–285.

[24] Rasmussen, R.V. (2008). Scheduling a triple round robin tournament for the best Danish soccer league. *European Journal of Operational Research*, 185(2), 795–810.

[25] Goossens, D., & Spieksma, F. (2009). Scheduling the Belgian soccer league. *Interfaces*, 39(2), 109–118.

[26] Ribeiro, C.C., & Urrutia, S. (2012). Scheduling the Brazilian soccer tournament: Solution approach and practice. *Interfaces*, 42(3), 260–272.

[27] Durán, G., Guajardo, M., Miranda, J., Sauré, D., Souyris, S., Weintraub, A., & Wolf, R. (2007). Scheduling the Chilean soccer league by integer programming. *Interfaces*, 37(6), 539–552.

[28] Bartsch, T., Drexl, A., & Krȯger, S. (2006). Scheduling the professional soccer leagues of Austria and Germany. *Computers and Operations Research*, 33(7), 1907–1937.

[29] Briskorn, D., & Drexl, A. (2009). IP models for round robin tournaments. *Computers and Operations Research*, 36(3), 837–852.

[30] Durán, G., Durán, S., Marenco, J., Mascialino, F., & Rey, P.A. (2019). Scheduling Argentina's professional basketball leagues: A variation on the Travelling Tournament Problem. *European Journal of Operational Research*, 275(3), 1126–1138.

[31] Haselgrove, J., & Leech, J. (1977). A tournament design problem. *The Amer. Math. Monthly*, 84(3), 198–201.

[32] de Werra, D. (1981). Scheduling in Sports. In: P. Hansen, ed. *Studies on Graphs and Discrete Programming.* North Holland, 381–395.

[33] Elf, M., & Junger, M. (2003). Minimizing breaks by maximizing cuts. *Operations Research Letters*, 31(5), 343–349.

[34] van't Hof, P., Post, G., & Briskorn, D. (2010). Constructing fair round robin tournaments with a minimum number of breaks. *Operations Research Letters*, 38(6), 592–596.

[35] Yi, X., Goossens, D., & Nobibon, F.T. (2020). Proactive and reactive strategies for football league timetabling. *European Journal of Operational Research*, 282(2), 772–785.

[36] Jane, W.J. (2014). The relationship between outcome uncertainties and match attendance: New evidence in the National Basketball Association. *Review of Industrial Organization*, 45(2), 177–200.

[37] Tainsky, S., & Winfree, J.A. (2010). Short-Run Demand and Uncertainty of Outcome in Major League Baseball. *Review of Industrial Organization*, 37(3), 197–214.

[38] Buraimo, B., & Simmons, R. (2009). A tale of two audiences: Spectators, television viewers and outcome uncertainty in Spanish football. *Journal of Economics and Business*, 61(4), 326–338.

[39] Szymanski, S. (2003). The economic design of sporting contests. *Journal of Economic Literature*, 41(4), 1137–1187.

[40] Benz, M.A., Brandes, L., & Franck, E. (2009). Do soccer associations really spend on a good thing? Empirical evidence on heterogeneity in the consumer response to match uncertainty of outcome. *Contemporary Economic Policy*, 27(2), 216–235.

[41] Goller, D., & Krumer, A. (2020). Let's meet as usual: Do games played on non-frequent days differ? Evidence from top European soccer leagues. *European Journal of Operational Research*, 286(2), 740–754.

[42] Garcia, J., & Rodriguez, P. (2002). The determinants of football match attendance revisited: Empirical evidence from the Spanish football league. *Journal of Sports Economics*, 3(1), 18–38.

[43] Forrest, D., & Simmons, R. (2006). New issues in attendance demand: The case of the English football league. *Journal of Sports Economics*, 7(3), 247–266.

[44] Baimbridge, M., Cameron, S., & Dawson, P. (1996). Satellite Television and the Demand for Football: A Whole New Ball Game? *Scottish Journal of Political Economy*, 43(3), 317–333.

[45] Turkish Football Federation (TFF) Website (2021). Available from: `https://www.tff.org` [Accessed 1 September 2021].

[46] Vollman, T., Berry, W., Whybark, D., & Jakobs, F. (2011). *Manufacturing Planning and Control for Supply Chain Management.* 5th ed. McGraw-Hill, New York.

[47] Yavuz, M., İnan, U.H., & Fığlalı, A. (2008). Fair referee assignments for professional football leagues. *Computers and Operations Research*, 35(9), 2937–2951.

[48] Knust, S., & von Thaden, M. (2006). Balanced home-away assignments. *Disrete Optimization*, 3(4), 354–365.

[49] Goossens, D.R., & Spieksma, F.C.R. (2012). Soccer schedules in Europe: an overview. *Journal of Scheduling*, 15, 641–651.

[50] Göçgün, Y. (2018). Dynamic Scheduling with Cancellations: An Application to Chemotherapy Appointment Booking. *An International Journal of Optimization and Control: Theories and Applications*, 8(2), 161-169.

[51] Göçgün, Y. (2021). Performance Comparison of Approximate Dynamic Programming Techniques for Dynamic Stochastic Scheduling. *An International Journal of Optimization and Control: Theories and Applications*, 11(2), 178-185.

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