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An algorithm for solution of an interval valued EOQ model

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Abstract. This paper deals with the problem of determining the economic order quantity (EOQ) in the interval sense. A purchasing inventory model with shortages and lead time, whose carrying cost, shortage cost, setup cost, demand quantity and lead time are considered as interval numbers, instead of real numbers. First, a brief survey of the existing works on comparing and ranking any two interval numbers on the real line is presented. A common algorithm for the optimum production quantity (Economic lot-size) per cycle of a single product (so as to minimize the total average cost) is developed which works well on interval number optimization under consideration. A numerical example is presented for better understanding the solution procedure. Finally a sensitive analysis of the optimal solution with respect to the parameters of the model is examined.

Keywords: Inventory; interval arithmetic; interval valued demand; interval lead time; interval valued inventory cost parameters.

AMS Classification: 90B05

1. Introduction

Today many names are crowded with the economic order quantity(EOQ) model, the most fundamental one in inventory control theory of which Harris [3] was the pioneer. Generally we face inventory problems in manufacturing maintenance service and business operations. Making companies succeed their goals and targets with regards to ensuring delivery, avoiding shortages, helping sales at competitive prices and solving many other problems, inventory control plays a particularly prominent role in supply chain management. A proper control of inventory is imperative when a company targets a good deal of profit. In spite of the fact that the EOQ model has played a crucial role in the field of control theory, we find difficulties in applying the EOQ model from paper to soil because, if we try to get the exact values of carrying cost, shortage cost, setup cost and demand quantity, the demand and these inventory costs often change slightly from one cycle to another. For example, inventory carrying cost may be different in rainy seasons from summer or winter seasons (costs of taking proper action to prevent deterioration of items in different seasons and also the labour charges in different seasons are different). Ordering cost being dependent on the transportation facilities may also vary from season to season. Changes in price of fuels, mailing and telephonic charges may also make the ordering cost variable. Unit purchase cost is highly dependent on the costs of raw materials and labour charges and it may fluctuate with time. Similarly the customer's demand also differs on various season. For example, the demands of soft-drinks, ice-cream, refrigerators, etc. generally increase in summer while

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rain-coat and umbrella are needed mostly in the rainy season. So we can know the values only approximately. Usually, researchers [18, 16, 17] consider parameters either as constant or dependent on time or probabilistic in nature. Generally, uncertainties are considered as randomness and are handled by probability theory in conventional inventory models. In stochastic approach, the parameters are assumed to be random variables with known probability distribution. But we cannot estimate the exact probability distribution due to lack of pre-existing data. To solve the problem with such imprecise numbers, fuzzy and fuzzy-stochastic approaches may be used. Recently, much attention has been focused on EOQ models with fuzzy carrying cost, fuzzy shortage cost, fuzzy set-up cost, fuzzy demand etc; this means that elements of carrying cost, shortage cost, setup cost and demand are fuzzy numbers [2, 4, 9, 10, 15]. In fuzzy approach, the parameters, constraints and goals are considered as fuzzy sets with known membership functions. On the other hand, in fuzzy-stochastic approach, some parameters are viewed as fuzzy sets and others, as random variables. However, in reality, it is not always easy to specify the membership function or probability distribution in an exact environment. We choose the interval numbers instead of the fuzzy numbers due to the following facts.

(i)If the membership function of the fuzzy variable is complex, for example, when a trapezoidal fuzzy number and a Gaussian fuzzy number coexist in a model, it is hard to obtain the exact membership function of the total cost, this lack of accuracy will affect the quality of the solution obtained.

(ii) An interval number can be throughout an extension of the concept of a real number and also a subset of a real line $\Re$ ([12]). Zimmermann [7] shows that α-cut of a fuzzy number is an interval number. As the coefficients of an interval signifies the extent of tolerance (or a region) that the parameter can possibly take.

(iii) To define a fuzzy number, three parameters are required when for an interval number, two parameters are used. The notation of interval numbers has the advantage of being simple and at the same time it is a better model to represent the values in the situation like "is or lies between α and β". So the interval numbers [12], serve our required purpose better.

Thus, the interval number theory, rather than the traditional probability theory and fuzzy set theory, is better suited to solve the inventory problem. According to the decision makers' points of view under changeable conditions, we may replace the real numbers by the interval valued numbers to solve the problems easily. Since, the optimal total average cost of the model should be interval-valued and no study has yet been carried out for interval valued purchasing inventory models with shortages and with nonzero lead time, we intend to examine the above mentioned problem in this paper.

In this paper, we have proposed an optimization technique based on the division criteria of prescribed/accepted search region to solve the problems with the help of finite interval arithmetic and interval order relations developed recently in Mahato and Bhunia [14].

We organize the paper as follows : In section 2, we give some basic definitions, notations and comparison on interval numbers. In section 3, we give the model formulation and we present the solution procedure in section 4. Finally a numerical example is presented and sensitive analysis of the optimal results with respect to the parameters of the model is performed in section 5.

2. Interval Number

Let $\Re$ be the set of all real numbers. An interval, Moore [12], may be expressed as

$$\overline{a} = [a_L, a_R] = \{x : a_L \leq x \leq a_R, a_L, a_R \in \Re\}$$

where a_L and a_R are called the lower and upper limits of the interval $\overline{a}$, respectively. If $a_L = a_R$ then $\overline{a} = [a_L, a_R]$ is reduced to a real number a, where $a = a_L = a_R$. Alternatively an interval $\overline{a}$ can be expressed in mean-width or center-radius form as $\overline{a} = \langle m(\overline{a}), w(\overline{a})\rangle$, where $m(\overline{a}) = \frac{1}{2}(a_L + a_R)$ and $w(\overline{a}) = \frac{1}{2}(a_R - a_L)$ are respectively the mid-point and half-width of the interval $\overline{a}$. The set of all interval numbers in $\Re$ is denoted by $I(\Re)$.

2.1. Properties of interval

The intervals are precisely the connected subsets of $\Re$. It follows that the image of an interval by any continuous function is also an interval. This is one formulation of the intermediate value theorem. The intervals are also the convex subsets of $\Re$. The interval enclosure of a subset $X \subseteq \Re$ is also the convex hull of X. The intersection of any collection of intervals is always an interval. The union of two intervals is an interval if and only if they have a non-empty intersection or an open end-point of one interval is a closed end-point of the other. Any element X of an interval I defines a partition of I into three disjoint intervals I_1, I_2, I_3 respectively, the elements of I that are less than X, the singleton $[x, x] = \{x\}$ and the elements that are greater than x. The parts I_1 and I_3 are both non-empty (and have non-empty interiors) if and only if x is in the interior of I. This is an interval version of the trichotomy principle.

2.2. Basic interval arithmetic

Let $\overline{a} = [a_L, a_R] = \langle m(\overline{a}), w(\overline{a})\rangle$ and $\overline{b} = [b_L, b_R] = \langle m(\overline{b}), w(\overline{b})\rangle \in I(\Re)$, then

$$\begin{aligned}\overline{a}+\overline{b} &= [a_L + b_L, a_R + b_R] \\ &= \langle m(\overline{a}) + m(\overline{b}), w(\overline{a}) + w(\overline{b})\rangle. \quad (1)\end{aligned}$$

The multiplication of an interval by a real number $c \neq 0$ is defined as

$$\begin{aligned}c\overline{a} &= [ca_L, ca_R]; \quad \text{if } c > 0 \\ &= [ca_R, ca_L]; \quad \text{if } c < 0 \quad (2)\end{aligned}$$

$$c\overline{a} = c\langle m(\overline{a}), w(\overline{a})\rangle = \langle cm(\overline{a}), |c|w(\overline{a})\rangle \quad (3)$$

The difference of these two interval numbers is

$$\overline{a} - \overline{b} = [a_L - b_R, a_R - b_L]. \quad (4)$$

The product of these two distinct interval numbers is given by

$$\begin{aligned}\overline{a}.\overline{b} = \; &[\min\{a_L.b_L, a_L.b_R, a_R.b_L, a_R.b_R\}, \\ &\max\{a_L.b_L, a_L.b_R, a_R.b_L, a_R.b_R\}] \quad (5)\end{aligned}$$

The division of these two interval numbers with $0 \notin \overline{b}$ is given by

$$\begin{aligned}\overline{a}/\overline{b} = \; &\left[\min\left\{\frac{a_L}{b_L}, \frac{a_L}{b_R}, \frac{a_R}{b_L}, \frac{a_R}{b_R}\right\},\right. \\ &\left.\max\left\{\frac{a_L}{b_L}, \frac{a_L}{b_R}, \frac{a_R}{b_L}, \frac{a_R}{b_R}\right\}\right] \quad (6)\end{aligned}$$

The power of an interval $\overline{a} = [a_L, a_R]$ is given by

$$\begin{aligned}\overline{a}^k &= [1, 1]; \text{ if } k = 0, \\ &= [a_L^k, a_R^k]; \text{ if } a_L \geq 0 \text{ or if } k \text{ is odd }, \\ &= [a_R^k, a_L^k]; \text{ if } a_R \leq 0 \text{ or if } k \text{ is even }, \\ &= [\min\{a_L.a_R^{k-1}, a_R.a_L^{k-1}\}, \{a_L^k, a_R^k\}]; \\ &\quad \text{if } a_L \leq 0 \leq a_R, k \text{ is even} \quad (7)\end{aligned}$$

The addition and multiplication operations are commutative, associative and sub-distributive.

2.3. Order relations of interval numbers

In this section, we shall discuss the developments of order relations of interval numbers. Any two closed intervals $\overline{A}$ and $\overline{B}$ may be of the following types.

Type I: Non-overlapping intervals.

Type II: Partially overlapping intervals.

Type III: Fully overlapping intervals.

Moore [12] first pointed out two transitive order relations of the interval numbers. Then Ishibuchi and Tanaka [6] defined the order relations of two closed intervals which are partially order related. Generalizing the definitions of Ishibuchi and Tanaka [6], Chanas and Kuchta [13] proposed the concept of t_0t_1-cut of an interval and defined new order relations. Sengupta and Pal [1] gives another approach of ranking of two closed intervals, defining by the acceptability function $\mathcal{A} : I \times I \to [0, \infty)$ for the intervals $\overline{A}$ and $\overline{B}$ as

$$\mathcal{A}(\overline{A}, \overline{B}) = \frac{m(\overline{b}) - m(\overline{a})}{w(\overline{b}) + w(\overline{a})}, \text{ where } w(\overline{b}) + w(\overline{a}) \neq 0$$

$\mathcal{A}(\overline{A}, \overline{B})$ may be regarded as a grade of acceptability of the 'first interval to be inferior to the second'. The acceptability index is only a value-based ranking index and it can be applied partially to select the best alternative from the pessimistic point of view of the decision maker. So, only the optimistic decision maker can use it completely. Recently Mahato and Bhunia [14] introduced the revised definition of order relations between interval costs (or times) for minimization problems and interval profits for maximization problems in the context of optimistic and pessimistic decision making. As usual, let the intervals $\overline{A}$ and $\overline{B}$ represent the uncertain interval costs (or times) or profits in center-radius form.

Optimistic decision making: For minimization problems the order relation '$\leq'_{o\min}$ between the intervals $\overline{A}$ and $\overline{B}$ is

(i) $\overline{A} \leq_{o\min} \overline{B}$ iff $a_L \leq b_L$,

(ii) $\overline{A} <_{o\min} \overline{B}$ iff $\overline{A} \leq_{o\min} \overline{B}$ and $\overline{A} \neq \overline{B}$.

This implies that $\overline{A}$ is superior to $\overline{B}$ and $\overline{A}$ is accepted. This order relation is not symmetric.

Pessimistic decision making: In this case, the decision maker expects the minimum cost/time for minimization problems according to the principle 'Less uncertainty is better than more uncertainty'.

For minimization problems the order relation ' $<'_{p\min}$ between the intervals $\overline{A} = [a_L, a_R] = \langle m(\overline{a}), w(\overline{a})\rangle$ and $\overline{B} = [b_L, b_R] = \langle m(\bar{b}), w(\bar{b})\rangle$ is

(i) $\overline{A} <_{p\min} \overline{B}$ iff $m(\overline{a}) < m(\bar{b})$, for type-I and type-II intervals,

(ii) $\overline{A} <_{p\min} \overline{B}$ iff $m(\overline{a}) \leq m(\bar{b})$ and $w(\overline{a}) < w(\bar{b})$, for type-III intervals.

However, for type-III intervals with $m(\overline{a}) < m(\bar{b})$ and $w(\overline{a}) > w(\bar{b})$, the pessimistic decision cannot be taken. Here, the optimistic decision is considered.

3. Model Formulation

The purpose of the EOQ model is to find the optimal order quantity of inventory items at each time such that the sum of the order cost, the carrying cost and the shortage cost, i.e., total cost is minimal.

Notations : For the sake of clarity, the following notations are used throughout the paper.

t_1, is the time for reordering for next cycle. i.e., reorder point ;

t_2, is the time of the inventory cycle when on hand inventory reaches zero ;

t_3, length of each cycle ;

$t_3 - t_2$, is the duration of the inventory cycle when stock out occurs ;

$\overline{Q}$, is the order quantity, which enters into inventory at time $t = 0$;

$\overline{Q_1}$, is the on hand inventory at time $t = t_1$, that is, at reorder point ;

$\overline{Q_2}$, is the shortages amount inventory after time $t = t_3$, that is, after one cycle ;

$\overline{C}(Q)$, total cost in the plan period ;

$\overline{D} = [d_L, d_R]$, demand rate per unit time ;

$\overline{C}_1 = [c_{1L}, c_{1R}]$, carrying cost or holding cost per unit item per unit time ;

$\overline{C}_2 = [c_{2L}, c_{2R}]$, shortage cost per unit item per unit time ;

$\overline{C}_3 = [c_{3L}, c_{3R}]$, ordering or setup cost per order ;

$\bar{l} = [l_L, l_R]$, represents the lead time ;

Assumptions : We have the following assumptions:

(i) Shortages are allowed.

(ii) The inventory planning horizon is infinite and the inventory system involves only one item and one stocking point.

(iii) Only a single order will be placed and the entire lot is delivered in one batch.

(iv) The lead time $\bar{l}$ is non-zero.

(iv) The quantities $\overline{C_1}, \overline{C_2}, \overline{C}_3, \overline{D}$ and $\bar{l}$ are assumed to be interval number, belongs to $I(\Re)$.

A typical behavior of the EOQ purchasing inventory model with uniform demand and with lead time and shortage is depicted in Figure 1. Replenishment is received at time $t = 0$ when inventory level reaches at its maximum, Q. At time $t = t_2$, the inventory level at zero. The time $t = t_1$ is the reorder point when order is placed for the next cycle. At time $t = t_1$, the inventory level reaches at $Q_1 (< Q)$. Thus order should be placed for the next cycle when on hand inventory become Q_1. The inventory level reaches zero at time $t = t_2$ and shortages starts and continues till time $t = t_3$ when the order for the next cycle, which was placed at time $t = t_1$ is added into the inventory. Thus the entire demand during the period $(t_3 - t_2)$ is backlogged. Let Q_2 be the shortages inventory level. In the Fig. 1 the area of $\triangle BCE$ represents the failure to meet the demand and the area of $\triangle FOB$ represents the inventory. Therefore, the total cost X (say) in the plan period $[0, t_3]$ can be expressed as (see in appendix)

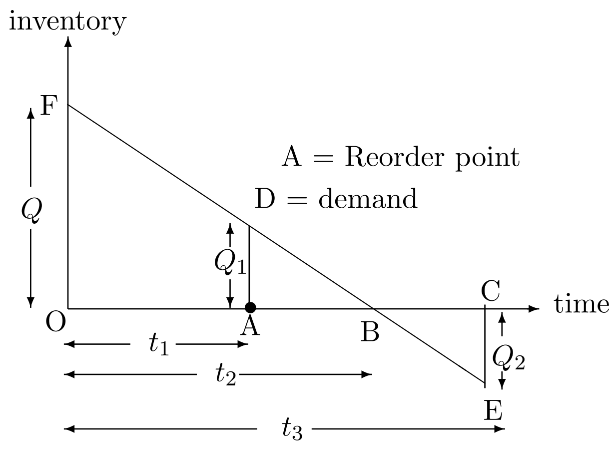

Figure 1. Graphical presentation of inventory system.

$$X = C_3 + \frac{1}{2} C_1 D t_2^2 + \frac{1}{2} C_2 D (t_3 - t_2)^2.$$

Therefore total average cost $C(t_1, t_2, t_3)$ is given by

$$C(t_1,t_2,t_3)=\frac{X}{t_3}$$
$$=\frac{1}{t_3}\left[C_3+\frac{1}{2}C_1Dt_2^2+\frac{1}{2}C_2D(t_3-t_2)^2\right].$$

Since $t_3=t_1+l$ so we have,

$$C(t_1,t_2)=\frac{1}{t_1+l}$$
$$\times\left[C_3+\frac{1}{2}C_1Dt_2^2+\frac{1}{2}C_2D(t_1+l-t_2)^2\right] \quad (8)$$

By using calculus, we optimize $C(t_1,t_2)$ and get optimum values of C, t_1 and t_2 as

$$C^*=\sqrt{\frac{2C_1C_2C_3D}{C_1+C_2}},$$
$$t_1^*=\sqrt{\frac{2C_3(C_1+C_2)}{C_1C_2D}}-l,$$
$$t_2^*=\sqrt{\frac{2C_2C_3}{C_1(C_1+C_2)D}}$$

by using these values we get the optimum values of Q_1, Q, t_3.

Usually, in mathematical programming we deal with the real numbers which are assumed to be fixed in value. In usual models costs and demand are always fixed in value. But in real life, business cannot be properly formulated in this way due to uncertainty. In such cases demand and other costs are assumed to be interval valued. But in interval oriented system, we cannot use the calculus method for optimization.

3.1. Deterministic representation of the proposed EOQ model

Let us assume interval valued demand by $\overline{D}=[d_L,d_R]$, carrying cost by $\overline{C}_1=[c_{1L},c_{1R}]$, shortage cost by $\overline{C}_2=[c_{2L},c_{2R}]$ and set up cost by $\overline{C}_3=[c_{3L},c_{3R}]$, where first term within the bracket denotes lower limit and 2nd term within the bracket denotes the upper limit of the variable. Replacing D by $[d_L,d_R]$, C_1 by $[c_{1L},c_{1R}]$, C_2 by $[c_{2L},c_{2R}]$ and C_3 by $[c_{3L},c_{3R}]$ in equation (8) we have,

$$\overline{C}(t_1,t_2) = \frac{1}{t_1+[l_L,l_R]}\left\{[c_{3L},c_{3R}]+\frac{1}{2}[c_{1L},c_{1R}]\right.$$
$$[d_L,d_R]t_2^2+\frac{1}{2}[c_{1L},c_{1R}][d_L,d_R]$$
$$\left.(t_1+[l_L,l_R]-t_2)^2\right\} \quad (9)$$

Addition and other composition rules (seen in the section 2.2 in this paper) on interval numbers are used in this equation. In interval oriented system as we cannot use the calculus method for optimization of $\overline{C}(t_1,t_2)$ we have presented a new method dependent on interval computing technique (multi-section method) to solve the unconstrained optimization problems, and for that if we take $t_1=[t_{1L},t_{1R}]$ and $t_2=[t_{2L},t_{2R}]$ then the expression (9) becomes,

$$\overline{C}(t_1,t_2) = \frac{1}{[t_{1L},t_{1R}]+[l_L,l_R]}\{[c_{3L},c_{3R}]$$
$$+\frac{1}{2}[c_{1L},c_{1R}][d_L,d_R][t_{2L}^2,t_{2R}^2]$$
$$+\frac{1}{2}[c_{2L},c_{2R}][d_L,d_R]([t_{1L},t_{1R}]$$
$$-[t_{2L},t_{2R}]+[l_L,l_R])^2\} \quad (10)$$

By using multi-section method we are to find $t_1^*=[t_{1L}^*,t_{1L}^*]$, and $t_2^*=[t_{2L}^*,t_{2L}^*]$, for which $\overline{C}(t_1,t_2)$ have the optimal (minimum) value.

4. Solution Procedure

Here, we use the multi-section algorithm, the basis of this method is the comparison of intervals (as described in the section 2.3 of this paper) according to the DM's point of view.

Let us consider a bound unconstrained optimization (maximization or minimization) problem with fixed coefficients as follows:

$$z=f(x), \quad l\le x\le u,$$

where $x=(x_1,x_2,\ldots,x_n), l=(l_1,l_2,\ldots,l_n), u=(u_1,u_2,\ldots,u_n)$, and n represents the number of decision variables, the j^{th} decision variable $x_j;(j=1,2,\ldots,n)$ lies in the prescribed interval $[l_j,u_j]$. Hence, the search space of the above problem is as follows:

$$S=x\in\Re^n: l_j\le x_j\le u_j, j=1,2,\ldots,n.$$

Suppose that, an industry divides the sales season into λ periods. Now our object is to split the accepted (reduced) region (for the first time, it is the given search space or assumed if the search space is not given) into finite number of distinct equal subregions $R_1,R_2,\ldots,R_\lambda$ to select the subregion containing the best function value.

Let $f(R_i) = [\underline{f_i}, \overline{f_i}]; i = 1, 2, \ldots, \lambda$ be the interval valued objective function $f(x)$ in the i^{th} subregion R_i, where $\underline{f_i}, \overline{f_i}$ denote the upper and lower bounds of $f(x)$ in R_i, computed by the application of finite interval arithmetic. Now, comparing all the interval-valued values of objective function, $f(x)$ in $R_i (i = 1, 2, \ldots, \lambda)$ with the help of interval order relations mentioned in earlier section, the subregion containing the best objective function value is accepted. Again, this accepted subregion is divided into other smaller distinct subregions $R'_i (i = 1, 2, \ldots, \lambda)$ by the aforesaid process and applying the same acceptance criteria, we get the reduced subregion. This process is terminated after reaching the desired degree of accuracy and finally, we get the best value of the objective function and the corresponding values of the decision variables in the form of closed intervals with negligible width.

Hence the overall procedure to solve the optimization problem (14) with an interval valued carrying cost, shortage cost, set up cost and demand, the interval valued cost function has the following structure.

4.1. Algorithm for interval optimization

Input: n(number of variables), λ(number of divisions), l_j and u_j, $(j = 1, 2, \cdots, n)$ are the lower and upper bounds respectively.

Output: The optimum values $t_1^*, t_2^*, \overline{Q}^*, \overline{Q}_1^*$ and $\overline{C}^*$.

Step 1: Initialize l_{min} and u_{min}, the lower and upper value of interval valued cost function

Step 2:[Calculation of step lengths]

Step 2.1: For $i = 0$ to $n - 1$
calculate $h_i = (u_i - l_i)/\lambda$
Set $l_i = a_i$
end for

Step 3:[Division of region S into equal subregions R_i]

Step 3.1: For $j = 0$ to $\lambda - 1$
Calculate
$l_0 = a_i + j * h_i$ and $u_0 = a_i + (j + 1) * h_i$

Step 3.2: For $j1 = 0$ to $\lambda - 1$
Calculate
$l_1 = a_i + j1 * h_i$ and $u_1 = a_i + (j1 + 1) * h_i$

Step 3.3:[**Call the function** f_l **and** f_u].

By using basic interval arithmetic defined in the section (2.2), Calculate f_l and f_u, the lower and upper values of the interval $\overline{C}(t_1, t_2)$ respectively, obtained by as in equation (10)

Step 3.4: Applying pessimistic order relation (defined in the section 2.2) between any two interval numbers $[fl, fu]$ and $[l_{\min}, u_{\min}]$ choose the optimal interval number.

end $j1$ loop

end j loop

Step 3.5: choose the subregion R^{opt} among $R_i (i = 1, 2, \ldots, \lambda)$ which has better objective function value by comparing the interval values $f(R_i), i = 1, 2, 3, \ldots, \lambda$ to each other.

Step 4: Calculation of widths

Step 4.1: For $i1 = 0$ to $n - 1$
Calculate widths $w_{i1} = u_{i1} - l_{i1}$

Step 4.2: While $w_{i1} > \varepsilon$

break

Step 4.3: Set $R^{opt} \leftarrow R_i$

Return to step 1.2

end for

end while.

Output

5. Computational Results

In this section, we illustrate that the solution procedure proposed in Algorithm 4.1 can be easily implemented on a computer and we show that, with such an implementation, the optimal solutions can be obtained. To serve our purpose, we make out a computer programming using C++ on a PENTIUM 4 personal computer.

The preceding solution procedure can be illustrated by the following numerical example. Consider an inventory model with the values of the parameters: $\overline{C}_1 = [2.5, 3.5], \overline{C}_2 =$

$[7.5, 8.5], \overline{C}_3 = [245, 255], \overline{D} = [77.5, 82.5]$ and $\bar{l} = [0.75, 0.85]$. The optimal solutions are $t_1^* = 0.9351, t_2^* = 1.2501, \overline{Q}^* = [96.8827, 103.1332], \overline{Q_1}^* = [24.4100, 25.9848]$ and $\overline{C}^* = [252.8625, 344.7752]$.

5.1. Sensitivity analysis

Based on the numerical example considered above, we now calculate the corresponding outputs for changing the input parameters one by one. While taking one parameter at a time to change we keep the remaining parameters unchanged as it is shown in the table 1. We now study the effects of changing the values of input parameters $\overline{C}_1, \overline{C}_2, \overline{C}_3, \bar{l}$ and $\overline{D}$ on the outputs $t_1^*, t_2^*, \overline{Q}^*, \overline{Q_1}^*$ and $\overline{C}^*$.

The sensitivity analysis is performed by changing mid value of each parameter $\overline{C_1}, \overline{C_2}, \overline{C_3}, \overline{D}$ and $\bar{l}$ by $+50\%, +25\%, -25\%$ and -50%; taking one parameter at a time and keeping the remaining parameters unchanged. The changes of $t_1^*, t_2^*, \overline{Q}^*, \overline{Q_1}^*$ and $\overline{C}^*$ are analyzed in the table 2.

From Table 2, it is seen that

(i) $t_1^*, t_2^*, \overline{Q}^*$ and $\overline{Q_1}^*$ is fairly sensitive while $\overline{C}^*$ is moderately sensitive to changes in the value of the carrying cost $\overline{C}_1$.

(ii) Each of $t_1^*, t_2^*, \overline{Q}^*$ and $\overline{C}^*$ are not much sensitive but $\overline{Q_1}^*$ is very high sensitive to changes in the value of the shortage cost $\overline{C}_2$.

(iii) Each of $t_1^*, t_2^*, \overline{Q}^*, \overline{Q_1}^*$ and $\overline{C}^*$ are moderately sensitive to changes in the value of the setup cost $\overline{C}_3$.

(iv) Changes in the demand rate $\overline{D}$ include less changes in $\overline{Q}^*$, and $\overline{C}^*$ in comparison with considerable changes of $t_1^*, t_2^*, \overline{Q_1}^*$.

(v) Each of t_1^* and $\overline{Q}_1^*$ are very high sensitive while t_2^*, $\overline{Q}^*$ and $\overline{C}^*$ are insensitive for the changes in the lead time $\bar{l}$.

6. Conclusion

In this paper, we have presented an inventory model with shortage, where carrying cost, shortage cost, ordering or setup cost and demand are assumed as interval numbers instead of crisp or probabilistic in nature. We have considered the nature of these quantities as interval numbers to make the inventory model more realistic. At the present time, the presence of inventory has motivational effect on the people around it. These observations are attracted by the interest of researchers in marketing and behavioral science. At first, we have formulated a solution procedure to optimize a general function with coefficients as interval valued numbers using interval arithmetic and then we have proposed optimization methods depending on splitting criteria of the accepted subregion or prescribed region (initially), finite interval arithmetic and the revised definitions of order relations. This technique does not require any derivative information of the objective function. It is also different from any stochastic method or any heuristic or metaheuristic methods. In this splitting criteria, the whole accepted subregion is divided into several equal distinct subregions with respect to all the edges simultaneously. From the numerical experiments, it is observed that the methods possess the merits of global exploration. Also, by using a C++ computer program on a PENTIUM 4 personal computer we can find the optimal solutions with small computation time. The algorithm has been tested using numerical example and which shows that our algorithm is rather accurate and rapid. For future research, one may apply the same methodology of interval computing technique for constrained optimization problems and different branches of Operations Research. If demand rate, lead time and the related inventory costs namely holding cost, shortage cost and setup cost are assumed as fuzzy numbers [5], by using Grzegorzewski [11] we can transform these parameters as interval numbers and then we can apply our proposed method serving us a better result in short time. When the replenishment rate is considered as finite, it gives another inventory model which can be solved by our proposed method. The proposed model can be extended in several ways. In addition, we could consider the demand as a function of time, whose coefficients are considered as interval numbers. Some numerical examples are studied to illustrate the theoretical results. To study the effect of the optimal time t_1^* and t_2^*, on the optimal order quantity $\overline{Q}^*, \overline{Q}_1^*$ and on the optimal annual total cost $\overline{C}^*$ there are some managerial phenomena from table 2 which are discussed previously.

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Table 1. Various outputs for the changes in the input parameters (taking one parameter at a time to change and keeping the remaining parameters unchanged).

% change	Values of parameter	t_1^*	t_2^*	$\overline{Q}^*$	$\overline{Q_1}^*$	$\overline{C}^*$
+50	$\bar{c}_1$=[4, 5]	0.709	0.951	[73.740, 78.498]	[18.768, 19.979]	[295.177, 391.460]
+25	$\bar{c}_1$=[3.25, 4.25]	0.767	1.041	[80.737, 85.946]	[21.258, 22.630]	[276.632, 370.005]
-25	$\bar{c}_1$=[1.75, 2.75]	1.263	1.632	[126.483, 134.644]	[28.606, 30.452]	[221.392, 317.066]
-50	$\bar{c}_1$=[1, 2]	1.828	2.274	[176.266, 187.638]	[34.613, 36.846]	[176.339, 286.610]
+50	$\bar{c}_2$=[11.5, 12.5]	0.972	1.415	[109.644, 116.718]	[34.289, 36.502]	[263.978, 365.566]
+25	$\bar{c}_2$=[9.5, 10.5]	0.853	1.250	[96.883, 103.133]	[30.790, 32.777]	[259.681, 355.244]
-25	$\bar{c}_2$=[5.5, 6.5]	1.069	1.250	[96.883, 103.133]	[14.033, 14.938]	[242.506, 330.175]
-50	$\bar{c}_2$=[3.5, 4.5]	1.250	1.167	[90.421, 96.255]	[-6.878, -6.462]	[224.306, 306.607]
+50	$\bar{c}_3$=[370, 380]	1.408	1.618	[125.407, 133.498]	[16.297, 17.348]	[313.710, 417.784]
+25	$\bar{c}_3$=[307.5, 317.5]	1.198	1.458	[113.029, 120.321]	[20.155, 21.456]	[284.771, 383.200]
-25	$\bar{c}_3$=[182.5, 192.5]	0.664	1.042	[80.737, 85.946]	[29.247, 31.134]	[216.582, 302.255]
-50	$\bar{c}_3$=[120, 130]	0.381	0.833	[64.591, 68.758]	[35.052, 37.313]	[173.050, 252.615]
+50	$\overline{D}$=[117.5, 122.5]	0.640	1.042	[122.408, 127.616]	[47.156, 49.162]	[307.271, 426.097]
+25	$\overline{D}$=[97.5, 102.5]	0.833	1.188	[115.789, 121.726]	[34.529, 36.299]	[281.674, 388.452]
-25	$\overline{D}$=[57.5, 62.5]	1.217	1.458	[83.860, 91.152]	[13.862, 15.067]	[219.472, 297.930]
-50	$\overline{D}$=[37.5, 42.5]	1.761	1.875	[70.316, 79.692]	[4.297, 4.870]	[178.735, 244.647]
+50	$\bar{l}$=[1.15, 1.25]	0.535	1.250	[96.883, 103.133]	[55.410, 58.985]	[252.866, 344.775]
+25	$\bar{l}$=[0.95, 1.05]	0.735	1.250	[96.883, 103.133]	[39.916, 42.491]	[252.866, 344.775]
-25	$\bar{l}$=[0.55, 0.65]	1.135	1.250	[96.883, 103.133]	[8.91, 9.485]	[252.866, 344.775]
-50	$\bar{l}$=[0.35, 0.45]	1.335	1.250	[96.883, 103.133]	[-7.015, -6.59]	[252.866, 344.775]

Table 2. Effect of changes in the input parameters.

Mid value of the parameter	% change	t_1^*	t_2^*	$m(\overline{Q}^*)$	$m(\overline{Q_1}^*)$	$m(\overline{C}^*)$
	+50	-24.15	-23.89	-23.89	-23.11	+14.89
$m(\overline{C}_1)$	+25	-17.92	-16.66	-16.66	-12.91	+8.20
	-25	+35.05	+30.55	+30.55	+17.19	-9.90
	-50	+95.48	+81.94	+81.95	+41.80	-22.54
	+50	+3.98	+13.17	+13.17	+40.47	+5.34
$m(\overline{C}_2)$	+25	-8.80	0.00	0.00	+26.14	+2.89
	-25	+14.32	0.00	0.00	-42.52	-4.18
	-50	+33.69	-6.67	-6.67	-126.47	-11.16
	+50	+50.56	+29.45	+29.44	-33.24	+22.40
$m(\overline{C}_3)$	+25	+28.16	+16.67	+16.67	-17.43	+11.77
	-25	-28.95	-16.66	-16.66	+19.82	-13.19
	-50	-59.23	-33.33	-33.33	+43.50	-28.78
	+50	-31.52	-16.66	+25.00	+91.13	+22.71
$m(\overline{D})$	+25	-10.875	-5.00	+18.75	+40.55	+12.13
	-25	+30.19	+16.66	-12.50	-42.59	-13.43
	-50	+88.27	+50.00	-25.00	-81.810	-29.157
	+50	-42.78	0.00	0.00	+127.00	0.00
$m(\bar{l})$	+25	-21.39	0.00	0.00	+63.52	0.00
	-25	+21.39	0.00	0.00	-63.50	0.00
	-50	+42.78	0.00	0.00	-127.00	0.00

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Appendix

From Fig. 1) we see that

$$l = (t_3 - t_1), \quad Q = Dt_2$$
$$Q_1 = D(t_2 - t_1), \quad Q_2 = D(t_3 - t_2).$$

Therefore the holding cost ($C_{holding}$) over the time period 0 to t_2 is

$$\begin{aligned} C_{holding} = C_1 \times \triangle FOB &= C_1 \times \frac{1}{2} \times t_2 \times Q \\ &= \frac{1}{2} C_1 D t_2^2 \end{aligned}$$

and the set-up cost (C_{set-up}) for the entire cycle is

$$C_{set-up} = C_3.$$

Again the shortage cost ($C_{shortage}$) during the interval (t_2, t_3) is

$$\begin{aligned} C_{shortage} &= C_2 \times \triangle BCE \\ &= C_2 \times \frac{1}{2} \times (t_3 - t_2) \times Q_2 \\ &= \frac{1}{2} C_2 D (t_3 - t_2)^2. \end{aligned}$$

Therefore the total cost(X)(say) in the plan period $[0, t_3]$ is given by

$$X = C_{holding} + C_{set-up} + C_{shortage}.$$

Therefore total average cost $C(t_1, t_2)$ is given by

$$C(t_1, t_2) = \frac{C_3 + \frac{1}{2} C_1 D t_2^2 + \frac{1}{2} C_2 D (t_1 + l - t_2)^2}{t_1 + l} \tag{11}$$

Differentiating partially w.r.to t_1 and t_2 we have

$$\begin{aligned} \frac{\partial C}{\partial t_1} = &- \frac{C_3 + \frac{1}{2} C_1 D t_2^2 + \frac{1}{2} C_2 D (t_1 + l - t_2)^2}{(t_1 + l)^2} \\ &+ \frac{C_2 D (t_1 + l - t_2)}{t_1 + l} \end{aligned} \tag{12}$$

and

$$\frac{\partial C}{\partial t_2} = \frac{1}{t_1+l}\left[C_1Dt_2 - C_2D(t_1+l-t_2.)\right].$$

From

$$\frac{\partial C}{\partial t_2} = 0$$

we have

$$\begin{aligned} & C_1Dt_2 = C_2D(t_1+l-t_2) \\ \Rightarrow \quad & t_2 = \frac{C_2(t_1+l)}{C_1+C_2}. \end{aligned} \tag{13}$$

Again from

$$\frac{\partial C}{\partial t_1} = 0$$

we have

$$\begin{aligned} & C_3 + \frac{1}{2}C_1Dt_2^2 + \frac{1}{2}C_2D(t_1+l-t_2)^2 \\ - \; & C_2D(t_1+l-t_2)(t_1+l) = 0. \end{aligned} \tag{14}$$

Using (13) and then after simplifying we have

$$t_2^* = \sqrt{\frac{2C_2C_3}{C_1(C_1+C_2)D}}. \tag{15}$$

From (13) we have

$$t_1 + l = t_3 = \frac{C_1+C_2}{C_2}t_2. \tag{16}$$

Substituting the value of t_2^* in (16) we have

$$\begin{aligned} & t_3^* = \sqrt{\frac{2C_3(C_1+C_2)}{C_1C_2D}} \\ \textit{and} \quad & t_1^* = \sqrt{\frac{2C_3(C_1+C_2)}{C_1C_2D}} - l. \end{aligned}$$

Substituting the optimal values of t_1 and t_2 in (12), the optimal value of C i.e C^* is given by

$$C^* = \sqrt{\frac{2C_1C_2C_3D}{C_1+C_2}}.$$

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