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Optimizing Human Diet Problem Based on Price and Taste Using Multi-Objective Fuzzy Linear Programming Approach

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Abstract. Low price and good taste of foods are regarded as two major factors for optimal human nutrition. Due to price fluctuations and taste diversity, these two factors cannot be certainly and determinately evaluated. This problem must be viewed from another perspective because of the uncertainty about the amount of nutrients per unit of foods and also diversity of people's daily needs to receive them.This paper discusses human diet problem in fuzzy environment. The approach deals with multi-objective fuzzy linear programming problem using a fuzzy programming technique for its solution. By prescribing a diet merely based on crisp data, some ofthe realities are neglected. For the same reason, we dealt with human diet problem through fuzzy approach. Results indicated uncertainty about factors of nutrition diet -including taste and price, amount of nutrients and their intake- would affect diet quality, making the proposed diet more realistic.

Keywords: Optimizing human diet; Multi-objective fuzzy linear programming; Triangular fuzzy number.
AMS Classification: 90C70, 90C29.

1. Introduction

In the modern world, people have variant nutrition requirements depending on different positions they have. After providing individuals with appropriate foods and a desirable diet, some priorities also emerge for selection of optimal nutrition regime for them. Examples of these priorities are taste and final cost of the eaten food. The higher quality of taste and flavor of the eaten food and the lower final price would contribute to more desirability for selecting a special diet. In addition to two above-mentioned factors, any of the limitations related to macro and micro nutrients could be minimized or maximized or stabilized to a certain amount depending on people's various requirements. In summary, it can be stated that, under the current circumstances, it seems vital to have an optimal diet in which the goals pursued by people and nutrition-science experts are met and also different tastes of people and their variant economical conditions are taken into account. Generally, a problem exists regarding the inaccurate values of nutrients in foods because the approximate amounts of nutrients available in a certain food are normally known but there is always a question of their exact amounts. If presence of a certain nutrient is doubted, its amount is assumed to be negligible which makes the problem of inaccurate near-zero amounts appear again. In addition, the exact price of foods cannot be determined due to imbalance in market and price fluctuations. Thus, the price of foods should be considered as a fuzzy number. Some studies have been conducted concerning application of fuzzy logic in nutrition: Wirsam et al. [14] demonstrated that a nutrient intake can be described in a differentiated way and evaluated by employing fuzzy decision making. Similarly, in a paper, Mamat et al. [10] investigated human diet problem using fuzzy price and considered other factors as

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crisp data; they just minimized the costs.

When foodsare treated and packed precisely and stably (like variety of oils, sugar, biscuits, etc); the amounts of their nutrients are mainly expressed with certainty and precision. For other foods, amounts of nutrients per 100 gram of a certain food (e.g. apple, white bread, potato) might vary within an interval; range of these variations might be rather large. Amount of carbohydrates in an apple is a function of apple type and level of maturation. Even besides checking maturation level and type of apple, other variations associated with soil and growth conditions are also present. For example, sugar percentage in 100 gram of a warm/arid region apple is less than that of a similar fruit cropped in a temperate mountainous area (due to deficiency of water contained in apple). Accordingly, necessity of incorporating fuzzy concept is further appreciated for expressing the respective values. On the other hand, it is not possible to determine a definite boundary for people's maximal and minimal daily usage of micronutrients and macronutrients owing to several factors such as geographical region, gender, age, etc, which cause fluctuation and obscurity in the usage amount of nutrients. For example, when it is said that the maximal permissible daily usage of sodium is 2300 mg, this value must be considered as a fuzzy number. Moreover, desirability of taste and flavor of food is among the items to be investigated in nutrition scope. This parameter has not been yet regarded as one of the criteria for food selection. As we know, taste of food is one of the parameters whose quality declines as more food is eaten. From another aspect, there is no exact scale for determining priority of tastes of foods in comparison to one another. Instead, such priorities are expressed as qualitative verbal criteria. It is rather prevalent to use colloquial expressions such as "very delicious", "relatively delicious", "ordinary", "tasteless", "unpleasant", and so on while they are not normally applied in prescription, preparation and supplying of nutrition diets. Therefore, fuzzy concept can be also used here with the intention of enabling usage of this relatively significant factor.

Fuzzy logic was first proposed by Zadeh in 1965. Fuzzy set theory has been applied to many disciplines such as control theory and management sciences, mathematical modeling and industrial applications. The concept of fuzzy linear programming (FLP) in general level was first introduced by Tanaka et al [3]. Fuzzy linear programming (FLP) is an especially useful and practical model for many real world problems. Concept of decision analysis in fuzzy environment was first proposed by Bellman and Zadeh [11]. Zimmermann [4, 5] first used the max-min operator of Bellman and Zadeh to solve FLP problems. Other researchers used this operator, too. (e.g. [2, 12]). Max-min operator has been used in solving other types of fuzzy programming. (e.g., [9, 6]) afterwards, many authors considered various types of the FLP problems and proposed several approaches for solving these problems. For example B. Jana and T. Kumar Roy examined the transportation model in fuzzy environment. Coefficients of target, supply, demand and capacity transfer functions were included as fuzzy numbers in their model [1].

In the current paper, the authors discussed about human diet problem. The research problem was formulated as a linear multi-objective fuzzy programming problem (MOFLPP) with mixed constraints where right hand side of the constraints are fuzzy numbers for which suitable solution is presented. Using Bellman and Zadeh's fuzzy decision-making process, the MOFLPP is converted into an equivalent crisp LPP. Then, it is solved by simplex method. Next, we also considered MOFLPP with coefficients of objective as well as constraintfunctions where right hand sides of constraints are Triangular Fuzzy Number (TFN). Converting the problem into an equivalent crisp non-linear programming problem, it is also solved by fuzzy decisive set method. Moreover, Human Diet Problem was assumed in the form of a fuzzy linear programming with two objective functions. All coefficients were assumed as triangular fuzzy numbers in this model. In the respective nutrition model which is in the form of a multi-objective fuzzy linear program, good results were achieved for presenting an optimum diet through minimizing final cost and also maximizing the taste felt in each meal.

2. Preliminaries

In this section, we recall some basic definitions used in our problemformulation.

Definition 2.1.*(Triangular Fuzzy Number (TFN)) Let $F(R)$ be a set of all triangular fuzzy number ina real line R. A triangular fuzzy number, $\tilde{A}$ $(\in F(R))$ represented with three points $\tilde{A} = (a_1, a_2, a_3)$ .This representation is interpreted as membership functions[1]:*

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1} & a_1 \le x \le a_2 \\ \dfrac{a_3 - x}{a_3 - a_2} & a_2 \le x \le a_3 \\ 0 & \text{elsewhere} \end{cases}$$

It is parameterized by a triplet $\tilde{A} = (a_1, a_2, a_3)$ where a_1, a_3 are the lower and upper limits of support of $\tilde{A}$ and a_2 is the pick (or center) value of $\tilde{A}$.

Remark 2.1. *We consider $\tilde{0}$= (0, 0, 0) as the zero triangular fuzzy number.*

Definition 2.2. *The left TFN $\tilde{A} = (a_1, a_2, a_2)$ is suitable to represent positive large or words with similar meaning. Provided that $a_2 > a_1$ it is represented by the following membership functions:*

$$\mu_{\tilde{A}}(x) = \begin{cases} 1 & x \ge a_2 \\ \dfrac{x - a_1}{a_2 - a_1} & a_1 \le x \le a_2 \\ 0 & x \le a_1 \end{cases}$$

Definition 2.3. *The right TFN $\tilde{A} = (a_2, a_2, a_3)$ is suitable to represent positive small or words with-similar meaning. Providedthat $a_3 > a_2$. It is represented by thefollowing membership functions:*

$$\mu_{\tilde{A}}(x) = \begin{cases} 1 & x \le a_2 \\ \dfrac{a_3 - x}{a_3 - a_2} & a_2 \le x \le a_3 \\ 0 & x \ge a_3 \end{cases}$$

Remark 2.2. *A TFN $\tilde{A} = (a_1, a_2, a_3)$ is positive (negative) if $a_1 \ge 0$ ($a_3 < 0$).*

Remark 2.3. *Consider the TFN $\tilde{A} = (a_1, a_2, a_3)$ then $-\tilde{A} = (-a_3, -a_2, -a_1)$ is a triangular fuzzy number.*

3. The Multi-Objective Fuzzy Linear Programming Problem (MOFLPP)

The multi-objective fuzzy linear programming problem (MOFLPP) with mixed constraints may be written as follows [1]:

Minimize $\tilde{Z}=[\tilde{Z}^1, \tilde{Z}^2, \tilde{Z}^3, \dots, \tilde{Z}^K]$ (3.1)

Subject to

$$\sum_{j=1}^{n} \tilde{a}_{ij} x_j \ge \tilde{b}_i \quad i = 1,2,3,\dots,m_1$$

$$\sum_{j=1}^{n} \tilde{a}_{ij} x_j \le \tilde{b}_i \quad i = m_1 + 1, m_1 + 2, \dots, m_2$$

$$\sum_{j=1}^{n} \tilde{a}_{ij} x_j = \tilde{b}_i \quad i = m_2 + 1, m_2 + 2, \dots, m$$

$$x_j \ge 0 \qquad j = 1,2,3,\dots,n$$

where

$$\tilde{Z}^k = \sum_{j=1}^{n} \tilde{c}_j^{\,k} x_j \; k = 1,2,3,\dots,K$$

Remark 3.1. *If some of the objective functions to be maximization (for $\tilde{Z}^k : k \in \mathcal{T} \subseteq \{1,2,\dots,K\}$) then the following conversion is used. ($\mathcal{T} \subseteq \{1,2,\dots,K\}$)Maximize $\tilde{Z}^k = -$Minimize $(-\tilde{Z}^k)$*

Assumption 1: Fuzzy objective and constraints coefficients are considered as the following positive TFN's: [1].

RightTFN $\tilde{c}_j^{\,k} = (c_j^{\,k}, c_j^{\,k}, c_j^{\,k} + p_j^{\,k})$with tolerance $p_j^{\,k} > 0$ for objective function $\sum_{j=1}^{n} \tilde{c}_j^{\,k} x_j$ for $k = 1,2,3,\dots,K$

Left TFN $\tilde{a}_{ij} = (a_{ij} - d_{ij}^{\,0}, a_{ij}, a_{ij})$ with tolerance $d_{ij}^{\,0} < a_{ij}$and $\tilde{b}_i = (b_i - b_i^{\,0}, b_i, b_i)$

With tolerance $b_i^{\,0} < b_i$ for $\sum_{j=1}^{n} \tilde{a}_{ij} x_j \ge \tilde{b}_i$ $i = 1,2,3,\dots,m_1$

Right TFN $\tilde{a}_{ij} = (a_{ij}, a_{ij}, a_{ij} + d_{ij}^{\,0})$ with tolerance $d_{ij}^{\,0} > 0$ and $\tilde{b}_i = (b_i, b_i, b_i + b_i^{\,0})$

With tolerance $b_i^{\,0} > 0$ for $\sum_{j=1}^{n} \tilde{a}_{ij} x_j \le \tilde{b}_i$ $i = m_1 + 1, m_1 + 2, \dots m_2$

TFN $\tilde{a}_{ij} = (a_{ij} - d_{ij}^{\,l}, a_{ij}, a_{ij} + d_{ij}^{\,r})$ with tolerance $d_{ij}^{\,l} < a_{ij}$, $d_{ij}^{\,r} > 0$ and

$\tilde{b}_i = (b_i - b_i^{\,l}, b_i, b_i + b_i^{\,r})$ With tolerance $b_i^{\,l} < b_i$, $b_i^{\,r} > 0$ for$\sum_{j=1}^{n} \tilde{a}_{ij} x_j = \tilde{b}_i$ $i = m_2 + 1, m_2 + 2, \dots, m$

Remark 3.2. *If some of the objective functions in (3.1) to be Maximization (for $\tilde{Z}^k : k \in \mathcal{T} \subseteq \{1,2,\dots,K\}$) then assumed $\tilde{c}_j^{\,k} = (c_j^{\,k} - p_j^{\,k}, c_j^{\,k}, c_j^{\,k})$. with tolerance $p_j^{\,k} < c_j^{\,k}$, $k \in \mathcal{T} \subseteq \{1,2,\dots,K\}$*

For the calculation of upper (U_k)and lower (L_k) bounds of the k-th (k = 1, 2, 3…K) objective function, we first construct the following eight sub-problems [1].

Minimize $Z^{k1} = \sum_{j=1}^{n} c_j^{\,k} x_j$ (3.2)

$$\text{S.t.} \sum_{j=1}^{n} a_{ij} x_j \ge b_i \quad i = 1,2,3,\dots,m_1$$

$$\sum_{j=1}^{n} a_{ij} x_j \le b_i \quad i = m_1 + 1, m_1 + 2, \dots, m_2$$

$$\sum_{j=1}^{n} a_{ij} x_j = b_i \quad i = m_2 + 1, m_2 + 2, \dots, m$$

$x_j \geq 0 \qquad j = 1,2,3,\dots,n$

Minimize $Z^{k2} = \sum_{j=1}^{n}(c_j^{\,k} + p_j^{\,k})x_j$ (3.3)
Subject to same constraints of (3.2)

Minimize $Z^{k3} = \sum_{j=1}^{n} c_j^{\,k} x_j$ (3.4)
Subject to

$$\sum_{j=1}^{n} a_{ij}x_j \geq b_i - b_i^{\,0} \quad i = 1,2,3,\dots,m_1$$

$$\sum_{j=1}^{n} a_{ij}x_j \leq b_i + b_i^{\,0} \quad i = m_1+1, m_1+2,\dots,m_2$$

$$\sum_{j=1}^{n} a_{ij}x_j \geq b_i - b_i^{\,l} \quad i = m_2+1, m_2+2,\dots,m$$

$$\sum_{j=1}^{n} a_{ij}x_j \leq b_i + b_i^{\,r} \quad i = m_2+1, m_2+2,\dots,m$$

$$x_j \geq 0 \qquad j = 1,2,3,\dots,n$$

Minimize $Z^{k4} = \sum_{j=1}^{n}(c_j^{\,k} + p_j^{\,k})x_j$ (3.5)
S.t. same constraints of (3.4)

Minimize $Z^{k5} = \sum_{j=1}^{n} c_j^{\,k} x_j$ (3.6)
Subject to

$$\sum_{j=1}^{n}(a_{ij} - d_{ij}^{\,0})x_j \geq b_i \quad i = 1,2,3,\dots,m_1$$

$$\sum_{j=1}^{n}(a_{ij} + d_{ij}^{\,0})x_j \leq b_i \quad i = m_1+1, m_1+2,\dots,m_2$$

$$\sum_{j=1}^{n}(a_{ij} - d_{ij}^{\,l})x_j \geq b_i \, i = m_2+1, m_2+2,\dots,m$$

$$\sum_{j=1}^{n}(a_{ij} + d_{ij}^{\,r})x_j \leq b_i \, i = m_2+1, m_2+2,\dots,m$$

$$x_j \geq 0 \quad i = 1,2,3,\dots,n$$

Minimize $Z^{k6} = \sum_{j=1}^{n}(c_j^{\,k} + p_j^{\,k})x_j$ (3.7)
Subject to same constraints of (3.6)

Minimize $Z^{k7} = \sum_{j=1}^{n} c_j^{\,k} x_j$ (3.8)
Subject to

$$\sum_{j=1}^{n}(a_{ij} - d_{ij}^{\,0})x_j \geq b_i - b_i^{\,0} \quad i = 1,2,3,\dots m_1$$

$$\sum_{j=1}^{n}(a_{ij} + d_{ij}^{\,0})x_j \leq b_i + b_i^{\,0} \quad i = m_1+1, m_1+2,\dots,m_2$$

$$\sum_{j=1}^{n}(a_{ij} - d_{ij}^{\,l})x_j \geq b_i - b_i^{\,l} \quad i = m_2+1, m_2+2,\dots,m$$

$$\sum_{j=1}^{n}(a_{ij} + d_{ij}^{\,r})x_j \leq b_i + b_i^{\,r} \quad i = m_2+1, m_2+2,\dots,m$$

$$x_j \geq 0 \qquad j = 1,2,3,\dots,n$$

Minimize $Z^{k8} = \sum_{j=1}^{n}(c_j^{\,k} + p_j^{\,k})x_j$ (3.9)
Subject to same constraints of (3.8).

3.1. Fuzzy programming technique for the solution of MOLPP with fuzzy coefficients and fuzzy resources

Let L_k and U_k be the lower and upper bound for the k-th objective, where L_k= aspired level of achievement for the k-th objective function, and U_k= highest acceptable level of achivement for the k-th objective function. When the aspiration levels for each objective have been specified, we formed a fuzzy model. Our next step is to transform the fuzzy model into a crisp model The foregoing steps may be presented as follows [1]:

Step-1: Solve the MOLPPs (3.2), (3.3), (3.4), (3.5), (3.6), (3.7), (3.8) and (3.9) for each kth objectives. (k = 1, 2, 3…, K).

Step-2: From the results of step-1, determine the corresponding value for every objective function at each solution.

Step-3: Find upper and lower bounds (U_k and L_k) for kth objective from the 8k objective values derived in step-2, as follows:

(3.10)

$$L_k = \min_{\substack{1 \leq r \leq K \\ 1 \leq s \leq 8}} \{Z^k (X^{rs^*})\} \qquad k = 1, 2, 3\dots, K$$

$$U_k = \max_{\substack{1 \leq r \leq K \\ 1 \leq s \leq 8}} \{Z^k (X^{rs^*})\} \qquad k = 1, 2, 3\dots, K$$

Step-4: The initial fuzzy model becomes: (in terms of aspiration levels with each objectives)

Find $\{x_j \geq 0 : j = 1,2,3,\dots,n\}$ (3.11)
So as to satisfy

$$Z_k \lessapprox L_k \qquad k = 1,2,3,\dots,K$$

$$\sum_{j=1}^{n} a_{ij}x_j \gtrapprox b_i \quad i = 1,2,3,\dots,m_1$$

$$\sum_{j=1}^{n} a_{ij}x_j \lessapprox b_i \quad i = m_1+1, m_1+2,\dots,m_2$$

$$\sum_{j=1}^{n} a_{ij}x_j \cong b_i \quad i = m_2+1, m_2+2,\dots,m$$

Here the membership functions for the fuzzy constraints of (3.11) are defined as (3.12):
(For Kth constraints $\tilde{G}_k$; $k = 1,2,3,\dots,K$))

$$\mu_{\tilde{G}_k}(U_k)=\begin{cases}0 & U_k<\sum_{j=1}^{n} c_j{}^k x_j\\ \dfrac{U_k-\sum_{j=1}^{n} c_j{}^k x_j}{\sum_{j=1}^{n} p_j{}^k x_j+N} & \sum_{j=1}^{n} c_j{}^k x_j\le U_k\le \sum_{j=1}^{n}(c_j{}^k+p_j{}^k)x_j+N\\ 1 & U_k\ge \sum_{j=1}^{n}(c_j{}^k+p_j{}^k)x_j+N\end{cases} \quad (3.12)$$

Where N= U_k-L_k , $k=1,2,3,\dots,K$

(For the ith constraints $\tilde{C}_i$; $i=1,2,3,\dots,m_1$))

$$\mu_{\tilde{G}_k}(U_k)=\begin{cases}0 & U_k<\sum_{j=1}^{n} c_j{}^k x_j\\ \dfrac{U_k-\sum_{j=1}^{n} c_j{}^k x_j}{\sum_{j=1}^{n} p_j{}^k x_j+N} & \sum_{j=1}^{n} c_j{}^k x_j\le U_k\le \sum_{j=1}^{n}(c_j{}^k+p_j{}^k)x_j+N\\ 1 & U_k\ge \sum_{j=1}^{n}(c_j{}^k+p_j{}^k)x_j+N\end{cases}$$

(For the *i*th constraints $\tilde{C}_i$; $i=m_1+1,m_1+2,\dots,m_2$)

$$\mu_{\tilde{C}_i}(b_i)=\begin{cases}0 & \sum_{j=1}^{n} a_{ij}x_j>b_i\\ \dfrac{b_i-\sum_{j=1}^{n} a_{ij}x_j}{\sum_{j=1}^{n} d_{ij}{}^0 x_j+b_i{}^0} & \sum_{j=1}^{n} a_{ij}x_j\le b_i\le \sum_{j=1}^{n}(a_{ij}+d_{ij}{}^0)x_j+b_i{}^0\\ 1 & b_i\ge \sum_{j=1}^{n}(a_{ij}+d_{ij}{}^0)x_j+b_i{}^0\end{cases}$$

(For the ith constraints $\tilde{C}_i$; $i=m_2+1,m_2+2,\dots,m$))

$$\mu_{\tilde{C}_i}(b_i)=\begin{cases}\dfrac{b_i-\sum_{j=1}^{n}(a_{ij}-d_{ij}{}^l)x_j+b_i{}^r}{\sum_{j=1}^{n} d_{ij}{}^l x_j+b_i{}^r} & \sum_{j=1}^{n}(a_{ij}-d_{ij}{}^l)x_j-b_i{}^r\le b_i\le \sum_{j=1}^{n} a_{ij}x_j\\ \dfrac{\sum_{j=1}^{n}(a_{ij}+d_{ij}{}^r)x_j+b_i{}^l-b_i}{\sum_{j=1}^{n} d_{ij}{}^r x_j+b_i{}^l} & \sum_{j=1}^{n} a_{ij}x_j\le b_i\le \sum_{j=1}^{n}(a_{ij}+d_{ij}{}^r)x_j+b_i{}^l\\ 0 & \text{elsewhere}\end{cases}$$

Remark 3.1.1. Suppose K_1-Th of the objective functions are maximization. In this case ($-Z_k\le -U_k$ for $k=1,2,3,\dots,K_1$) is replaced instead of the first K_1-Th of constraints in (3.12). In this case the initial fuzzy model becomes:

Find $\{x_j\ge 0: j=1,2,3$ So as to satisfy

$$Z_k \gtrapprox U_k \quad k=1,2,3,\dots,K_1$$

$$Z_k \lessapprox L_k \quad k=K_1+1,K_1+2,,\dots,K$$

$$\sum_{j=1}^{n} a_{ij}x_j \gtrapprox b_i \quad i=1,2,3,\dots,m_1$$

$$\sum_{j=1}^{n} a_{ij}x_j \lessapprox b_i \quad i=m_1+1,m_1+2,\dots,m_2$$

$$\sum_{j=1}^{n} a_{ij}x_j \cong b_i \quad i=m_2+1,m_2+2,\dots,m$$

Here the membership functions for the first K_1. The fuzzy constraints of (3.14) by replacing $-\sum_{j=1}^{n} a_{ij}x_j$ with $\sum_{j=1}^{n} a_{ij}x_j$ and $-U_k$ with L_k in (3.13) are defined as: (for Kth constraints $\tilde{G}_k$; $k=1,2,3,..K_1$))

$$\mu_{\tilde{G}_k}(l_k)=\begin{cases}0 & \sum_{j=1}^{n} c_j{}^k x_j<L_k\\ \dfrac{N+L_k-\sum_{j=1}^{n}(c_j{}^k-p_j{}^k)x_j}{\sum_{j=1}^{n} p_j{}^k x_j+N} & \sum_{j=1}^{n}(c_j{}^k-p_j{}^k)x_j-N\le L_k\le \sum_{j=1}^{n} c_j{}^k x_j\\ 1 & \sum_{j=1}^{n}(c_j{}^k-p_j{}^k)x_j-N\ge L_k\end{cases}$$

Where N=U_k-L_k $k=1,2,3,\dots,K_1$

<u>Step-5:</u> Using the max-min operator (e.g., Zimmermann) crisp LPP for (3.2) (Suppose K_1th of the first objective functions are maximization) is formulated as follows [6]:

Maximize λ (3.14)

Subject to

$$\sum_{j=1}^{n}(c_j{}^k-\lambda p_j{}^k)x_j-\lambda(U_k-L_k)\ge L_k \quad k=1,2,3,\dots,K_1$$

$$\sum_{j=1}^{n}(c_j{}^k+\lambda p_j{}^k)x_j+\lambda(U_k-L_k)\le U_k \quad k=K_1+1,\dots,K$$

$$\sum_{j=1}^{n}(a_{ij}-\lambda d_{ij}{}^0)x_j-\lambda b_i{}^0\ge b_i \quad i=1,2,3,\dots,m_1$$

$$\sum_{j=1}^{n}(a_{ij}+\lambda d_{ij}{}^0)x_j+\lambda b_i{}^0\le b_i \quad i=m_1+1,m_1+2,\dots,m_2$$

$$\sum_{j=1}^{n}(a_{ij}-(1-\lambda)d_{ij}{}^l)x_j+\lambda b_i{}^r\le b_i+b_i{}^r \quad i=m_2+1\dots,m$$

$$\sum_{j=1}^{n}(a_{ij}+(1-\lambda)d_{ij}{}^r)x_j-\lambda b_i{}^l\ge b_i-b_i{}^l \quad i=m_2+1..,m$$

$$0\le\lambda\le 1,\quad x_j\ge 0 \quad j=1,2,3,\dots,n$$

Remark 3.1.2.The constraints in problem (3.14) containing cross product terms λx_j, ($j=1,2,\dots,n$) which are not convex. Therefore the solution of this problem requires the Special approach adopted for solving general non-convex application problems. It may be solved by fuzzy decisive set method [7]. This method is based on the idea that, for a fixed value of, the problem (3.14) is linear programming problems. Obtaining the optimal solution $\lambda^*$to the problem (3.14) is equivalent to determining the maximum value of λ so that the feasible set is nonempty. The algorithm of this method for the problem (3.14) is presented below.

3.2. Algorithm

Step-1: Set λ =1 and test whether a feasible set satisfying the constraints of the Problem (3.14) exists or not, using phase one of the Simplex method [8]. If a feasible set exists, set λ=1, otherwise, set $\lambda^l = 0$ and λ^R=1.

Step-2: For the value of $\lambda=\frac{\lambda^l+\lambda^R}{2}$ update the value of $\lambda^l$and λ^R using the bisection method as follows: $\lambda^l = \lambda$ If feasible set is nonempty forλ and$\lambda^R = \lambda$, if feasible set is empty forλ.

Consequently, for eachλ, test whether a feasible set of the problem (3.14) exists or not using has one of the Simplex methods and determine the maximum value of λ^* satisfying the constraints of the problem (3.14).

3.3. Numerical example 1:

Minimize $\tilde{Z}^1 = \tilde{c}_1{}^1 x_1 + \tilde{c}_2{}^1 x_2$ (3.15)

Maximize $\tilde{Z}^2 = \tilde{c}_1{}^2 x_1 + \tilde{c}_2{}^2 x_2$

Subject to

$$\tilde{a}_{11}x_1 + \tilde{a}_{12}x_2 \le \tilde{b}_1, \dots, n\}$$
$$\tilde{a}_{21}x_1 + \tilde{a}_{22}x_2 \ge \tilde{b}_2$$
$$x_1, x_2 \ge 0$$

where

$\tilde{c}_1{}^1 = \tilde{5} = (5,5,6)$;$\tilde{c}_2{}^1 = \tilde{3} = (3,3,3)$ and

$\tilde{c}_1{}^2 = \tilde{2}$=(1,2,2); $\tilde{c}_2{}^2 = \tilde{1} = (.5,1,1)$

respectively for objective coefficients and

$\tilde{a}_{11} = \tilde{2}$=(2,2,2.5); $\tilde{a}_{12} = \tilde{5} = (5,5,5)$;

$\tilde{a}_{21} = \tilde{1} = (0,1,1)$; $\tilde{a}_{22} = \tilde{3} = (1.5,3,3)$

respectively for technological coefficients,

$\tilde{b}_1 = \widetilde{20} = (20,20,23)$; $\tilde{b}_2 = \tilde{3} = (2,3,3)$

respectively for constraint goals. To solve the problem (1), we first solve the following sub-problems: (by matlab software)

Min $Z^{11} = 5x_1 + 3$ (3.16)

Subject to $2x_1 + 5x_2 \le 20$
$x_1 + 3x_2 \ge 3$
$x_1, x_2 \ge 0$

Min $Z^{12} = 6x_1 + 3x_2$ (3.17)

Subject to same constraints of (3.16)

Min $Z^{13} = 5x_1 + 3x_2$ (3.18)

Subject to $2x_1 + 5x_2 \le 23$
$x_1 + 3x_2 \ge 2$
$x_1, x_2 \ge 0$

Min $Z^{14} = 6x_1 + 3x_2$ (3.19)

subject to same constraints of (3.18)

Min $Z^{15} = 5x_1 + 3x_2$ (3.20)

Subject to $2.5x_1 + 5x_2 \le 20$
$1.5x_2 \ge 3$
$x_1, x_2 \ge 0$

Min $Z^{16} = 6x_1 + 3x_2$ (3.21)

subject to same constraints of (3.20)

Min $Z^{17} = 5x_1 + 3x_2$ (3.22)

Subject to $2.5x_1 + 5x_2 \le 23$
$1.5x_2 \ge 2$
$x_1, x_2 \ge 0$

Min $Z^{18} = 6x_1 + 3x_2$ (3.23)

subject to same constraints of (3.22)

Min $Z^{21} = -2x_1 - x_2$ (3.24)

Subject to $2x_1 + 5x_2 \le 20$
$x_1 + 3x_2 \ge 3$
$x_1, x_2 \ge 0$

Min $Z^{22} = -x_1 - .5x_2$ (3.25)

subject to same constraints of (3.24)

Min $Z^{23} = -2x_1 - x_2$ (3.26)

Subject to $2x_1 + 5x_2 \le 23$
$x_1 + 3x_2 \ge 2$
\ $x_1, x_2 \ge 0$

Min $Z^{24} = -x_1 - .5x_2$ (3.27)

subject to same constraints of (3.26)

Min $Z^{25} = -2x_1 - x_2$ (3.28)

Subject to $2.5x_1 + 5x_2 \le 20$
$1.5x_2 \ge 3$
$x_1, x_2 \ge 0$

Min $Z^{26} = -x_1 - .5x_2$ (3.29)

subject to same constraints of (3.28)

Min $Z^{27} = -2x_1 - x_2$ (3.30)

Subject to $2.5x_1 + 5x_2 \le 23$
$1.5x_2 \ge 2$
$x_1, x_2 \ge 0$

Min $Z^{28} = -x_1 - .5x_2$ (3.31)

subject to same constraints of (3.30)

Now, using the Remark 4, we replace $-Z^{2j}$ with Z^{2j} for j=1, 2…, 8 . So the optimal solutions of the sub-problems ((3.16)–(3.31)) are shown in Table1.

Table 1. The optimal solutions of the sub-problems ((3.16)-(3.31))

$X^{11*} = (0,1)$ $Z^{11*} = 3$	$X^{21*} = (10,0)$ $Z^{21*} = 20$
$X^{12*} = (0,1)$ $Z^{12*} = 3$	$X^{22*} = (10,0)$ $Z^{22*} = 10$
$X^{13*} = (0,.6)$ $Z^{13*} = 2.000$	$X^{23*} = (11.5,0)$ $Z^{23*} = 23$
$X^{14*} = (0,.6667)$ $Z^{14*} = 2.0001$	$X^{24*} = (11.5,0)$ $Z^{24*} = 11.5$
$X^{15*} = (0,2)$ $Z^{15*} = 6$	$X^{25*} = (4,2)$ $Z^{25*} = 10$
$X^{16*} = (0,2)$ $Z^{16*} = 6$	$X^{26*} = (4,2)$ $Z^{26*} = 5$
$X^{17*} = (0,1.3)$ $Z^{17*} = 3.9$	$X^{27*} = (6.53,1.3)$ $Z^{27*} = 14.34$
$X^{18*} = (0,1.3)$ $Z^{18*} = 3.99$	$X^{28*} = (6.53,1.3)$ $Z^{28*} = 7.2$

So;

$$L_1 = \min \{Z^1(X^{rs*})\} = 2 \quad 1 \le r \le 2 \quad 1 \le s \le 8$$

$$U_1 = \max \{Z^1(X^{rs*})\} = 57.5 \quad 1 \le r \le 2 \quad 1 \le s \le 8$$

$$L_2 = \min \{Z^2(X^{rs*})\} = 0.6667 \quad 1 \le r \le 2 \quad 1 \le s \le 8$$

$$U_2 = \max \{Z^2(X^{rs*})\} = 23 \quad 1 \le r \le 2 \quad 1 \le s \le 8.$$

According to step-4 formulating membership functions and following step-5, Crisp LPP of (3.15) is formulated as follows:

Max λ (3.32)

Subject to

$$(5+\lambda)\, x_1 + 3x_2 + 55.5\lambda \le 57.5$$
$$(2-\lambda)x_1 + (1-.5\lambda)x_2 - 22.333333\lambda \ge 0.6667$$
$$(2+.5\lambda)x_1 + 5x_2 + 3\lambda \le 20$$
$$(1-\lambda)x_1 + (3-1.5\lambda)x_2 - \lambda \ge 3$$
$$0 \le \lambda \le 1, \quad x_1, x_2 \ge 0$$

The problem (3.32) may be solved by the fuzzy decisive set method. For λ=1, the problem can be written as:

$$6x_1 + 3x_2 \le 2 \qquad (3.33)$$
$$x_1 + 0.5x_2 \ge 23$$
$$2.5x_1 + 5x_2 \le 17$$
$$1.5x_2 \ge 4$$
$$x_1, x_2 \ge 0$$

Since the feasible set is empty, by taking $\lambda^l = 0$, $\lambda^R = 1$, the new value of ($\lambda = \frac{0+1}{2}$=0.5) is tried. For $\lambda = 0.5$ the problem (3.32) can be written as:

$$5.5x_1 + 3x_2 \le 29.75 \quad (3.34)$$
$$1.5x_1 + 0.75x_2 \ge 11.833333$$
$$2.25x_1 + 5x_2 \le 18.5$$
$$0.5x_1 + 2.25x_2 \ge 3.5$$
$$x_1, x_2 \ge 0$$

Since the feasible set is empty, by taking $\lambda^l = 0$, λ^R=0.5, the new value of $\lambda = \frac{0+0.5}{2} = 0.25$, is tried and so on. The following values of $\lambda$are obtained in the next 17^{th} iterations:

$\lambda = 0.25$; $\lambda = 0.375$; $\lambda = 0.4375$;
$\lambda = 0.40625$; $\lambda = 0.421875$; $\lambda = 0.414063$;
$\lambda = 0.417969$; $\lambda = 0.419922$; $\lambda = 0.418946$;
$\lambda = 0.418458$; $\lambda = 0.418214$; $\lambda = 0.418336$;
$\lambda = 0.418275$; $\lambda = 0.418306$; $\lambda = 0.418291$;
$\lambda = 0.418283$; $\lambda = 0.418287$;

Consequently, we obtain the optimal value λ^*=0.418287 at the 19^{th} iteration by using the fuzzy decisive set method and solutions of the problem (16) are $x_1^* = 6.32759$, $x_2^* = 0.00002$, Z^{1*} =31.63801, $Z^{2*} = 12.6552$ and aspiration Level λ^*=0.418287.

4. Application in Human Diet Problem

The linear programming model proposed in this paper is intended to achieve an optimal diet for individuals in fuzzy space. Human body requires daily intakes of vitamins, salts and minerals as well as carbohydrates, fats and proteins [13]. As discussed before, the values related to amounts of nutrient (either major or minor nutrients) available in foods are not certain and specified numbers. Also, no specific number can be assigned as the exact price of foods because of price fluctuations in the market. Besides, results of a simple search indicated that cost of buying foods in large volume is less than buying foods separately. These evidences are enough to regard price of foods as a fuzzy number. We act as follows for assigning a fuzzy number to taste and flavor of the cooked foods. List of foods were prepared and given to different individuals as questionnaires. The respondents were said to grade every food in the range 0-1 based on their personal tastes. The grades nearer to 1 would signify better taste and flavor of that food, and conversely; grades closer to 0 are indicative of worse taste quality of that food.

Suppose that "n" represents number of foods existing in the diet. Assume the i-th food (where i: 1, 2,…, n). Now, sum up the grades related to "taste" of i-th food obtained from polls, and by computing the arithmetic mean, specify the resultant number as the fuzzy number of "taste" of i-th food. A general Multi-Objective nutrition model with mixed constraints, written as follows:

$$\text{Minimize } Z^1 = \sum_{j=1}^{n} c_j^{\,1} x_j \qquad (4.1)$$

$$\text{Maximize } Z^2 = \sum_{j=1}^{n} c_j^{\,2} x_j$$

Subject to

$$\sum_{j=1}^{n} a_{ij} x_j \geq b_i \qquad i = 1,2,3, \ldots,$$

$$\sum_{j=1}^{n} a_{ij} x_j \leq d_i \qquad i = 1,2,3, \ldots, m$$

$$x_j \geq 0 \qquad j = 1,2, \ldots, n$$

x_j: 100 g of food j eaten per day

$c_j^{\,1}$: amount of price per100 grams of food j

$c_j^{\,2}$: amount of taste for food j

a_{ij}: The amount of nutrient i in 100 g of food j

b_i: The required daily amount of nutrient i

d_i: The maximum daily amount of nutrient i

m: The number of nutrients

n: The number of food.

Then the above model (4.1) in fuzzy environment may be rewritten as:

$$\text{Minimize } \tilde{Z}^1 = \sum_{j=1}^{n} \tilde{c}_j^{\,1} x_j$$

$$\text{Maximize } \tilde{Z}^2 = \sum_{j=1}^{n} \tilde{c}_j^{\,2} x_j$$

Subject to:

$$\sum_{j=1}^{n} \tilde{a}_{ij} x_j \geq \tilde{b}_i \qquad i = 1,2,3, \ldots, m$$

$$\sum_{j=1}^{n} \tilde{a}_{ij} x_j \leq \tilde{d}_i \qquad i = 1,2,3, \ldots, m$$

$$x_j \geq 0 \qquad j = 1,2, \ldots, n$$

It is a multi-objective fuzzy linear programming problem (MOFLPP). It can be solved as before.

4.1. Numerical example 2: (Fuzzy multi-objective nutrition model)

In the following example 20 useful food are considered including a variety of fruits, low fat dairy products, meat, protein and useful vegetables.These foods are apple, bread, chicken, fish, carrots, honey, lettuce, low-fat cheese, low-fat yogurt, low-fat milk, olive oil, orange, pomegranate, potato, soybeans, spinach, tangerines, tomatoes, walnut, white-rice. Due to market fluctuations, the prices for 100 grams of each food are introduced as fuzzy numbers using food costs in stores A, B, and C in different locations of Mashhad City, Iran. The maximal and minimal values of required daily amounts of nutrients are included in table 2 for poorly active women with body mass index of 25 kg/m^2. Though the values in table 2 are presented by crisp numbers but they have been applied as triangular fuzzy numbers in the fuzzy linear programming problem. The acceptable macronutrient distribution range (AMDR) for carbohydrate is 45-65%, fat is 20-35% and protein is 10-35% of calories [15] for 1982 Kcal of energy at 4Kcal/g, 65% of calories correspond to maximum of 322g of carbohydrate, 35% of calories correspond to maximum of 173g of protein while for 1982 Kcal of energy at 9 Kcal/g 35% of calories correspond to a maximum of 77.078 of fat.

Table 2: Approximate amount of nutrient and calery requirement per day [11] (poorly active Female, 55 years old, sedentary, BMI 25 kg/m^2).

Nutrient	Min	Max	Nutrient	Min	Max
Energy(kcal/d)	1982	N.D[1]	Thiamin(B_1)(mg/d)	1.1	N.D
Carbohydrat(g/d)	130	322	Folate(B_9)(µg/d)	400	1000
Total fiber(g/d)	20	35	Vitamin B_{12}(µg/d)	2.4	N.D
Sugar(g/d)	N.D	124	Calcium(mg/d)	1000	2500
Fat(g/d)	N.D	77.078	Iron(mg/d)	18	45
Protein(g/d)	46	173.425	Mangnesium(mg/d)	320	350
Vitamin A(IU)	2333	10000	Selenium(µg/d)	55	4000
Vitamin C(mg/d)	75	2000	Potassium(mg/d)	4700	N.D
Vitamin E(mg/d)	12	1000	Sodium(mg/d)	1500	2300

1:N.D= "Amount not determinable due to lack of data of adverse effects. Source of intake should be from food only to prevent high levels of intake

Table 2 shows the minimum, maximum and actual nutrient requirements. The limitation related to amount of nutrient intake which is undefined in table 2 will be disregarded. The minimum cost a person can pay for purchasing daily foods is his revenue function. Thus, we also consider level of minimal payable cost as one of the limitations. Suppose $2.5 for day.

As shown in Table 2, approximate amount of nutrients and energy requirement per day are given as crisp numbers. Decision variables i.e. the same amount of food consumed each day (in terms of 100 grams) and minimal-maximal permissible daily amounts for intaking vitamins and minerals are included as the constraints. In order to create triangular fuzzy number by using crisp data, one needs sampling and evaluating the fluctuation level which in turn requires further research and laboratory work, not performed by any researcher in the literature as of today. Nonetheless, we are forced to accept the related raw data gathered by nutrition authorities and alimental industries with slight deviation due to variations in price,taste and amount ofnutrients in foods besides lack of certain boundaries for maximal and minimal amount of daily intake of nutrients for differentindividuals. Therefore, if deviation of raw data “a” is assumed to be “d”, the related fuzzy number will be as follows :$\tilde{a} = (a - d, a, a + d)$.

Minimize $\tilde{Z}^1 = \sum_{j=1}^{n} \tilde{c}_j{}^1 x_j$

Maximize $\tilde{Z}^2 = \sum_{j=1}^{n} \tilde{c}_j{}^2 x_j$

$$\sum_{j=1}^{n} \tilde{a}_{ij} x_j \geq \tilde{b}_i \qquad i = 1,2,3, \dots ,17$$

$$\sum_{j=1}^{n} \tilde{a}_{ij} x_j \leq \tilde{d}_i \qquad i = 1,2,3, \dots ,14$$

$$x_j \geq 0, \qquad j = 1,2, \dots ,20$$

where all coefficients in this MOFLPP are given in Appendix. Using the previous method, the optimal solution of the above fuzzy nutrition problem is shown in table 3.

Table 4 shows the amount of foods in crisp and fuzzy solutions. The main objective of research that are to optimize the cost and taste (low-cost and good-tast) have been achieved.while table 5 demonstrates the intake of the nutrients by using the optimal human diet in crisp and fuzzy solutions. Table 6 providesvalue of the objective functions. Comparing the cost between crisp and fuzzy solutions,the results showed significant difference.

The following reasons are the causes forhaving different solutions between normal diet problem using the model of cost minimization and taste maximization and the diet problem with uncertain food price and variety of tastes using the model of MOLPP with fuzzy objective cofficient:

1) There are 11 kinds of food in crisp solution and 12 kinds of food in the fuzzy solution.
2) The difference of the cost is about 1.44$ per day, total difference for a month roughly becomes 43.2$
3) Implementation of human diet problem in fuzzy environment is not suggestive of total violation of its classic state; however, fuzzy approach –by considering the parameters in their real values (through taking into account all possible states) - would result in enhancement of nutritional quality. This causes the decisions to be more realistic in this regard.

The cost minimization diet problem with fuzzy price and fuzzy taste presented a minimum cost diet problem for human. By minimizing the cost, people still can fulfill their nutrient requirenments every day. The method used MOFLPP objective coffictients as the model.

5. Conclusions

Comparing the results of the previous models and the novel model using the multi-objectives fuzzy programming problem in this study, the effectiveness of the proposed approach is clearly evident. As observed before, the values used in the optimal nutrition model of human diet problem in the fuzzy environment are fuzzy numbers based on which the uncertainty in the previous data was covered and considered with a real aspect of nutrition issue. In some cases, there are ambiguities about the comparison of "fuzzy logic" and "reality". For example, are solutions of various problems closer to reality in fuzzy environment than in classical environment? Are the answers obtained in the fuzzy environment close to those obtained in the classical environment?

In this research and similar studies, it was found that real world phenomena should be dealt with in the fuzzy environment due to their uncertainty. Also, when earlier aspects are changed, then the decision making and action criteria should be considered in the fuzzy environment and the second question should be posed in another way: are the answers obtained in the classic environment close to the real answers?

Table 3: The amount of optimal solution parameters of fuzzy LP

$x_1 =$	1.4835	$x_7 =$	1.0794	$x_{13} =$	0	$x_{19} =$	0
$x_2 =$	3.0527	$x_8 =$	0	$x_{14} =$	1.0706	$x_{20} =$	0.4372
$x_3 =$	0	$x_9 =$	0	$x_{15} =$	0.2801		
$x_4 =$	1.4245	$x_{10} =$	3.1593	$x_{16} =$	0	$\lambda =$ (aspiration level)	0.6016228
$x_5 =$	0	$x_{11} =$	0.5672	$x_{17} =$	0		
$x_6 =$	0.3871	$x_{12} =$	1.5399	$x_{18} =$	10		

Since x_j is a 100 g of food j eaten per day, we have:

Table 4: The amount of foods(in terms of gram) in crisp and fuzzy solution

Food	Crisp solution	Fuzzzy solution
Apple	-	148.35
Bread	308.6	305.27
Chicken	116.06	-
Fish	64.29	142.45
Carrot	-	-
Honey	17.23	38.71
Lettuce	-	107.94
Low-fat cheese	-	-
Low-fat yogurt	-	-
Low-fat milk	368.29	315.93
Olive oil	53.68	56.72
Orange	548.78	153.99
Pomegranate	-	-
Potato	1.97	107.06
Soybeans	24.94	28.01
Spinach	-	-
Tangerines	-	-
Tomatoes	960.29	100
Walnut	-	-
White-rice	133.78	43.72

Table 5: Intake the nutrients by using the optimal human diet

Nutrients	in Crisp solution	in Fuzzzy solution
Energy(kcal/d)	2350.8	2215.7
Carbohydrat(g/d)	322	318.43
Total fiber(g/d)	34.99	32.43
Sugar(g/d)	124	121.57
Fat(g/d)	77.08	76.08
Protein(g/d)	103.92	79.56
Vitamin A(IU)	10000	9969.4
Vitamin C(mg/d)	416.47	233.85
Vitamin E(mg/d)	15.03	15.51
Thiamin(B_1)(mg/d)	2.26	1.84
Folate(B_9)(μg/d)	693.88	642.88
Vitamin B_{12}(μg/d)	2.4	2.55
Calcium(mg/d)	1270.9	1085.2
Iron(mg/d)	17.99	18.6
Mangnesium(mg/d)	349.99	336.55
Selenium(μg/d)	133.15	121.34
Potassium(mg/d)	4700	4728.2
Sodium(mg/d)	2300	2269.7

Table 6: The optimal solution for the cost & taste

	Cost ($)	Taste (Scor)
Crisp	3.5871532	13.81214
Fuzzy	5.025919	11.76121
Difference	1.438766	2.05093

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Appendix

$\tilde{C}^1 =$

(1.07,1.07,1.1) (.29,.29,.3) (.25,.25,.26) (.43,.43,.45) (.5,.5,.5) (1.07,1.07,1.08) (.07,.07,.08)
(.32,.32,.34) (.11,.11,.12) (.07,.07,.08) (.36,.36,.38) (.11,.11,.12) (.11,.11,.12) (.06,.06,.06)
(.11,.11,.11) (.07,.07,.08) (.11,.11,.12) (.07,.07,.08) (1.43,1.43,1.43) (.14,.14,.14)

$\tilde{C}^2 =$

(.4,.5,.5) (.4,.5,.5) (.4,.6,.6) (.4,.5,.5) (.1,.1,.1) (.7,.8,.8) (.2,.2,.2) (.4,.5,.5) (.2,.3,.3) (.1,.2,.2)
(.1,.1,.1) (.6,.7,.7) (.4,.5,.5) (.1,.2,.2) (.2,.2,.2) (.2,.2,.2) (.4,.5,.5) (.6,.6,.6) (.3,.4,.4) (.4,.5,.5)

Here $\tilde{c}_1{}^1 =$(1.07,1.07,1.1) ,$\tilde{c}_1{}^2 =$(.4,.5,.5),$\tilde{c}_2{}^1$(.29,.29,.3),$\tilde{c}_2{}^2 =$(.4,.5,.5) and similar representation for other elements and in continues, resources or the maximum and minimum values of required daily amounts of nutrients are given as follows:

$\tilde{b} =$

(1970,1982,1982) (126,130,130) (19,20,20) (45,46,46) (2330,2333,2333) (74,75,75)
(11,12,12) (1,1.1,1.1) (390,400,400) (2.3,2.4,2.4) (900,1000,1000) (17,18,18) (318,320,320)
(53,55,55) (4680,4700,4700) (1490,1500,1500) (2.5,2.6,2.6)

$\tilde{d} =$

(322,322,325) (35,35,36) (124,124,125) (77.078,77.078,78) (173.425,173.125,174) (10000,10000,10001)
(2000,2000,2100) (1000,1000,1100) (1000,1000,1100) (2500,2500,2550) (45,45,46) (350,350,360)
(400,400,420) (2300,2300,2350)

Here $\tilde{b}_1 =$ (1970,1982,1982), $\tilde{b}_2 =$(126,30,130),$\tilde{d}_1 =$(322,325,325), $\tilde{d}_2$=(35,36,36) and similar representation for other elements and in continue , fuzzy coefficients in constraints are given.

Note:The crisp data related amount of nutrients in 100 gram of foods was provided by USDA[2] SR-21.Because of the uncertainty in this numbers , their fuzzy form is assumed as following:

The Columns of the constraints matrix associated with the inequality "≥ ", is as follows:

Columns 1 through 10

(51,52,52)	(264,266,266)	(164,165,165)	(80,82,82)	(38,414,41)	(303,304,304)	(12,14,14)	(175,179,179)	(60,63,63)	(41,42,42)
(13.7,13.8,13.8)	(50.2,50.6,50.6)	(0,0,0)	(0,0,0)	(10,10,10)	(82.2,82.4,82.4)	(3.1,3.2,3.2)	(3.3,3.4,3.4)	(6.5,7,7)	(5.2,5.2,5.2)
(2.3,2.4,2.4)	(2.2,2.4,2.4)	(0,0,0)	(0,0,0)	(3,3,3)	(.15,.2,.2)	(1.1,1.2,1.2)	(0,0,0)	(0,0,0)	(0,0,0)
(.2,.3,.3)	(7.4,7.6,7.6)	(30,31,31)	(17.7,17.9,17.9)	(1,1,1)	(.2,.3,.3)	(.8,.9,.9)	(27.8,28.4,28.4)	(5,5.2,5.2)	(3.3,3.4,3.4)
(52,54,54)	(0,0,0)	(20,21,21)	(24,27,27)	(16704,16705,16705)	(0,0,0)	(499,502,502)	(150,152,152)	(49,51,51)	(195,196,196)
(4.4,4.6,4.6)	(0,0,0)	(0,0,0)	(2.8,2.9,2.9)	(6,6,6)	(.4,.5,.5)	(2.7,2.8,2.8)	(0,0,0)	(.7,.8,.8)	(0,0,0)
(.15,.2,.2)	(.2,.2,.2)	(.25,.3,.3)	(.6,.6,.6)	(1,1,1)	(0,0,0)	(.16,.2,.2)	(.1,.1,.1)	(0,0,0)	(0,0,0)
(0,0,0)	(.4,.5,.5)	(.1,.1,.1)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
(2,3,3)	(107,111,111)	(3,4,4)	(5,7,7)	(19,19,19)	(1,2,2)	(27,29,29)	(3,6,6)	(10,11,11)	(3,5,5)
(0,0,0)	(0,0,0)	(.25,.3,.3)	(.8,.9,.9)	(0,0,0)	(0,0,0)	(0,0,0)	(1.6,1.7,1.7)	(.5,.6,.6)	(.4,.4,.4)
(4,6,6)	(146,151,151)	(13,15,15)	(6,7,7)	(31,33,33)	(3,6,6)	(17,18,18)	(959,961,961)	(178,183,183)	(118,119,119)
(.1,.1,.1)	(3.7,3.7,3.7)	(.9,1,1)	(.3,.3,.3)	(0,0,0)	(.4,.4,.4)	(.4,.4,.4)	(.2,.2,.2)	(.1,.1,.1)	(0,0,0)
(4,5,5)	(21,23,23)	(28,29,29)	(22,24,24)	(11,12,12)	(2,2,2)	(7,7,7)	(34,36,36)	(16,17,17)	(11,11,11)
(0,0,0)	(17.1,17.3,17.3)	(26.2,27.6,27.6)	(36.5,36.5,36.5)	(0,0,0)	(.8,.8,.8)	(.1,.1,.1)	(12.7,12.7,12.7)	(3.3,3.3,3.3)	(3.3,3.3,3.3)
(106,107,107)	(99,100,100)	(254,256,256)	(173,174,174)	(318,320,320)	(52,52,52)	(140,141,141)	(108,111,111)	(233,234,234)	(148,150,150)
(1,1,1)	(681,681,681)	(73,74,74)	(71,71,71)	(69,69,69)	(4,4,4)	(10,10,10)	(255,260,260)	(69,70,70)	(4,4,4)
(1.04,1.07,1.07)	(.28,.29,.29)	(.24,.25,.25)	(.41,.43,43)	(0,1,1)	(1.06,1.07,1.07)	(.06,.07,.07)	(.3,.32,.32)	(.1,.11,.11)	(.06,.06,.06)

[2]: United States Department of Agriculture

Columns 11 through 20

(884,884,884)	(45,47,47)	(83,83,83)	(92,93,93)	(171,173,173)	(23,23,23)	(52,53,53)	(17,18,18)	(652,654,654)	(97,97,97)
(0,0,0)	(11.6,11.7,11.7)	(18.5,18.7,18.7)	(21.1,21.2,21.2)	(9.8,9.9,9.9)	(3.3,3.6,3.6)	(13.3,13.3,13.3)	(3.8,3.9,3.9)	(13.7,13.7,13.7)	(20.9,21.1,21.1)
(0,0,0)	(2.4,2.4,2.4)	(3.9,4,4)	(2.1,2.2,2.2)	(5,6,6)	(2,2.2,2.2)	(1.7,1.8,1.8)	(1,1.2,1.2)	(6.4,6.7,6.7)	(1,1,1)
(0,0,0)	(.8,.9,.9)	(1.5,1.7,1.7)	(2.2,2.5,2.5)	(16,16.6,16.6)	(2.8,2.9,2.9)	(.7,.8,.8)	(.7,.9,.9)	(14.9,15.2,15.2)	(1.8,2,2)
(0,0,0)	(222,225,225)	(0,0,0)	(9,10,10)	(8,9,9)	(9374,9376,9376)	(679,681,681)	(830,833,833)	(19,20,20)	(0,0,0)
(0,0,0)	(52.9,53.9,53.9)	(9.9,10.2,10.2)	(9.2,9.6,9.6)	(1.5,1.7,1.7)	(27.8,28.1,28.1)	(26.4,26.7,26.7)	(12.4,12.7,12.7)	(1.2,1.3,1.3)	(0,0,0)
(14.3,14.3,14.3)	(.2,.2,.2)	(.6,.6,.6)	(0,0,0)	(.4,.4,.4)	(2,2,2)	(.2,.2,.2)	(.5,.5,.5)	(.6,.7,.7)	(0,0,0)
(0,0,0)	(.1,.1,.1)	(.1,.1,.1)	(.1,.1,.1)	(.15,.2,.2)	(.1,.1,.1)	(.1,.1,.1)	(0,0,0)	(.2,.3,.3)	(0,0,0)
(0,0,0)	(28,30,30)	(38,38,38)	(27,28,28)	(54,54,54)	(193,194,194)	(14,16,16)	(12,15,15)	(97,98,98)	(0,0,0)
(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
(1,1,1)	(38,40,40)	(9,10,10)	(12,15,15)	(101,102,102)	(96,99,99)	(35,37,37)	(9,10,10)	(96,98,98)	(2,2,2)
(.6,.6,.6)	(.1,.1,.1)	(.3,.3,.3)	(1.1,1.1,1.1)	(5.1,5.1,5.1)	(2.7,2.7,2.7)	(.2,.2,.2)	(.3,.3,.3)	(2.8,2.9,2.9)	(.1,.1,.1)
(0,0,0)	(10,10,10)	(12,12,12)	(27,28,28)	(83,86,86)	(78,79,79)	(10,12,12)	(11,11,11)	(156,158,158)	(5,5,5)
(0,0,0)	(.5,.5,.5)	(.5,.5,.5)	(.4,.4,.4)	(6.6,7.3,7.3)	(1,1,1)	(.1,.1,.1)	(0,0,0)	(3.8,4.9,4.9)	(5.2,5.6,5.6)
(0,0,0)	(180,181,181)	(234,236,236)	(534,535,535)	(513,515,515)	(557,558,558)	(165,166,166)	(236,237,237)	(439,441,441)	(9,10,10)
(0,0,0)	(0,0,0)	(3,3,3)	(10,10,10)	(1,1,1)	(79,79,79)	(2,2,2)	(5,5,5)	(2,2,2)	(4,5,5)
(.34,.36,.36)	(.1,.11,.11)	(.1,.11,.11)	(.06,.06,.06)	(.11,.11,.11)	(.1,.1,.1)	(.1,.11,.11)	(.06,.07,.07)	(1.43,1.43,1.43)	(.14,.14,.14)

The Columns of the constraints matrix associated with the inequality "$\leq$ ", is as follows:

Columns 1 through 10

(13.8,13.8,13.9)	(50.6,50.6,51)	(0,0,0)	(0,0,0)	(10,10,10)	(82.4,82.4,82.6)	(3.2,3.2,3.3)	(3.4,3.4,3.5)	(7,7,7.5)	(5.2,5.2,5.2)
(2.4,2.4,2.5)	(2.4,2.4,2.6)	(0,0,0)	(0,0,0)	(3,3,3)	(.2,.2,.25)	(1.2,1.2,1.3)	(0,0,0)	(0,0,0)	(0,0,0)
(10.4,10.4,10.6)	(4.3,4.3,4.4)	(0,0,0)	(0,0,0)	(4.7,4.7,4.8)	(82.1,82.1,82.3)	(2,2,2)	(1.3,1.3,1.4)	(7,7,7)	(5.2,5.2,5.5)
(.2,.2,.25)	(3.3,3.3,3.4)	(3.6,3.6,3.7)	(.6,.6,.65)	(.2,.2,.2)	(0,0,0)	(.1,.1,.1)	(5.1,5.1,5.2)	(1.5,1.5,1.5)	(1,1,1)
(.3,.3,.4)	(7.6,7.6,7.8)	(31,31,32)	(17.9,17.9,18.1)	(1,1,1)	(.3,.3,.4)	(.9,.9,1)	(28.4,28.4,29)	(5.2,5.2,5.4)	(,3.4,3.4,3.5)
(54,54,56)	(0,0,0)	(21,21,22)	(27,27,30)	(16705,16705,16706)	(0,0,0)	(502,502,505)	(152,152,154)	(51,51,54)	(196,196,197)
(4.6,4.6,4.8)	(0,0,0)	(0,0,0)	(2.9,2.9,3)	(6,6,6)	(.5,.5,.6)	(2.8,2.8,2.9)	(0,0,0)	(.8,.8,.9)	(0,0,0)
(.2,.2,.25)	(.2,.2,.2)	(.3,.3,.35)	(.6,.6,.6)	(1,1,1)	(0,0,0)	(.2,.2,2.4)	(.1,.1,.1)	(0,0,0)	(0,0,0)
(3,3,4)	(111,111,117)	(4,4,5)	(7,7,9)	(19,19,19)	(2,2,3)	(29,29,31)	(6,6,3)	(11,11,12)	(5,5,7)
(6,6,8)	(151,151,156)	(15,15,17)	(7,7,8)	(33,33,35)	(6,6,9)	(18,18,19)	(961,961,963)	(183,183,188)	(119,119,120)
(.1,.1,.1)	(3.7,3.7,3.7)	(1,1,1.1)	(.3,.3,.3)	(0,0,0)	(.4,.4,.4)	(.4,.4,.4)	(.2,.2,.2)	(.1,.1,.1)	(0,0,0)
(5,5,6)	(23,23,25)	(29,29,30)	(24,24,26)	(12,12,13)	(2,2,2)	(7,7,7)	(36,36,38)	(17,17,18)	(11,11,11)
(0,0,0)	(17.3,17.3,17.5)	(,27.6,27.6,29)	(36.5,36.5,36.5)	(0,0,0)	(.8,.8,.8)	(.1,.1,.1)	(12.7,12.7,12.7)	(3.3,3.3,3.3)	(3.3,3.3,3.3)
(1,1,1)	(681,681,681)	(74,74,75)	(71,71,71)	(69,69,69)	(4,4,4)	(10,10,10)	(260,260,265)	(70,70,71)	(4,4,4)

Columns 11 through 20

(0,0,0)	(11.6,11.7,11.7)	(18.5,18.7,18.7)	(21.1,21.2,21.2)	(9.8,9.9,9.9)	(3.3,3.6,3.6)	(13.3,13.3,13.3)	(3.8,3.9,3.9)	(13.7,13.7,13.7)	(20.9,21.1,21.1)
(0,0,0)	(2.4,2.4,2.4)	(4,4,4.1)	(2.2,2.2,2.3)	(6,6,7)	(2.2,2.2,2.4)	(1.8,1.8,1.9)	(1.2,1.2,1.4)	(6.7,6.7,6.8)	(1,1,1)
(0,0,0)	(9.4,9.4,9.6)	(13.7,13.7,14)	(1.2,1.2,1.3)	(3,3,3)	(.4,.4,.5)	(10.6,10.6,10.8)	(2.6,2.6,2.7)	(2.6,2.6,2.7)	(.1,.1,.1)
(100,100,100)	(.1,.1,.1)	(1.2,1.2,1.2)	(.1,.1,.1)	(9,9,10))	(.4,.4,.4)	(.3,.3,.4)	(.2,.2,.2)	(65.2,65.2,66.2)	(.2,.2,.2)
(0,0,0)	(.9,.9,1)	(1.7,1.7,1.9)	(2.5,2.5,2.8)	(16.6,16.6,17.2)	(2.9,2.9,3)	(.8,.8,.9)	(.9,.9,1.1)	(15.2,15.2,15.5)	(2,2,2.2)
(0,0,0)	(225,225,228)	(0,0,0)	(10,10,11)	(9,9,10)	(9376,9376,9378)	(681,681,683)	(833,833,836)	(20,20,21)	(0,0,0)
(0,0,0)	(53.9,53.9,54.9)	(10.2,10.2,10.5)	(9.6,9.6,10)	(1.7,1.7,1.9)	(28.1,28.1,28.4)	(26.7,26.7,27)	(12.7,12.7,13)	(1.3,1.3,1.4)	(0,0,0)
(14.3,14.3,14.3)	(.2,.2,.2)	(.6,.6,.6)	(0,0,0)	(.4,.4,.4)	(2,2,2)	(.2,.2,.2)	(.5,.5,.5)	(.7,.7,.8)	(0,0,0)
(0,0,0)	(.1,.1,.1)	(.1,.1,.1)	(.1,.1,.1)	(.2,.2,.25)	(.1,.1,.1)	(.1,.1,.1)	(0,0,0)	(.3,.3,.4)	(0,0,0)
(0,0,0)	(30,30,32)	(38,38,38)	(28,28,29)	(54,54,54)	(194,194,195)	(16,16,18)	(15,15,18)	(98,98,99)	(0,0,0)
(1,1,1)	(40,40,42)	(10,10,11)	(15,15,18)	(102,102,103)	(99,99,102)	(37,37,39)	(10,10,11)	(98,98,100)	(2,2,2)
(0,0,0)	(10,10,10)	(12,12,12)	(28,28,29)	(86,86,89)	(79,79,80)	(12,12,14)	(11,11,11)	(158,158,160)	(5,5,5)
(0,0,0)	(.5,.5,.5)	(.5,.5,.5)	(.4,.4,.4)	(7.3,7.3,8)	(1,1,1)	(.1,.1,.1)	(0,0,0)	(4.9,4.9,6)	(5.6,5.6,6)
(0,0,0)	(0,0,0)	(3,3,3)	(10,10,10)	(1,1,1)	(79,79,79)	(2,2,2)	(5,5,5)	(2,2,2)	(5,5,6)

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