

RESEARCH ARTICLE

## Simultaneous state and fault estimation for Takagi-Sugeno implicit models with Lipschitz constraints

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### ABSTRACT

This paper presents a state and fault observer design for a class of Takagi-Sugeno implicit models (TSIMs) with unmeasurable premise variables satisfying the Lipschitz constraints. The fault variable is constituted by the actuator and sensor faults. The actuator fault affects the state and the sensor fault affects the output of the system. The approach is based on the separation between dynamic and static relations in the TSIM. Firstly, the method begins by decomposing the dynamic equations of the algebraic equations. Secondly, the fuzzy observer design that satisfies the Lipschitz conditions and permits to estimate simultaneously the unknown states, actuator and sensor faults is developed. The aim of this approach for the observer design is to construct an augmented model where the fault variable is added to the state vector. The exponential convergence of the state estimation error is studied by using the Lyapunov theory and the stability condition is given in term of only one linear matrix inequality (LMI). Finally, numerical simulation results are given to highlight the performances of the proposed method by using a TSIM of a single-link flexible joint robot.



## 1. Introduction

Due to the growth of the industrial demand for high reliability and safety, the field of the observer design for many chemical and physical processes by using different approaches has attracted much attention of the researchers for a long time. This is due to its wide and successful use in the areas of control, fault detection and diagnosis (FDD) and fault tolerant control (FTC). We may cite e.g. [1–6] and the references therein.

In few decades, fuzzy control systems based on the Takagi-Sugeno (T-S) approach has been received great attention in researches and recognized as a powerful tool to describe the global behaviors of nonlinear systems. The interest of

this formalism is to apprehend the global behavior of a nonlinear process by a set of local linear models, [7, 8]. Once the T-S fuzzy models are obtained, some linear control methodologies can be used see e.g. [9, 10]. Many research works on observation and control of T-S models have been discussed in literature [11–13].

Likewise, due to the fact that many industrial systems are modeled by implicit models which are also called singular models or descriptor models, several researches have been done on implicit systems. Implicit systems are applied to many applications such as power systems, economic systems, electrical systems, chemical systems and so on, see e.g. [14–17]. Such systems are used to describe a larger class of systems than the normal

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system model and to design controller and observer in different areas of research. Notice that, in [18, 19], a fuzzy implicit model is defined by extending the ordinary T-S fuzzy model [7].

In this paper, based on the T-S approach, we study the problem of observer design for simultaneous state and fault estimation for a class of continuous-time TSIMs in presence of actuator and sensor faults. In the literature, many works on fuzzy observer design and its application to FDD and FTC for T-S fuzzy models exist in both explicit and implicit cases see e.g. [20–27] for T-S explicit models case and [28–37] for T-S implicit models. It should be noted that, generally an interesting way to solve the various observer problems raised previously is to write the convergence conditions on the LMI form [38].

The main contribution of this paper is to develop a new methodology of simultaneous state and fault estimation for a class of TSIMs with unmeasurable premise variables satisfying the Lipschitz constraints. Based on the separation between the dynamic and static relations in the TSIM, the augmented system constituted of dynamic equations and fault vector is constructed. The exponential stability of the augmented state estimation error is studied by using the Lyapunov theory and the stability condition is given in term of only one LMI. Besides, the proposed fuzzy observer is synthesized by only an explicit structure.

The rest of this paper is organized as follows: Section 2 presents the class of TSIMs including actuator and sensor faults. In section 3, the fuzzy observer is designed allowing the simultaneous state and fault estimation. Finally, Section 4 applies this result on the TSIM of the single-link flexible joint robot which allows a clear evaluation of the performance of the proposed method.

The next notations are useful:

- $A^T$  represents the transposed matrix of  $A$ .
- $A > 0$  indicates that  $A$  is a matrix symmetric and positive definite.
- $I$  and  $0$  indicate respectively the identity matrix and the zero matrix with the appropriate dimension.
- $\mathbf{R}^n$  and  $\mathbf{R}^{n \times m}$  denote the spaces of  $n$ -dimensional real vectors and  $n \times m$  real matrices, respectively.

## 2. Takagi-Sugeno implicit model description

The following class of T-S fuzzy implicit models with unmeasurable premise variables in presence of actuator and sensor faults is considered:

$$\begin{cases} M\dot{z} &= \sum_{i=1}^q \varphi_i(z)(A_i z + B_i u + F_{ai} f_a) \\ y &= Cz + Du + D_a f_a + F_s f_s \end{cases} \quad (1)$$

where  $z = [Z_1^T \ Z_2^T]^T \in \mathbf{R}^n$  is the state vector with  $Z_1 \in \mathbf{R}^r$  is the vector of differential variables,  $Z_2 \in \mathbf{R}^{n-r}$  is the vector of algebraic variables,  $u \in \mathbf{R}^m$  is the control input,  $y \in \mathbf{R}^p$  is the measured output vector.  $q$  is the number of sub-models.  $f_a \in \mathbf{R}^{n_a}$  and  $f_s \in \mathbf{R}^{n_s}$  are the actuator fault and sensor fault, respectively.  $A_i \in \mathbf{R}^{n \times n}$ ,  $B_i \in \mathbf{R}^{n \times m}$ ,  $C \in \mathbf{R}^{p \times n}$ ,  $D \in \mathbf{R}^{p \times m}$ ,  $D_a \in \mathbf{R}^{p \times n_a}$ ,  $F_{ai} \in \mathbf{R}^{n \times n_a}$ ,  $F_s \in \mathbf{R}^{p \times n_s}$  and  $M \in \mathbf{R}^{n \times n}$  are real known constant matrices with:

$$\begin{cases} A_i = \begin{pmatrix} A_i^{11} & A_i^{12} \\ A_i^{21} & A_i^{22} \end{pmatrix}; \quad B = \begin{pmatrix} B_i^1 \\ B_i^2 \end{pmatrix} \\ F_{ai} = \begin{pmatrix} F_{ai}^1 \\ F_{ai}^2 \end{pmatrix}; \quad C = \begin{pmatrix} C^1 & 0 \end{pmatrix} \end{cases} \quad (2)$$

where  $A_i^{22}$  are supposed invertible.  $M$  is assumed to be of the form:

$$M = \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix} \quad (3)$$

and satisfies  $\text{rank}(M) = r < n$ . Notice that if  $M$  is not of the form (3), without loss of generality, we can always find two nonsingular matrices  $X \in \mathbb{R}^{n \times n}$  and  $Y \in \mathbb{R}^{n \times n}$  such that the following relation is satisfied [17]:

$$XMY = \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix} \quad (4)$$

$\varphi_i(z)$  are the weighting functions that ensure the transition between the contribution of each sub model:

$$\begin{cases} M\dot{z} &= A_i z + B_i u + F_{ai} f_a \\ y &= Cz + Du + D_a f_a + F_s f_s \end{cases} \quad (5)$$

They verify the so-called convex sum property:

$$\sum_{i=1}^q \varphi_i(z) = 1; \quad 0 \leq \varphi_i(z) \leq 1 \quad (6)$$

**Assumption 1.** Suppose that [17]:

- $(M, A_i)$  is regular ( $\det(sM - A_i) \neq 0 \forall s \in \mathbf{C}$ ).
- All sub-models (5) are impulse observable and detectable.

As mentioned above, our approach consists to separate the differential equations from the algebraic equations in each sub-model (5) and the global

fuzzy model is obtained by aggregation of the resulting sub-models. Indeed, from (2) and (3), sub-model (5) can be rewritten as follows:

$$\begin{cases} \dot{Z}_1 &= A_i^{11}Z_1 + A_i^{12}Z_2 + B_i^1u + F_{ai}^1f_a \\ 0 &= A_i^{21}Z_1 + A_i^{22}Z_2 + B_i^2u + F_{ai}^2f_a \\ y &= C^1Z_1 + Du + D_a f_a + F_s f_s \end{cases} \quad (7)$$

which is equivalent to the following form:

$$\begin{cases} \dot{Z}_1 &= M_i Z_1 + N_i u + P_{ai} f_a \\ Z_2 &= J_i Z_1 + K_i u + L_{ai} f_a \\ y &= C^1 Z_1 + Du + D_a f_a + F_s f_s \end{cases} \quad (8)$$

where

$$\begin{cases} M_i &= A_i^{11} + A_i^{12} J_i \\ N_i &= B_i^1 + A_i^{12} K_i \\ P_{ai} &= F_{ai}^1 + A_i^{12} L_{ai} \\ J_i &= -(A_i^{22})^{-1} A_i^{21} \\ K_i &= -(A_i^{22})^{-1} B_i^2 \\ L_{ai} &= -(A_i^{22})^{-1} F_{ai}^2 \end{cases} \quad (9)$$

Let

$$f = \begin{pmatrix} f_a \\ f_s \end{pmatrix} \quad (10)$$

So, system (8) can be rewritten under the equivalent state representation given by:

$$\begin{cases} \dot{Z}_1 &= M_i Z_1 + N_i u + P_i f \\ Z_2 &= J_i Z_1 + K_i u + L_i f \\ y &= C^1 Z_1 + Du + T f \end{cases} \quad (11)$$

where

$$\begin{cases} P_i &= \begin{pmatrix} P_{ai} & 0 \end{pmatrix} \\ L_i &= \begin{pmatrix} L_{ai} & 0 \end{pmatrix} \\ T &= \begin{pmatrix} D_a & F_s \end{pmatrix} \end{cases} \quad (12)$$

So, from (11) it follows that:

$$\varphi_i(z) = \varphi_i(Z_1, Z_2) = \varphi_i(\lambda) \quad (13)$$

with  $\lambda = [Z_1^T \ u^T \ f^T]^T$ .

Then, the following equivalent form of TSIM (1) can be obtained:

$$\begin{cases} \dot{Z}_1 &= \sum_{i=1}^q \varphi_i(\lambda)(M_i Z_1 + N_i u + P_i f) \\ Z_2 &= \sum_{i=1}^q \varphi_i(\lambda)(J_i Z_1 + K_i u + L_i f) \\ y &= C^1 Z_1 + Du + T f \end{cases} \quad (14)$$

**Assumption 2.** : Suppose that the fault  $f$  is of the following form:

$$f = \alpha_0 + \alpha_1 t + \dots + \alpha_{n_f} t^{n_f} \quad (15)$$

where the  $(n_f + 1)^{th}$  time derivative of  $f$  is null (i.e.  $f^{(n_f+1)} = 0$ ) and  $\alpha_j, j = 0, 1, \dots, n_f$  are unknown constant parameters.

Let

$$\eta_j = f^{(j-1)} \quad j = 1, \dots, n_f + 1 \quad (16)$$

Then, we have:

$$\begin{cases} \dot{\eta}_1 &= \eta_2 \\ \dot{\eta}_2 &= \eta_3 \\ &\vdots \\ \dot{\eta}_{n_f} &= \eta_{n_f+1} \\ \dot{\eta}_{n_f+1} &= 0 \end{cases} \quad (17)$$

Thus, the equivalent augmented form of system (14) can be written:

$$\begin{cases} \dot{\xi}_1 &= \sum_{i=1}^q \varphi_i(\mu)(\Phi_i \xi_1 + \Psi_i u) \\ \xi_2 &= \sum_{i=1}^q \varphi_i(\mu)(\Omega_i \xi_1 + K_i u) \\ y &= R \xi_1 + Du \end{cases} \quad (18)$$

where

$$\begin{cases} \xi_1 &= \begin{pmatrix} Z_1^T & \eta_1^T & \dots & \eta_{n_f}^T \end{pmatrix}^T \\ \xi_2 &= Z_2 \\ \Phi_i &= \begin{pmatrix} M_i & P_i & 0 & \dots & 0 \\ 0 & 0 & I & \dots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & I \\ 0 & 0 & \dots & 0 & 0 \end{pmatrix} \\ \Psi_i^T &= \begin{pmatrix} N_i^T & 0 & \dots & 0 \end{pmatrix} \\ \Omega_i &= \begin{pmatrix} J_i & L_i & 0 & \dots & 0 \end{pmatrix} \\ R &= \begin{pmatrix} C^1 & T & 0 & \dots & 0 \end{pmatrix} \end{cases} \quad (19)$$

and

$$\mu = [\xi_1^T \ u^T]^T \quad (20)$$

In order to design an observer for system (1), we introduce the following matrices:

$$\begin{cases} \Phi_0 &= \frac{1}{q} \sum_{i=1}^q \Phi_i \\ \bar{\Phi}_i &= \Phi_i - \Phi_0 \\ \Psi_0 &= \frac{1}{q} \sum_{i=1}^q \Psi_i \\ \bar{\Psi}_i &= \Psi_i - \Psi_0 \end{cases} \quad (21)$$

By using (21), system (18) takes the following equivalent form:

$$\begin{cases} \dot{\xi}_1 &= \Phi_0 \xi_1 + \Psi_0 u \\ &+ \sum_{i=1}^q \varphi_i(\mu)(\bar{\Phi}_i \xi_1 + \bar{\Psi}_i u) \\ \xi_2 &= \sum_{i=1}^q \varphi_i(\mu)(\Omega_i \xi_1 + K_i u) \\ y &= R \xi_1 + Du \end{cases} \quad (22)$$

### 3. Main result

Based on the form (22), the proposed fuzzy observer design of TSIM (1) takes the following

structure:

$$\begin{cases} \hat{\xi}_1 &= \Phi_0 \hat{\xi}_1 + \Psi_0 u - H(\hat{y} - y) \\ &+ \sum_{i=1}^q \varphi_i(\hat{\mu})(\bar{\Phi}_i \hat{\xi}_1 + \bar{\Psi}_i u) \\ \hat{\xi}_2 &= \sum_{i=1}^q \varphi_i(\hat{\mu})(\Omega_i \hat{\xi}_1 + K_i u) \\ \hat{y} &= R \hat{\xi}_1 + Du \end{cases} \quad (23)$$

where  $(\hat{\xi}_1, \hat{\xi}_2)$ ,  $\hat{y}$  and  $\hat{\mu}$  denote the estimate of  $(\xi_1, \xi_2)$ ,  $y$  and  $\eta$  respectively. Matrix  $H$  is to be determined such that  $(\hat{\xi}_1, \hat{\xi}_2)$  converges toward  $(\xi_1, \xi_2)$  exponentially.

In order to establish the asymptotic convergence condition of the observer (23), we define:

$$\varepsilon = \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \end{pmatrix} = \begin{pmatrix} \hat{\xi}_1 - \xi_1 \\ \hat{\xi}_2 - \xi_2 \end{pmatrix} \quad (24)$$

From systems (22) and (23), the estimation error dynamics is given by:

$$\begin{cases} \dot{\varepsilon}_1 &= \Gamma_0 \varepsilon_1 + \Delta \\ \varepsilon_2 &= \sum_{i=1}^q (\varphi_i(\hat{\mu}) - \varphi_i(\mu))(\Omega_i \xi_1 + K_i u) \\ &+ \sum_{i=1}^q \varphi_i(\hat{\mu}) \Omega_i \varepsilon_1 \end{cases} \quad (25)$$

where

$$\Gamma_0 = \Phi_0 - HR \quad (26)$$

and

$$\Delta = \sum_{i=1}^q (\bar{\Phi}_i \Delta_1 + \Delta_2 u) \quad (27)$$

with

$$\begin{cases} \Delta_1 &= \varphi_i(\hat{\mu}) \hat{\xi}_1 - \varphi_i(\mu) \xi_1 \\ \Delta_2 &= \bar{\Psi}_i (\varphi_i(\hat{\mu}) - \varphi_i(\mu)) \end{cases} \quad (28)$$

**Assumption 3.** : Assume that the following conditions hold:

$$\begin{cases} |\Delta_1| &< \delta_i |\varepsilon_1| \\ |\Delta_2| &< \beta_i |\varepsilon_1| \\ |u| &< \gamma \end{cases} \quad (29)$$

where  $\delta_i, \beta_i$  are positives scalars Lipschitz constants and  $\gamma > 0$ .

Let

$$\theta = \sum_{i=1}^q (\sigma(\bar{\Phi}_i) \delta_i + \beta_i \gamma) \quad (30)$$

where  $\sigma(\bar{\Phi}_i)$  denotes the maximum singular value of the matrix  $\bar{\Phi}_i$ .

Then, by using Assumption 3, the term  $\Delta$  can be bounded as follows:

$$|\Delta| < \theta |\varepsilon_1| \quad (31)$$

The following result can be stated.

**Theorem 1.** : Under above Assumption 3, the system (25) is globally exponentially stable if given  $\rho > 0$  there exists matrices  $P > 0$ ,  $Q > 0$  and  $W$  verifying the following LMI:

$$\begin{pmatrix} \Sigma + \theta^2 Q + 2\rho P & P \\ P & -Q \end{pmatrix} < 0 \quad (32)$$

where

$$\Sigma = \Phi_0^T P - R^T W^T + P \Phi_0 - WR \quad (33)$$

The gain  $H$  of the observer (23) is computed by:

$$H = P^{-1}W \quad (34)$$

**Proof of Theorem 1** : Notice that to prove the global asymptotic stability toward zero of the system (25), it suffices to prove that  $\varepsilon_1$  converges toward zero. Indeed, the following Lyapunov function is considered:

$$V = \varepsilon_1^T P \varepsilon_1 \quad P > 0 \quad (35)$$

From (25), it follows that:

$$\dot{V} = \varepsilon_1^T (\Gamma_0^T P + P \Gamma_0) \varepsilon_1 + \Delta^T P \varepsilon_1 + \varepsilon_1^T P \Delta \quad (36)$$

**Lemma 1.** : For any matrices  $X$  and  $Y$  with appropriate dimensions, the following property holds for any invertible matrix  $J$ :

$$X^T Y + Y^T X \leq X^T J^{-1} X + Y^T J Y \quad (37)$$

For  $Q > 0$ , by applying Lemma 1 and Assumption 3, (36) becomes:

$$\dot{V} < \varepsilon_1^T (\Gamma_0^T P + P \Gamma_0 + P Q^{-1} P) \varepsilon_1 + \Delta^T Q \Delta \quad (38)$$

Taking into account (31), (38) becomes:

$$\dot{V} < \varepsilon_1^T (\Gamma_0^T P + P \Gamma_0 + P Q^{-1} P + \theta^2 Q) \varepsilon_1 \quad (39)$$

Let  $\rho > 0$ , to ensure the exponential convergence of the estimation error, the following condition must be guaranteed (see [39] as cited in [9]):

$$\dot{V} < \varepsilon_1^T (\Gamma_0^T P + P \Gamma_0 + P Q^{-1} P + \theta^2 Q) \varepsilon_1 < -2\rho V \quad (40)$$

That leads to the following condition:

$$\Gamma_0^T P + P \Gamma_0 + P Q^{-1} P + \theta^2 Q + 2\rho P < 0 \quad (41)$$

Then from (26), we can establish the LMI condition (32) of Theorem 1 by using the Schur complement and the following change of variables:

$$W = PH \quad (42)$$

Thus, from the Lyapunov stability theory, if the LMI condition (32) is satisfied, the system (25)

is exponentially asymptotically stable. This completes the proof of Theorem 1.

### 4. Numerical illustration

In this section, the proposed fuzzy observer design (23) is applied to a single-link flexible joint robot in order to estimate on-line the unknown states and the faults of actuator and sensor simultaneously. The TSIM that we consider here is given in [36] which is supposed to be affected by an actuator and sensor faults as follows:

$$\begin{cases} M\dot{z} &= \sum_{i=1}^2 \varphi_i(z)(A_i z + Bu + F_a f_a) \\ y &= Cz + F_s f_s \end{cases} \quad (43)$$

where  $z = (z_1, z_2, z_3, z_4, z_5, z_6)^T$  is the state vector with  $z_1$  and  $z_2$  are the angles of rotations of the motor and the link respectively,  $z_3$  and  $z_4$  are their angular velocities,  $z_5$  and  $z_6$  are their angular accelerations.  $u$  is the control variable,  $y$  is the output measurement vector,  $f_a$  is the actuator fault and  $f_s$  is the sensor fault.

$$A_1 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{-k_1}{J_m} & \frac{k_1}{J_m} & \frac{-\beta}{J_m} & 0 & -1 & 0 \\ \frac{k_1}{J_L} & \nu_{max} & 0 & 0 & 0 & -1 \end{pmatrix}$$

$$A_2 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{-k_1}{J_m} & \frac{k_1}{J_m} & \frac{-\beta}{J_m} & 0 & -1 & 0 \\ \frac{k_1}{J_L} & \nu_{min} & 0 & 0 & 0 & -1 \end{pmatrix}$$

$$B^T = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{k_2}{J_m} & 0 \end{pmatrix}$$

$$C = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

$$M = \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix} \text{ with } rank(M) = 4$$

$$F_a = B; F_s = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

The weighting functions are given by:

$$\begin{cases} \varphi_1(z) = \frac{\nu - \nu_{min}}{\nu_{max} - \nu_{min}} \\ \varphi_2(z) = \frac{\nu_{max} - \nu}{\nu_{max} - \nu} \end{cases} \quad (44)$$

where  $\nu$  is the premise variable having the expression below:

$$\nu = -\frac{k_1}{J_L} - \frac{mgb \sin(z_2)}{J_L z_2} \in [\nu_{min}, \nu_{max}] \quad (45)$$

The expression of control variable and values of physical parameters are given in [36] and under assumption 2, the expression of unknown fault signals  $f_a$  and  $f_s$  are defined as in Figure 4.

Therefore, to apply the proposed fuzzy observer (23) for the one-link flexible joint robot as stated in Theorem 1, it suffices to rewrite the model (43) into its equivalent form (22) as mentioned above. Thus, by Theorem 1 with  $\rho = 2.5$  the following observer gain  $H$  is obtained:

$$H = \begin{pmatrix} 37.5 & -4.1 \\ -25.0 & -60.2 \\ 472.3 & -37.5 \\ 246.9 & 55.6 \\ 488.9 & 151.7 \\ -453.1 & 83.4 \\ 1617.8 & 675.8 \\ -1993.7 & 463.2 \\ 1986.6 & 977.5 \\ -3001.7 & 850.7 \end{pmatrix}$$

Simulation results with initial conditions:

$$\begin{aligned} \xi_1(0) &= [0 \ 0.1047 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]^T \\ \hat{\xi}_1(0) &= [0 \ 0.1147 \ 0 \ 0.001 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]^T \\ \xi_2(0) &= [6.1756 \ -5.6158]^T \\ \hat{\xi}_2(0) &= [6.6620 \ -6.1507]^T \end{aligned}$$

are given in Figure 1 to Figure 6 for which faults  $f_a$  and  $f_s$  are applied during the time intervals [5, 40] and [50, 85] respectively.

These simulation results show the performances of the proposed fuzzy observer (23) with the gain  $H$  where the dashed lines denote the state variables, unknown faults and their derivatives estimated by the observer. They show that the observer gives a good estimation of unmeasurable states  $z_2, z_4, z_5, z_6$ , and unknown faults  $f_a, f_s$  and their derivatives  $\dot{f}_a, \dot{f}_s, \ddot{f}_a, \ddot{f}_s$  of the considered flexible robot.

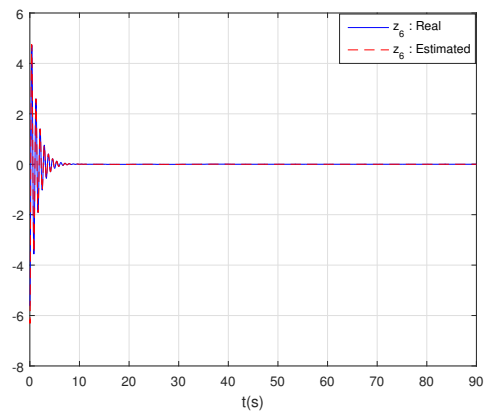
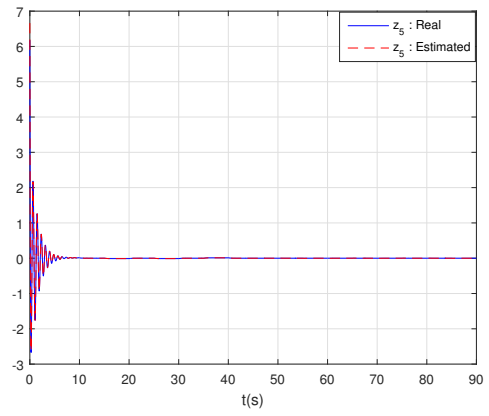
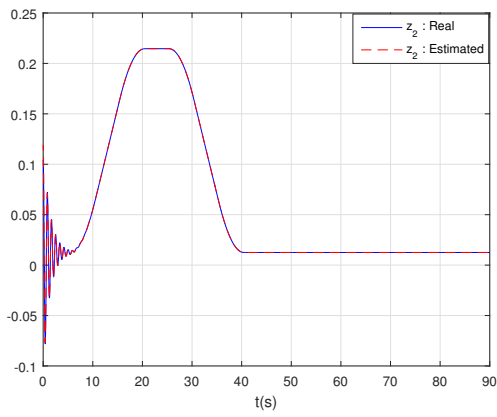
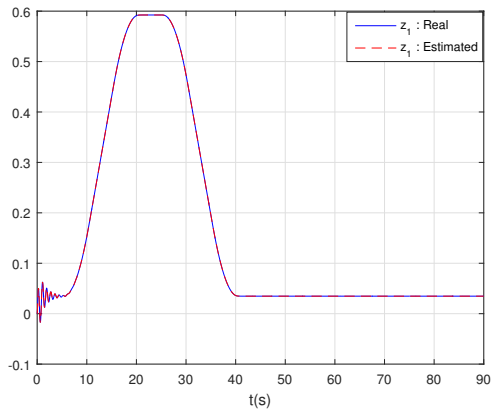


Figure 1.  $z_1$  and  $z_2$  with their estimates.

Figure 3.  $z_5$  and  $z_6$  with their estimates.

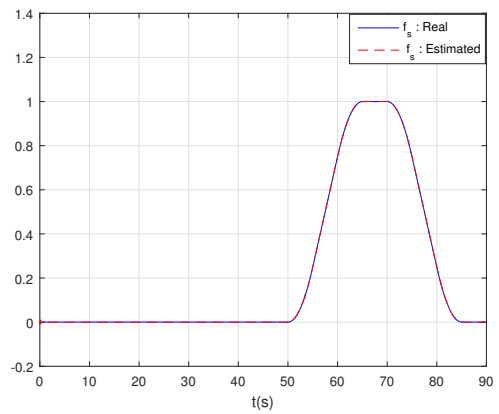
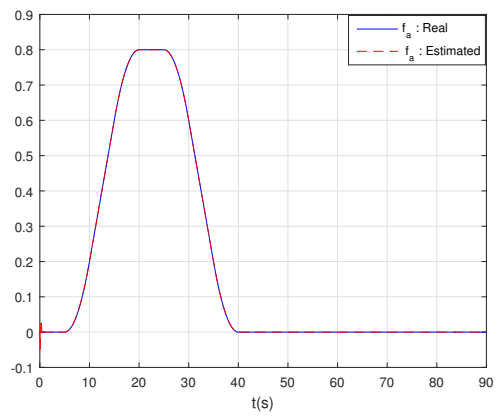
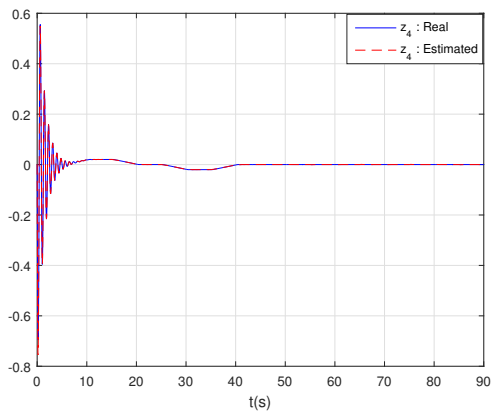
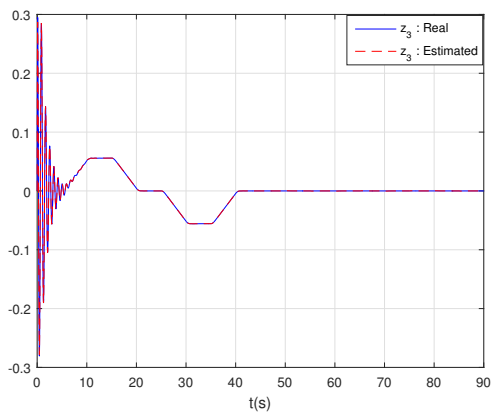


Figure 2.  $z_3$  and  $z_4$  with their estimates.

Figure 4.  $f_a$  and  $f_s$  with their estimates.

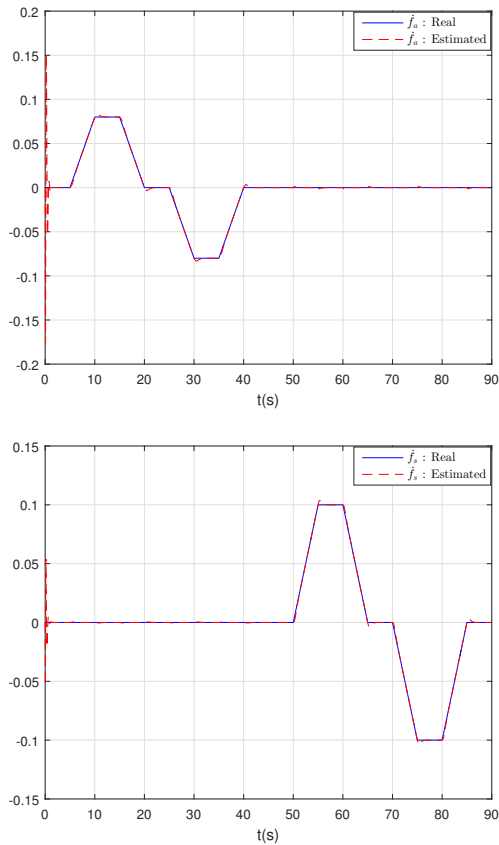


Figure 5.  $\dot{f}_a$  and  $\dot{f}_s$  with their estimates.

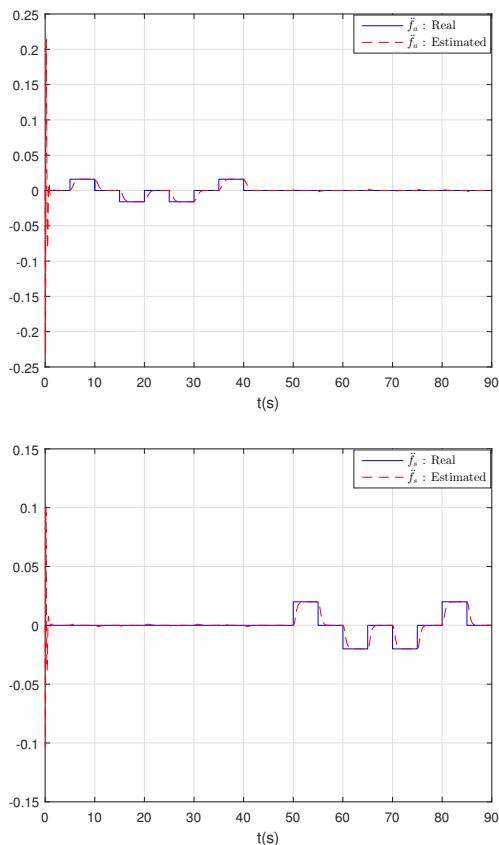


Figure 6.  $\ddot{f}_a$  and  $\ddot{f}_s$  with their estimates.

## 5. Conclusion





This paper develops a new methodology of fuzzy observer design, as a new contribution to simultaneously estimate fault variables as well as system states variables, for a class of TSIMs with unmeasurable premise variables satisfying the Lipschitz constraints in presence of the actuator and sensor faults. The approach is based on the separation between dynamic and static relations in the considered TSIM. The convergence of the state estimation error is studied by using the Lyapunov theory and the exponential stability conditions are given in term of only one LMI. In order to prove the performance of this observer design, a TSIM of the one-link flexible joint robot is studied. Simulation results are given and illustrated the effectiveness of the proposed approach.

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