RESEARCH ARTICLE

A Boiti-Leon Pimpinelli equations with time-conformable derivative

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ARTICLE INFO	ABSTRACT
	In this paper, we derive some new soliton solutions to $(2+1)$-Boiti-Leon Pempinelli equations with conformable derivative by using an expansion technique based on the Sinh-Gordon equation. The obtained solutions have different expression such as trigonometric, complex and hyperbolic functions. This powerful and simple technique can be used to investigate solutions of other nonlinear partial differential equations.

1. Introduction

Partial differential equations play an important role in interpretation and modeling of many phenomena appearing in applied mathematics and physics including flu d mechanics, electrical circuits, diffusion, damping laws, relaxation processes, optimal control theory, solid mechanics, propagation of waves, chemistry, biology, and so on. Therefore, seeking solutions for partial differential equations is an important aspect of scientifi research.
Besides, many scientists have focused on new find ngs to the nonlinear partial differential equations, such as traveling wave solutions, complex funtions, trigonometric functions, Jacobi elliptic functions, and so on. For constructing such solutions, there exist numerous efficient techniques. For example, Sumudu homotopy perturbation transform method [1]- [4], Lie symmetry method [5], $\tan(\phi(\xi)/2)-$ expansion method [6,7], generalized trigonometry functions [8], Riccati equation expansion technique [9], Jacobi elliptic function technique [10] and extended Jacobian

elliptic function technique [11], etc. For more informations about the analytical methods, we refer the reader to the following references [12]- [20].

In this article, we adopt a transformation method based on a sinh-Gordon expansion equation to obtain new soliton solutions of Boiti-Leon Pimpinelli equations (BLP) with conformable derivative. For more details on BLP equation we refer the reader to the references [21]- [23].
On the other hand, the following equation

$$\frac{\partial^2 u}{\partial x \partial t} = \alpha \sinh u, \qquad (1)$$

is called Sinh-Gordon equation and arises in various areas of nonlinear sciences, where α is an arbitrary constant.
Using the traveling wave transformation

$$\begin{cases} u(x,t) = U(\xi) \\ \xi = \mu\left(x + y - \lambda t\right), \end{cases} \qquad (2)$$

equation (1) is converted to

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$$\frac{\partial^2 U}{\partial \xi^2} = -\frac{\alpha}{\mu^2 \lambda} \sinh U, \qquad (3)$$

where the coefficients μ and λ stands for the wave number and wave speed, respectively. Now, integrating (3) yields to

$$\left(\frac{d}{d\xi}\frac{1}{2}U\right)^2 = -\frac{\alpha}{\mu^2 \lambda} \sinh^2\left(\frac{1}{2}U\right) + c, \qquad (4)$$

where c is an integration constant. Consider the following

$$c = 0, \ \alpha = -\mu^2 \lambda \ \text{ and } \ \frac{1}{2}U = w,$$

equation (4) takes the form

$$\frac{dw(\xi)}{d\xi} = \sinh w(\xi). \qquad (5)$$

To construct Jacobi elliptic function solutions, we convert equation (3) into the following

$$\frac{d^2 w}{d\xi^2} = \frac{1}{2} \sinh 2w, \qquad (6)$$

under the assumptions $\phi = 2w$ and $-\frac{\alpha}{\mu^2 \lambda} = 1$. Equation (6) can be also written as

$$\left(\frac{dw}{d\xi}\right)^2 = \sinh^2 w + c, \qquad (7)$$

which can be used in the adopted method, where c is an integration constant. Therefore, Equation (7) has the following solutions

$$\sinh[w(\xi)] = \text{cs}(\xi; m), \qquad (8)$$

$$\cosh[w(\xi)] = \text{ns}(\xi; m), \qquad (9)$$

where m is the modulus of the Jacobian elliptic functions :

$$\text{cs}(\xi; m) = \frac{\text{cn}(\xi; m)}{\text{sn}(\xi; m)}$$

$$\text{ns}(\xi; m) = \frac{1}{\text{sn}(\xi; m)},$$

with the properties

$$\frac{d\,\text{cs}(\xi; m)}{d\xi} = -\text{ns}(\xi; m)\,\text{ds}(\xi; m),$$

$$\frac{d\,\text{ns}(\xi; m)}{d\xi} = -\text{cs}(\xi; m)\,\text{ds}(\xi; m).$$

Substitution of (8) and (9) in (7) reveals that the constant c must satisfy

$$c = 1 - m^2, \qquad (10)$$

which is used throughout this work.

The plan of this paper is as follows: In section 2 some properties of conformable derivative are given. In section 3, we describe the sinh-Gordon expansion technique. Section 4 is devoted to construct exact solutions of $(2+1)$-Boiti-Leon Pimpinelli equations with time-conformable derivatives. Finally, a conclusion is given in section 5.

2. Conformable derivative

Recently, Khalil and his co-workers [24] presented a novel derivative called conformable. This section is devoted to provide some properties on it.

Definition 1. *The conformable derivative with order α for a function $f : [0, \infty) \to R$ is given by*

$$T_\alpha(f)(t) = \lim_{\epsilon \to 0} \frac{f\left(t + \epsilon t^{1-\alpha}\right) - f(t)}{\epsilon} \qquad (11)$$

where $t > 0, \alpha \in (0, 1)$.

Now, we recall some of its properties :

$T_\alpha(af + bg) = aT_\alpha(f) + bT_\alpha(g)$ for all real constant a and b,

$T_\alpha(fg) = fT_\alpha(g) + gT_\alpha(f),$

$T_\alpha(t^r) = rt^{r-\alpha}$ for all r,

$T_\alpha\left(\frac{g}{f}\right) = \frac{fT_\alpha(g) - gT_\alpha(f)}{f^2},$

$T_\alpha(C) = 0$. Where C is a constant.

Moreovere, if f is differentiable, then

$$T_\alpha(f) = t^{1-\alpha}\frac{df}{dt}(t).$$

Theorem 1. *Suppose that $f : [0, \infty)$ is differentiable and conformable-differerentiable with order α and the function g is also differentiable. Then, we have the next property*

$$T_\alpha(fog) = t^{1-\alpha}g'(t)f'(g(t)). \qquad (12)$$

3. Description of the method

The analytical method, called sinh-Gordon equation expansion technique [25], is an efficient tool

to construct new explicit solutions for many problems arising in various branches of sciences and engineering. The algorithm of this method is based on equation (6) or equation (7) and it can be described as follows

- Consider the following nonlinear equation in the sense of conformable derivative:

$$\mathbf{N}\left(u, T_t^\alpha u, T_x^\alpha u, T_y^\alpha u, ...\right) = 0. \quad (13)$$

- Using the following transformation

$$u(x,y,t) = U(\xi), \qquad \xi = \mu\left(\frac{x^\alpha}{\alpha} + \frac{y^\alpha}{\alpha} - \lambda\frac{t^\alpha}{\alpha}\right).$$

Equation (13) is converted to an ordinary differential equation

$$\mathbf{Q}\left(U, U', \mu U', -\lambda U', U'', \mu^2 U'', ...\right) = 0. \quad (14)$$

- Now, we assume that the solution of (14) is as follows

$$U(w) = A_0 + \sum_{i=1}^{n} \cosh^{i-1} w \left[A_i \sinh w + B_i \cosh w\right], \quad (15)$$

where $w = w(\xi)$ satisfes (6) or (7) and (10), A_i, B_i for $i = 0, 1, 2, ..., n$, are constants to be fxed later.

- By virtue of the balance principle, we take the nonlinear terms and the highest-order derivatives in (14) to determine the value of integer n. Now, let the coefficients of $\sinh^i w \cosh^j w$ that have same power to be zero, to get a system of equations with the unknowns:

$$\mu, \quad \lambda, \quad A_i \quad \text{and} \quad B_j \quad \text{for} \quad i = 0, 1, ..., n.$$

- Finally, we solve the obtained system with Maple software, then we substitute $A_0, A_1, B_1, ..., A_n, B_n, \mu$ and λ in (15).

Remark 1. When $m \to 1$, we have

$$\text{cs}(\xi, m) \to \text{csch}(\xi), \quad \text{ns}(\xi, m) \to \coth(\xi), \quad (16)$$

Similarly, when $m \to 0$, it comes

$$\text{cs}(\xi, m) \to \cot(\xi), \qquad \text{ns}(\xi, m) \to \csc(\xi). \quad (17)$$

4. Application of the method

In this section, we apply the above described method to solve the $(2+1)$-Boiti-Leon Pempinelli equations def ned as follows [26]:

$$\begin{cases} T_t^\alpha u_y = (u^2 - u_x)_{xy} + 2v_{xxx}, \\ \\ T_t^\alpha v_y = v_{xx} + 2uv_x. \end{cases} \quad (18)$$

Accordingly, we consider the following wave transformation

$$\begin{cases} u(x,y,t) = U(\xi), \\ \\ v(x,y,t) = V(\xi), \\ \\ \xi = \mu\left(x + y - \lambda\frac{t^\alpha}{\alpha}\right), \end{cases} \quad (19)$$

where λ, μ are constants to be f xed later. The previous wave transformation reduces (20) to the following system of ODEs

$$\begin{cases} T_t^\alpha(u_y) = -\lambda\mu^2 U'', \\ \\ (u^2 - u_x)_{xy} = \mu^2\left[(U^2)'' - \mu U'''\right], \\ \\ 2v_{xxx} = 2\mu^3 V''', \\ \\ T_t^\alpha v = -\lambda\mu V', \\ \\ v_{xx} = \mu^2 V'', \\ \\ 2uv_x = 2\mu UV'. \end{cases} \quad (20)$$

Then, the new system becomes

$$\begin{cases} -\lambda\mu^2 U'' = \mu^2(U^2)'' - \mu^3 U''' + 2\mu^3 V''', \\ -\lambda\mu V' = \mu^2 V'' + 2\mu UV'. \end{cases} \quad (21)$$

After simplif cation, we get

$$-\lambda U'' = (U^2)'' - \mu U''' + 2\mu V''', \quad (22)$$

$$-\lambda V' = \mu V'' + 2UV'. \quad (23)$$

integrating equation (22) twice and taking zero as constants of integration, yields to

$$V' = \frac{U'}{2} - \frac{U^2 + \lambda U}{2\mu}. \quad (24)$$

Injecting equation (24) into equation (23), gives the following nonlinear differential equation

$$\mu^2 U'' - 2U^3 - 3\lambda U^2 - \lambda^2 U = 0. \quad (25)$$

Now, balancing the terms U'' and U^3, yields $n = 1$. Therefore, the solutions of equation (25) is converted to the following form

$$U(\xi) = A_0 + A_1 \sinh(w(\xi)) + B_1 \cosh(w(\xi)). \tag{26}$$

Substituting (26) into (25), we get a set of algebraic equations for $\lambda, \mu, A_0, A_1,$ and B_1 as follows

$$\begin{cases} eq1 = -6 A_1{}^2 B_1 - 2 B_1{}^3 + 2 B_1 \mu^2, \\ eq2 = -2 A_1{}^3 - 6 A_1 B_1{}^2 + 2 A_1 \mu^2, \\ eq3 = -6 A_0 A_1{}^2 - 6 A_0 B_1{}^2 - 3 A_1{}^2 \lambda - 3 B_1{}^2 \lambda, \\ eq4 = -12 A_0 A_1 B_1 - 6 A_1 B_1 \lambda, \\ eq5 = B_1 c\mu^2 - 6 A_0{}^2 B_1 - 6 A_0 B_1 \lambda + 6 A_1{}^2 B_1 \\ \qquad -2 B_1 \mu^2 - B_1 \lambda^2, \\ eq6 = A_1 c\mu^2 - 6 A_0{}^2 A_1 - 6 A_0 A_1 \lambda + 2 A_1{}^3 \\ \qquad -A_1\mu^2 - A_1 \lambda^2, \\ eq7 = -2 A_0{}^3 - 3 \lambda A_0{}^2 + 6 A_0 A_1{}^2 - \lambda^2 A_0 \\ \qquad +3 \lambda A_1{}^2. \end{cases} \tag{27}$$

Solving the set of above equations, we get

Case I:

$$\begin{cases} A_0 = -\frac{\lambda}{2}, \qquad B_1 = \frac{\lambda}{\sqrt{2 m^2 + 2}}, \\ \mu = -\frac{\lambda}{\sqrt{2 m^2 + 2}}, \qquad A_1 = 0. \end{cases}$$

By using (28) and (26), we attain

$$U_1(\xi) = -\frac{1}{2}\lambda + \frac{\lambda\,\mathrm{ns}(\xi, m)}{\sqrt{2 m^2 + 2}}, \tag{28}$$

and

$$V_1(\xi) = -1/4 \frac{\lambda m^2 \xi}{\sqrt{2 m^2 + 2}} - 1/2 \frac{\lambda \,\mathrm{dn}(\xi,m)\mathrm{cn}(\xi,m)}{\sqrt{2 m^2 + 2}\,\mathrm{sn}(\xi,m)}$$

$$-1/2 \frac{\lambda\, \mathrm{E}(\mathrm{sn}(\xi,m),m)}{\sqrt{2 m^2 + 2}} + 1/4 \frac{\lambda \xi}{\sqrt{2 m^2 + 2}}$$

$$+1/2 \frac{\lambda}{\sqrt{2 m^2 + 2}\,\mathrm{sn}(\xi,m)}. \tag{29}$$

where $\xi = \mu\left(x + y - \lambda\frac{t^\alpha}{\alpha}\right)$.

Case II:

$$\begin{cases} A_0 = -\frac{\lambda}{2}, \qquad A_1 = \frac{\lambda}{\sqrt{2 m^2 - 4}}, \\ \mu = -\frac{\lambda}{\sqrt{2 m^2 - 4}}, \qquad B_1 = 0. \end{cases} \tag{30}$$

From (30) and (26), yields

$$U_2(\xi) = -\frac{1}{2}\lambda + \frac{\lambda\,\mathrm{cs}(\xi, m)}{\sqrt{2 m^2 - 4}}, \tag{31}$$

and

$$V_2(\xi) = -1/2 \frac{\lambda m^2 \mathrm{cn}(\xi,m)\mathrm{sn}(\xi,m)}{\sqrt{2 m^2 - 4}\left((\mathrm{dn}(\xi,m))^2 - 1\right)} + 1/8 \frac{\lambda^2 \xi}{\mu}$$

$$+1/4 \frac{\lambda^2 \sqrt{2 m^2 - 4}\,\ln(\mathrm{ns}(\xi,m) - \mathrm{ds}(\xi,m))}{\mu\,(m^2 - 2)}$$

$$+1/4 \frac{\lambda^2 \mathrm{ds}(\xi,m)\mathrm{cs}(\xi,m)}{\mu\,(m^2 - 2)\mathrm{ns}(\xi,m)} + 1/4 \frac{\lambda^2 \mathrm{E}(\mathrm{sn}(\xi,m),m)}{\mu\,(m^2 - 2)}$$

$$-1/2 \frac{\lambda^2 \ln(\mathrm{ns}(\xi,m) - \mathrm{ds}(\xi,m))}{\mu\sqrt{2 m^2 - 4}}, \tag{32}$$

where $\xi = \mu\left(x + y - \lambda\frac{t^\alpha}{\alpha}\right)$.

Case III:

$$\begin{cases} A_0 = -\frac{1}{2}\lambda, \qquad A_1 = \frac{1}{2}\frac{\lambda}{\sqrt{2 m^2 - 1}}, \\ B_1 = \frac{1}{2}\frac{\lambda}{\sqrt{2 m^2 - 1}}, \qquad \mu = \frac{\lambda}{\sqrt{2 m^2 - 1}}. \end{cases} \tag{33}$$

By using (33) and (26), we get

$$U_3(\xi) = -\frac{1}{2}\lambda + \frac{1}{2}\frac{\lambda\,\mathrm{cs}(\xi, m)}{\sqrt{2 m^2 - 1}} + \frac{1}{2}\frac{\lambda\,\mathrm{ns}(\xi, m)}{\sqrt{2 m^2 - 1}}, \tag{34}$$

where $\xi = \mu\left(x + y - \lambda\frac{t^\alpha}{\alpha}\right)$.

Remark 2. The expression of V_3 is too long to be mentionned here.

If $m \to 0$, the following solitary wave solutions of (20) are generated from (28),(31) and (34), namely

$$U_4(\xi) = -\frac{1}{2}\lambda + \frac{1}{2}\lambda\,\csc(\xi)\sqrt{2}, \tag{35}$$

$$V_4(\xi) = \frac{1}{4}\lambda\,\csc(\xi)\sqrt{2} - \frac{1}{8}\lambda\sqrt{2}\xi$$

$$-\frac{1}{2}\lambda\ln(\csc(\xi) - \cot(\xi))$$

$$-\frac{1}{4}\frac{\lambda\sqrt{2}\cos(\xi)}{\sin(\xi)} - \frac{1}{2}\lambda\ln(\csc(\xi) + \cot(\xi)), \tag{36}$$

$$U_5(\xi) = -\frac{1}{2}\lambda - \frac{1}{2}i\lambda\cot(\xi), \tag{37}$$

$$V_5(\xi) = -\tfrac{1}{4}\,i\lambda\,\xi - \tfrac{1}{8}\,i\lambda\,\pi + \tfrac{1}{4}\,i\lambda\,\mathrm{arccot}\,(\cot(\xi))$$

$$+\tfrac{1}{4}\,\lambda\,\ln\left((\cot(\xi))^2 + 1\right) + \tfrac{1}{2}\,\lambda\,\ln(\sin(\xi))\,, \tag{38}$$

$$U_6(\xi) = -\frac{1}{2}\,\lambda - \frac{1}{2}\,i\lambda\,\cot(\xi) - \frac{1}{2}\,i\lambda\,\csc(\xi)\,, \tag{39}$$

$$V_6(\xi) = \tfrac{1}{4}\,i\lambda\,\xi - \tfrac{3}{8}\,i\lambda\,\cot(\xi) + \tfrac{1}{8}\,i\lambda\,\pi$$

$$-\tfrac{1}{4}\,i\lambda\,\mathrm{arccot}\,(\cot(\xi)) - \tfrac{1}{4}\,i\lambda\,\csc(\xi)$$

$$-\frac{\tfrac{1}{4}\,i\lambda}{\sin(\xi)}\,\frac{-\tfrac{1}{8}\,i\lambda\,\cos(\xi)}{\sin(\xi)} + \tfrac{1}{4}\,\lambda\,\ln(\csc(\xi) - \cot(\xi))$$

$$+\tfrac{1}{4}\,\lambda\,\ln(\csc(\xi) + \cot(\xi))\,, \tag{40}$$

where $\xi = \mu\left(x + y - \lambda\frac{t^\alpha}{\alpha}\right)$.

If $m \to 1$, we get from (28),(31) and (34), new solutions of (20)

$$U_7(\xi) = -\frac{1}{2}\,\lambda + \frac{1}{2}\,\lambda\,\coth(\xi)\,, \tag{41}$$

$$V_7(\xi) = -1/4\,\lambda\,\xi + 1/8\,\lambda\,\ln(\cosh(\xi) - \sinh(\xi))$$

$$+3/8\,\lambda\,\ln(\cosh(\xi) + \sinh(\xi))\,, \tag{42}$$

$$U_8(\xi) = -\frac{1}{2}\,\lambda - \frac{1}{2}\,i\lambda\,\mathrm{csch}\,(\xi)\,\sqrt{2}\,, \tag{43}$$

$$V_8(\xi) = -1/4\,i\lambda\,\sqrt{2}\mathrm{csch}\,(\xi) - 1/8\,i\lambda\,\sqrt{2}\xi +$$

$$\lambda\,\mathrm{arctanh}\,(e^\xi) + \frac{1/4\,i\lambda\,\sqrt{2}\cosh(\xi)}{\sinh(\xi)} +$$

$$1/2\,\lambda\,\ln\left(\frac{\cosh(\xi)-1}{\sinh(\xi)}\right). \tag{44}$$

$$U_9(\xi) = -\frac{1}{2}\,\lambda + \frac{1}{2}\,\lambda\,\mathrm{csch}\,(\xi) + \frac{1}{2}\,\lambda\,\coth(\xi)\,, \tag{45}$$

$$V_9(\xi) = 1/4\,\lambda\,\mathrm{csch}\,(\xi) + 1/4\,\lambda\,\xi + 3/8\,\lambda\,\coth(\xi)$$

$$+\lambda\,/8\,(\ln(\coth(\xi) - 1) - \ln(\coth(\xi) + 1))$$

$$-1/2\,\lambda\,\mathrm{arctanh}\,(e^\xi) + 1/8\,\frac{\lambda\,\cosh(\xi)}{\sinh(\xi)}$$

$$+1/4\,\frac{\lambda\,(\cosh(\xi))^2}{\sinh(\xi)} - 1/4\,\lambda\,\sinh(\xi)$$

$$-1/4\,\lambda\,\ln\left(\frac{\cosh(\xi)-1}{\sinh(\xi)}\right), \tag{46}$$

where $\xi = \mu\left(x + y - \lambda\frac{t^\alpha}{\alpha}\right)$.

5. Conclusion

In this paper, we have obtained some new solitary wave solutions to the $(2 + 1)$-dimensional-Boiti-Leon Pempinelli equations with time-conformable derivative. It is clear to see that our obtained solutions through the suggested method are interesting and new comparing to the existing literature. Moreover, the obtrained solitons have various structures such hyperbolic, trigonometric and complex, which signifes that they have an important physical meanings.

Acknowledgments

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