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Anti-Synchronization of Tigan and Li Systems with Unknown Parameters via Adaptive Control

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Abstract. In this paper, the adaptive nonlinear control method has been deployed to derive new results for the anti-synchronization of identical Tigan systems (2008), identical Li systems (2009) and non-identical Tigan and Li systems. In adaptive anti-synchronization of identical chaotic systems, the parameters of the master and slave systems are unknown and the feedback control law has been derived using the estimates of the system parameters. In adaptive anti-synchronization of non-identical chaotic systems, the parameters of the master system are known, but the parameters of the slave system are unknown and accordingly, the feedback control law has been derived using the estimates of the parameters of the slave system. Our adaptive synchronization results derived in this paper for the uncertain Tigan and Li systems are established using Lyapunov stability theory. Numerical simulations are shown to demonstrate the effectiveness of the adaptive anti-synchronization schemes for the uncertain chaotic systems addressed in this paper.

Keywords: Adaptive control, Anti-synchronization, Chaos, Tigan system, Li system.
AMS Classification: 34H10, 93C10, 93C15

1. Introduction

Chaotic systems are nonlinear systems that are highly sensitive to initial conditions. This sensitivity is popularly known as the *butterfly effect* [1]. The first chaotic system was discovered by Lorenz [2] when he was studying weather patterns.

Since the pioneering work by Pecora and Carroll [3] chaos synchronization and anti-synchronization problems have been studied extensively and intensively in the chaos literature [3-31].

Chaos theory has been applied successfully to a variety of fields such as physical systems [4], chemical systems [5], ecological system [6], secure communications [7-8], etc.

In the last two decades, various schemes have been applied for chaos synchronization such as the OGY method [9], the active control method [10-17], the adaptive control method [18-23], the time-delay feedback method [24], the backstepping design method [25-26], the sampled-data feedback synchronization method [27], the sliding mode control method [28-31] and others.

In most of the chaos synchronization approaches, the *master-slave* or *drive-response* formalism is used. If a particular chaotic system is called a *master* or *drive* system and another chaotic system is called a *slave* or *response* system, then the goal of anti-synchronization is to use the output of the master system to control the slave system so that the states of the slave system have the same amplitude but opposite signs as the states of the master system asymptotically.

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In this paper, we discuss the anti-synchronization of identical hyperchaotic Tigan systems [32], identical Li systems [33], and non-identical Tigan and Li systems. Our synchronization results are established using the Lyapunov stability theory [34].

In adaptive synchronization of identical chaotic systems, the parameters of the master and slave systems are unknown and we devise feedback control laws using the estimates of the system parameters.

In adaptive synchronization of non-identical chaotic systems, the parameters of the master system are known, but the parameters of the slave system are unknown and we devise feedback control laws using the estimates of the parameters of the slave system.

This paper has been organized as follows. In Section 2, we discuss the adaptive anti-synchronization of identical Tigan systems [32]. In Section 3, we discuss the adaptive anti-synchronization of identical Li systems [33]. In Section 4, we discuss the adaptive anti-synchronization of non-identical Tigan and Li systems. In Section 5, we summarize the main results obtained in this paper.

2. Adaptive Anti-Synchronization of Identical Tigan Systems

This section details the adaptive anti-synchronization of identical Tigan systems [32], when the parameters of the master and slave systems are unknown.

2.1. Theoretical Results

As the master system, we consider the Tigan dynamics described by

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) \\ \dot{x}_2 &= (c-a)x_1 - ax_1x_3 \\ \dot{x}_3 &= -bx_3 + x_1x_2 \end{aligned} \tag{1}$$

where x_1, x_2, x_3 are the state variables and a, b, c are unknown parameters of the system.

As the slave system, we consider the controlled Tigan dynamics described by

$$\begin{aligned} \dot{y}_1 &= a(y_2 - y_1) + u_1 \\ \dot{y}_2 &= (c-a)y_1 - ay_1y_3 + u_2 \\ \dot{y}_3 &= -by_3 + y_1y_2 + u_3 \end{aligned} \tag{2}$$

where y_1, y_2, y_3 are the state variables and u_1, u_2, u_3 are the nonlinear control inputs to be designed.

The Tigan systems (1) and (2) are chaotic when the parameter values are chosen as

$$a = 2.1, \; b = 0.6, \;\; c = 30$$

The strange chaotic attractor of the system (1) is depicted in Figure 1.

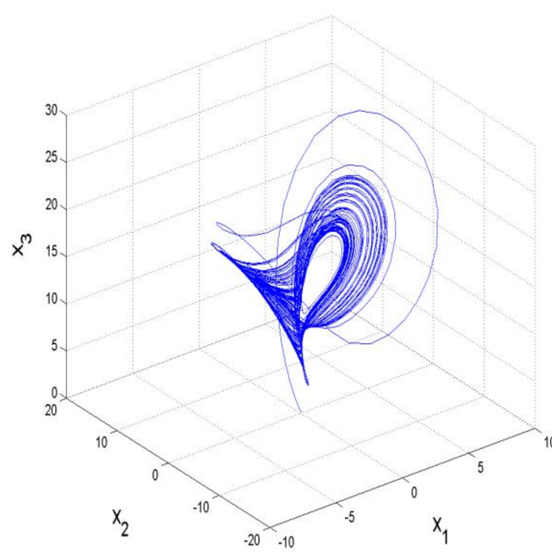

Figure 1. Strange Attractor of the Tigan System

The anti-synchronization error is defined as

$$e_i = y_i + x_i, \quad (i = 1, 2, 3). \tag{3}$$

The error dynamics is obtained as

$$\begin{aligned} \dot{e}_1 &= a(e_2 - e_1) + u_1 \\ \dot{e}_2 &= (c-a)e_1 - ay_1y_3 - ax_1x_3 + u_2 \\ \dot{e}_3 &= -be_3 + y_1y_2 + x_1x_2 + u_3. \end{aligned} \tag{4}$$

We define the adaptive control functions as

$$\begin{aligned} u_1(t) &= -\hat{a}(e_2 - e_1) - k_1 e_1 \\ u_2(t) &= -(\hat{c} - \hat{a})e_1 + \hat{a}y_1 y_3 + \hat{a}y_1 y_3 - k_2 e_2 \\ u_3(t) &= \hat{b}e_3 - y_1 y_2 - x_1 x_2 - k_3 e_3 \end{aligned} \tag{5}$$

where $\hat{a}, \hat{b}, \hat{c}$ are estimates of $a, b, c,$ respectively and k_1, k_2, k_3 are positive constants.

Substituting (5) into (4), the closed-loop error dynamics is obtained as

$$\begin{aligned} \dot{e}_1 &= (a - \hat{a})(e_2 - e_1) - k_1 e_1 \\ \dot{e}_2 &= (c - \hat{c})e_1 - (a - \hat{a})(e_1 + y_1 y_3 + x_1 x_3) \\ &\quad - k_2 e_2 \\ \dot{e}_3 &= -(b - \hat{b})e_3 - k_3 e_3. \end{aligned} \tag{6}$$

We define the parameter estimation errors as

$$e_a = a - \hat{a}, \quad e_b = b - \hat{b}, \quad e_c = c - \hat{c}. \tag{7}$$

Using (7), the error dynamics is simplified as

$$\begin{aligned} \dot{e}_1 &= e_a(e_2 - e_1) - k_1 e_1 \\ \dot{e}_2 &= e_c e_1 - e_a(e_1 + y_1 y_3 + x_1 x_3) - k_2 e_2 \\ \dot{e}_3 &= -e_b e_3 - k_3 e_3. \end{aligned} \tag{8}$$

For the derivation of the update law for adjusting the estimates of parameters, the Lyapunov method is used.

We consider the quadratic Lyapunov function defined by

$$V = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_a^2 + e_b^2 + e_c^2\right), \tag{9}$$

which is a positive definite function on R^6.

We note that

$$\dot{e}_a = -\dot{\hat{a}}, \quad \dot{e}_b = -\dot{\hat{b}}, \quad \dot{e}_c = -\dot{\hat{c}} \tag{10}$$

Differentiating (9) along the trajectories of (8) and noting (10), we find that

$$\begin{aligned} \dot{V} &= -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 \\ &\quad + e_a\left[-e_1^2 - e_2(y_1 y_3 + x_1 x_3) - \dot{\hat{a}}\right] \\ &\quad + e_b\left[-e_3^2 - \dot{\hat{b}}\right] + e_c\left[e_1 e_2 - \dot{\hat{c}}\right] \end{aligned} \tag{11}$$

In view of (11), the estimated parameters are updated by the following law:

$$\begin{aligned} \dot{\hat{a}} &= -e_1^2 - e_2(y_1 y_3 + x_1 x_3) + k_4 e_a \\ \dot{\hat{b}} &= -e_3^2 + k_5 e_b \\ \dot{\hat{c}} &= e_1 e_2 + k_6 e_c \end{aligned} \tag{12}$$

where k_4, k_5, k_6 are positive constants.

Now, we state and prove the following result.

Theorem 1. *The identical uncertain Tigan systems (1) and (2) are globally and exponentially anti-synchronized by the adaptive control law (5), where the update law for the parameter estimates* $\hat{a}, \hat{b}, \hat{c}$ *is given by (12) and* $k_i, (i = 1, 2, \ldots, 6)$ *are positive constants. The errors for parameter estimates* e_a, e_b, e_c *decay to zero exponentially as* $t \to \infty$.

Proof. This result is a simple consequence of the Lyapunov stability theory.

We know that V as defined in (9) is a positive definite function on R^6.

Substituting (12) into (11), we obtain

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - k_4 e_a^2 - k_5 e_b^2 - k_6 e_c^2, \tag{13}$$

which is a negative definite function on R^6.

Hence, by the Lyapunov stability theory [34], it follows that $e_i(t) \to 0$ exponentially as $t \to \infty$ for $i = 1, 2, 3$ and $e_a \to 0, e_b \to 0, e_c \to 0$ as $t \to \infty$.

This completes the proof. ■

2.2. Numerical Results

For the numerical simulations, the fourth-order Runge-Kutta method with time-step $h = 10^{-6}$ is used to solve the two systems of differential equations (1) and (2) with the adaptive nonlinear controller (5) and update law of estimates (12).

We take

$$k_i = 4 \text{ for } i = 1, 2, \ldots, 6.$$

The parameters of the Tigan systems are chosen so that the system (1) and (2) are chaotic, *i.e.*

$$a = 2.1, \quad b = 0.6 \text{ and } c = 30.$$

The initial values of the parameter estimates are chosen as

$$\hat{a}(0) = 1, \quad \hat{b}(0) = 2 \text{ and } \hat{c}(0) = 5.$$

The initial values of the master system (1) are chosen as

$$x_1(0) = 7, \quad x_2(0) = 12 \text{ and } x_3(0) = 8.$$

The initial values of the slave system (2) are chosen as

$$y_1(0) = 4, \quad y_2(0) = -8 \text{ and } y_3(0) = 7.$$

Figure 2 shows the anti-synchronization of the Tigan systems (1) and (2).

Figure 3 shows the time-history of the anti-synchronization errors $e_1, e_2, e_3.$

Figure 4 shows the time-history of the parameter estimates $\hat{a}, \hat{b}, \hat{c}.$

Figure 5 shows the time-history of the parameter estimation errors $e_a, e_b, e_c.$

Figure 6 shows the time-history of the applied control inputs $u_1, u_2, u_3.$

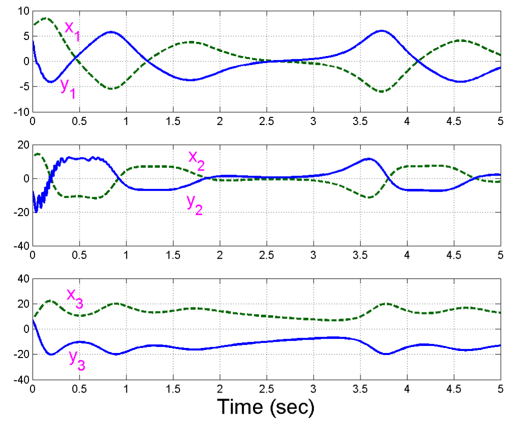

Figure 2. Anti-Synchronization of Identical Tigan Chaotic Systems

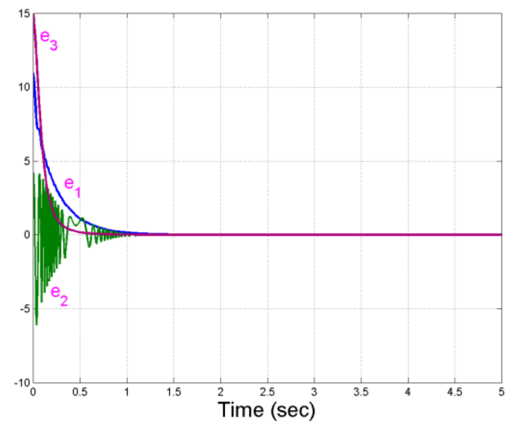

Figure 3. Time History of the Error States e_1, e_2, e_3

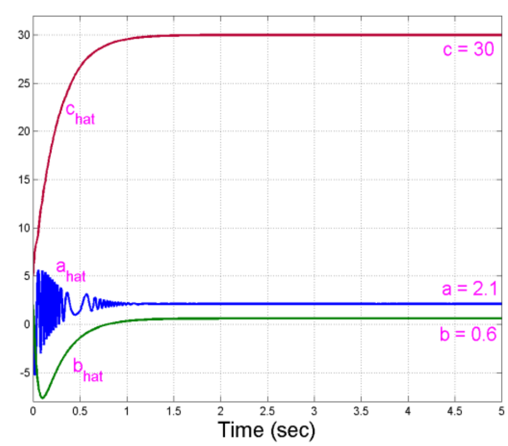

Figure 4. Time History of the Estimates $\hat{a}, \hat{b}, \hat{c}$

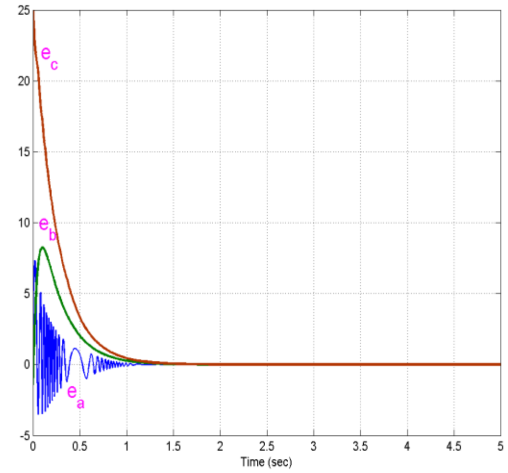

Figure 5. Time History of the Estimation Errors

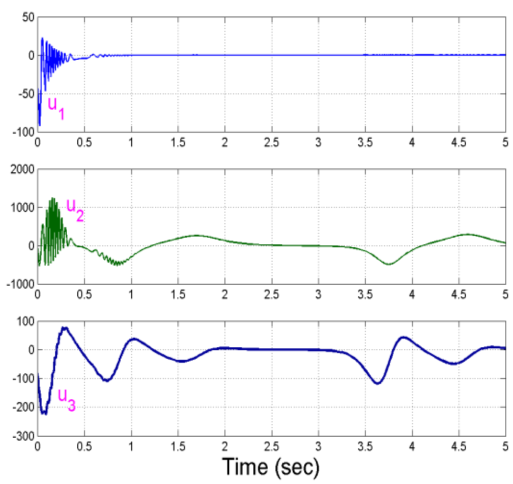

Figure 6. Time History of the Applied Control Inputs u_1, u_2, u_3

3. Adaptive Anti-Synchronization of Identical Li Systems

This section details the adaptive anti-synchronization of identical Li systems [33], when the parameters of the master and slave systems are unknown.

3.1. Theoretical Results

As the master system, we consider the Li dynamics described by

$$\begin{aligned} \dot{x}_1 &= \alpha(x_2 - x_1) \\ \dot{x}_2 &= x_1 x_3 - x_2 \\ \dot{x}_3 &= \beta - x_1 x_2 - \gamma x_3 \end{aligned} \tag{14}$$

where x_1, x_2, x_3 are the state variables and α, β, γ are unknown parameters of the system.

As the slave system, we consider the controlled Li dynamics described by

$$\begin{aligned} \dot{y}_1 &= \alpha(y_2 - y_1) + u_1 \\ \dot{y}_2 &= y_1 y_3 - y_2 + u_2 \\ \dot{y}_3 &= \beta - y_1 y_2 - \gamma y_3 + u_3 \end{aligned} \tag{15}$$

where y_1, y_2, y_3 are the state variables and u_1, u_2, u_3 are the nonlinear control inputs to be designed.

The Li systems (14) and (15) are chaotic when the parameter values are chosen as;

$$\alpha = 5, \quad \beta = 16, \quad \gamma = 1.$$

The strange chaotic attractor of the system (14) is depicted in Figure 7.

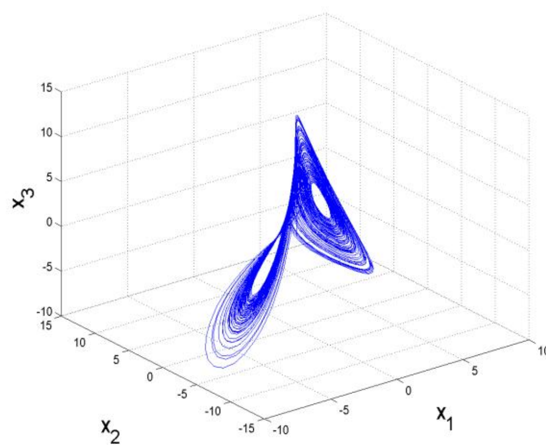

Figure 7. Strange Attractor of the Li System

The anti-synchronization error is defined as

$$e_i = y_i + x_i, \quad (i = 1, 2, 3). \tag{16}$$

The error dynamics is obtained as

$$\begin{aligned}\dot{e}_1 &= \alpha(e_2 - e_1) + u_1 \\ \dot{e}_2 &= -e_2 + y_1 y_3 + x_1 x_3 + u_2 \\ \dot{e}_3 &= -\gamma e_3 - y_1 y_2 - x_1 x_2 + 2\beta + u_3.\end{aligned} \tag{17}$$

We define the adaptive control functions as

$$\begin{aligned}u_1(t) &= -\hat{\alpha}(e_2 - e_1) - k_1 e_1 \\ u_2(t) &= e_2 - y_1 y_3 - x_1 x_3 - k_2 e_2 \\ u_3(t) &= \hat{\gamma} e_3 + y_1 y_2 + x_1 x_2 - 2\beta - k_3 e_3\end{aligned} \tag{18}$$

where $\hat{\alpha}, \hat{\beta}, \hat{\gamma}$ are estimates of $\alpha, \beta, \gamma,$ respectively and k_1, k_2, k_3 are positive constants.

Substituting (18) into (17), the closed-loop error dynamics is obtained as

$$\begin{aligned}\dot{e}_1 &= (\alpha - \hat{\alpha})(e_2 - e_1) - k_1 e_1 \\ \dot{e}_2 &= -k_2 e_2 \\ \dot{e}_3 &= -(\gamma - \hat{\gamma}) e_3 + 2(\beta - \hat{\beta}) - k_3 e_3.\end{aligned} \tag{19}$$

We define the *parameter estimation errors* as;

$$e_\alpha = \alpha - \hat{\alpha}, \;\; e_\beta = \beta - \hat{\beta}, \;\; e_\gamma = \gamma - \hat{\gamma}. \tag{20}$$

Using (20), the error dynamics is simplified as

$$\begin{aligned}\dot{e}_1 &= e_\alpha (e_2 - e_1) - k_1 e_1 \\ \dot{e}_2 &= -k_2 e_2 \\ \dot{e}_3 &= -e_\gamma e_3 + 2 e_\beta - k_3 e_3.\end{aligned} \tag{21}$$

For the derivation of the update law for adjusting the estimates of parameters, the Lyapunov method is used.

We consider the quadratic Lyapunov function defined by

$$V = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_\alpha^2 + e_\beta^2 + e_\gamma^2\right), \tag{22}$$

which is a positive definite function on R^6.

We note that

$$\dot{e}_\alpha = -\dot{\hat{\alpha}}, \;\; \dot{e}_\beta = -\dot{\hat{\beta}}, \;\; \dot{e}_\gamma = -\dot{\hat{\gamma}} \tag{23}$$

Differentiating (22) along the trajectories of (21) and noting (23), we find that

$$\begin{aligned}\dot{V} = &-k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 + e_\alpha \left[e_1(e_2 - e_1) - \dot{\hat{\alpha}} \right] \\ &+ e_\beta \left[2e_3 - \dot{\hat{\beta}} \right] + e_\gamma \left[-e_3^2 - \dot{\hat{\gamma}} \right].\end{aligned} \tag{24}$$

In view of (24), the estimated parameters are updated by the following law:

$$\begin{aligned}\dot{\hat{\alpha}} &= e_1(e_2 - e_1) + k_4 e_\alpha \\ \dot{\hat{\beta}} &= 2e_3 + k_5 e_\beta \\ \dot{\hat{\gamma}} &= -e_3^2 + k_6 e_\gamma\end{aligned} \tag{25}$$

where k_4, k_5, k_6 are positive constants.

Now, we state and prove the following result.

Theorem 2. *The identical uncertain Li systems (14) and (15) are globally and exponentially anti-synchronized by the adaptive control law (18), where the update law for the parameter estimates* $\hat{\alpha}, \hat{\beta}, \hat{\gamma}$ *is given by (25) and* $k_i, (i = 1, 2, \ldots, 6)$ *are positive constants. The errors for parameter estimates* $e_\alpha, e_\beta, e_\gamma$ *decay to zero exponentially as* $t \to \infty$.

Proof. This resut is a simple consequence of the Lyapunov stability theory. We know that V as defined in (22) is a positive definite function on R^6. Substituting (25) into (24), we obtain

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - k_4 e_\alpha^2 - k_5 e_\beta^2 - k_6 e_\gamma^2, \tag{26}$$

which is a negative definite function on R^6.

Hence, by the Lyapunov stability theory [34], it follows that $e_i(t) \to 0$ exponentially as $t \to \infty$ for $i = 1, 2, 3$ and $e_\alpha \to 0, \;\; e_\beta \to 0, \;\; e_\beta \to 0,$ as $t \to \infty$. ■

3.2. Numerical Results

For the numerical simulations, the fourth-order Runge-Kutta method with time-step $h = 10^{-6}$ is used to solve the two systems of differential equations (14) and (15) with the adaptive nonlinear controller (18) and update law of estimates (25).

We take $k_i = 4$ for $i = 1, 2, \ldots, 6.$

The parameters of the Li systems are chosen so that the system (14) and (15) are chaotic, *i.e.*

$$\alpha = 5, \ \beta = 16 \text{ and } \gamma = 1.$$

The initial values of the parameter estimates are chosen as $\hat{\alpha}(0) = 4, \ \hat{\beta}(0) = 5$ and $\hat{\gamma}(0) = 12.$

The initial values of the master system (14) are chosen as

$$x_1(0) = 4, \ \ x_2(0) = 8 \ \text{ and } \ x_3(0) = 10.$$

The initial values of the slave system (15) are chosen as

$$y_1(0) = 12, \ y_2(0) = 15 \ \text{ and } \ y_3(0) = 7.$$

Figure 8 shows the anti-synchronization of the Li systems (14) and (15). Figure 9 shows the time-history of the anti-synchronization errors $e_1, e_2, e_3.$ Figure 10 shows the time-history of the parameter estimates $\hat{\alpha}, \hat{\beta}, \hat{\gamma}.$ Figure 11 shows the time-history of the parameter estimation errors $e_\alpha, e_\beta, e_\gamma.$ Figure 12 shows the time-history of the applied control inputs $u_1, u_2, u_3.$

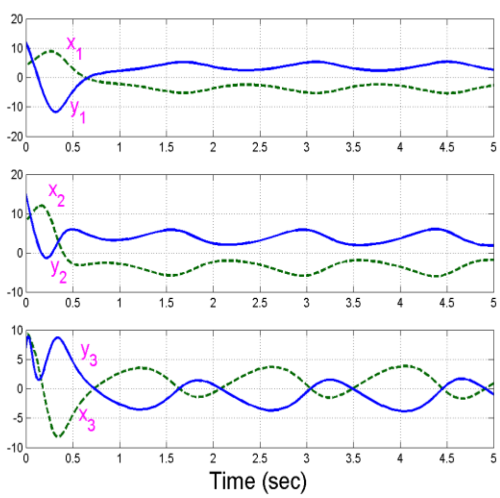

Figure 8. Anti-Synchronization of Identical Li Chaotic Systems

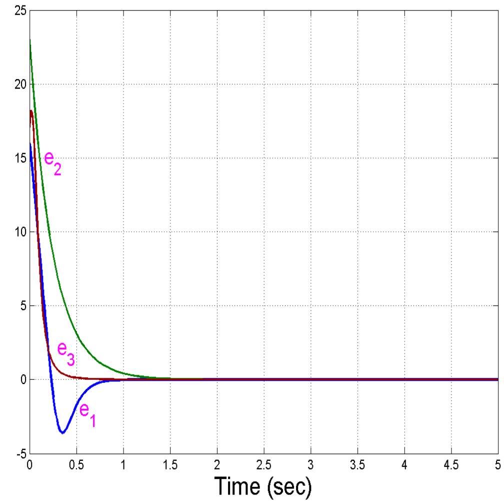

Figure 9. Time History of the Error States e_1, e_2, e_3

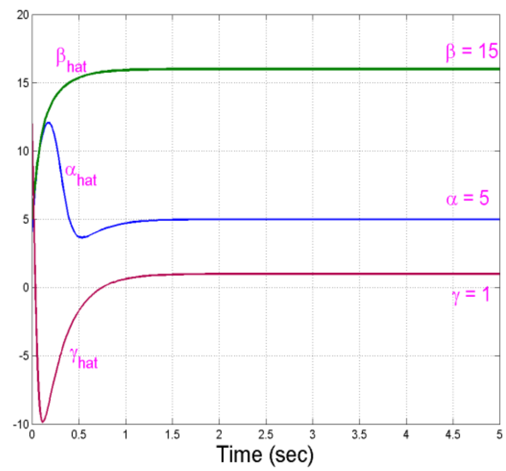

Figure 10. Time History of the Estimates $\hat{\alpha}, \hat{\beta}, \hat{\gamma}$

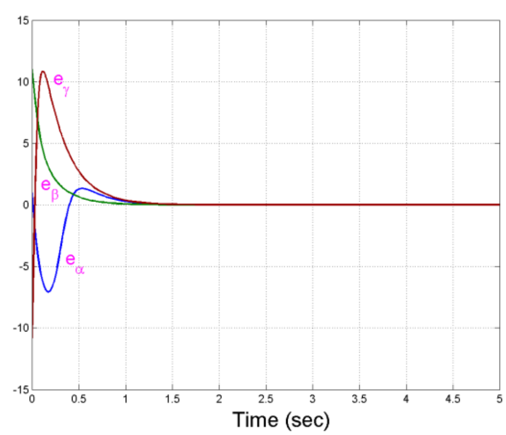

Figure 11. Time History of the Estimation Errors

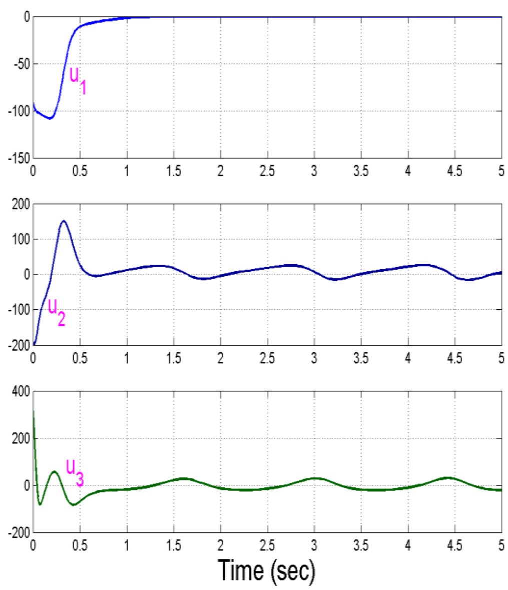

Figure 12. Time History of the Applied Control Inputs u_1, u_2, u_3

4. Adaptive Anti-Synchronization of Tigan and Li Chaotic Systems

This section details the adaptive anti-synchronization of Tigan and Li systems. Here, we consider the Tigan system [32] as the master system, whose parameters are known. Also, we consider the Li system [33] as the slave system, whose parameters are unknown.

4.1. Theoretical Results

As the master system, we consider the Tigan dynamics described by

$$\begin{aligned}\dot{x}_1 &= a(x_2 - x_1)\\ \dot{x}_2 &= (c-a)x_1 - ax_1x_3\\ \dot{x}_3 &= -bx_3 + x_1x_2\end{aligned} \tag{27}$$

where x_1, x_2, x_3 are the state variables and a, b, c are known parameters of the system.

As the slave system, we consider the controlled Li dynamics described by

$$\begin{aligned}\dot{y}_1 &= \alpha(y_2 - y_1) + u_1\\ \dot{y}_2 &= y_1y_3 - y_2 + u_2\\ \dot{y}_3 &= \beta - y_1y_2 - \gamma y_3 + u_3\end{aligned} \tag{28}$$

where y_1, y_2, y_3 are the state variables, α, β, γ are unknown parameters of the system and u_1, u_2, u_3 are the nonlinear control inputs to be designed.

The anti-synchronization error is defined as

$$e_i = y_i + x_i, \quad (i = 1, 2, 3). \tag{29}$$

The error dynamics is obtained as

$$\begin{aligned}\dot{e}_1 &= \alpha(y_2 - y_1) + a(x_2 - x_1) + u_1\\ \dot{e}_2 &= -y_2 + (c-a)x_1 + y_1y_3 - ax_1x_3 + u_2\\ \dot{e}_3 &= \beta - \gamma y_3 - bx_3 - y_1y_2 + x_1x_2 + u_3.\end{aligned} \tag{30}$$

We define the adaptive control functions as

$$\begin{aligned}u_1(t) &= -\hat{\alpha}(y_2 - y_1) - a(x_2 - x_1) - k_1e_1\\ u_2(t) &= y_2 - (c-a)x_1 - y_1y_3 + ax_1x_3 - k_2e_2\\ u_3(t) &= -\hat{\beta} + \hat{\gamma}y_3 + bx_3 + y_1y_2 - x_1x_2 - k_3e_3\end{aligned} \tag{31}$$

where $\hat{\alpha}, \hat{\beta}, \hat{\gamma}$ are estimates of α, β, γ, respectively and k_1, k_2, k_3 are positive constants.

Substituting (31) into (30), the closed-loop error dynamics is obtained as

$$\begin{aligned}\dot{e}_1 &= (\alpha - \hat{\alpha})(y_2 - y_1) - k_1e_1\\ \dot{e}_2 &= -k_2e_2\\ \dot{e}_3 &= (\beta - \hat{\beta}) - (\gamma - \hat{\gamma})y_3 - k_3e_3.\end{aligned} \tag{32}$$

We define the parameter estimation errors as

$$\begin{aligned}e_\alpha &= \alpha - \hat{\alpha}\\ e_\beta &= \beta - \hat{\beta}\\ e_\gamma &= \gamma - \hat{\gamma}.\end{aligned} \tag{33}$$

Using (33), the error dynamics is simplified as

$$\begin{aligned} \dot{e}_1 &= e_\alpha (y_2 - y_1) - k_1 e_1 \\ \dot{e}_2 &= -k_2 e_2 \\ \dot{e}_3 &= e_\beta - e_\gamma y_3 - k_3 e_3. \end{aligned} \tag{34}$$

For the derivation of the update law for adjusting the estimates of parameters, the Lyapunov method is used.

We consider the quadratic Lyapunov function defined by

$$V = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_\alpha^2 + e_\beta^2 + e_\gamma^2\right), \tag{35}$$

which is a positive definite function on R^6.

We note that

$$\begin{aligned} \dot{e}_\alpha &= -\dot{\hat{\alpha}} \\ \dot{e}_\beta &= -\dot{\hat{\beta}} \\ \dot{e}_\gamma &= -\dot{\hat{\gamma}}. \end{aligned} \tag{36}$$

Differentiating (35) along the trajectories of (34) and noting (36), we find that

$$\begin{aligned} \dot{V} = &-k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 \\ &+ e_\alpha \left[e_1 (y_2 - y_1) - \dot{\hat{\alpha}} \right] \\ &+ e_\beta \left[e_3 - \dot{\hat{\beta}} \right] + e_\gamma \left[-e_3 y_3 - \dot{\hat{\gamma}} \right]. \end{aligned} \tag{37}$$

In view of (37), the estimated parameters are updated by the following law:

$$\begin{aligned} \dot{\hat{\alpha}} &= e_1 (y_2 - y_1) + k_4 e_\alpha \\ \dot{\hat{\beta}} &= e_3 + k_5 e_\beta \\ \dot{\hat{\gamma}} &= -e_3 y_3 + k_6 e_\gamma \end{aligned} \tag{38}$$

where k_4, k_5, k_6 are positive constants.

Now, we state and prove the following result.

Theorem 3. *The Tigan system (27) with known parameters and the Li system (28) with unknown parameters are globally and exponentially anti-synchronized by the adaptive control law (31), where the update law for the parameter estimates* $\hat{\alpha}, \hat{\beta}, \hat{\gamma}$ *is given by (38) and* $k_i, (i = 1, 2, \ldots, 6)$ *are positive constants. The errors for parameter estimates* $e_\alpha, e_\beta, e_\gamma$ *decay to zero exponentially as* $t \to \infty$.

Proof. This resut is a simple consequence of the Lyapunov stability theory.

We know that V as defined in (35) is a positive definite function on R^6.

Substituting (38) into (37), we obtain

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - k_4 e_\alpha^2 - k_5 e_\beta^2 - k_6 e_\gamma^2, \tag{39}$$

which is a negative definite function on R^6.

Hence, by the Lyapunov stability theory [34], it follows that $e_i(t) \to 0$ exponentially as $t \to \infty$ for $i = 1, 2, 3$ and $e_a \to 0,\ e_b \to 0,\ e_c \to 0,$ as $t \to \infty$. This completes the proof. ■

4.2. Numerical Results

For the numerical simulations, the fourth-order Runge-Kutta method with time-step $h = 10^{-6}$ is used to solve the two systems of differential equations (27) and (28) with the adaptive nonlinear controller (31) and update law of estimates (38).

We take $k_i = 4$ for $i = 1, 2, \ldots, 6.$

The parameters of the Tigan system (27) are chosen so that the system is chaotic, namely,

$a = 2.1,\ b = 0.6$ and $c = 30.$

The parameters of the Li system (28) are chosen so that the system is chaotic, namely,

$\alpha = 5,\ \beta = 16$ and $\gamma = 1.$

The initial values of the parameter estimates are chosen as

$\hat{\alpha}(0) = 3,\ \ \hat{\beta}(0) = 1$ and $\hat{\gamma}(0) = 7.$

The initial values of the master system (27) are chosen as

$x_1(0) = 4,\ x_2(0) = 8$ and $x_3(0) = 5.$

The initial values of the slave system (28) are chosen as

$$y_1(0) = 6,\ y_2(0) = -4 \text{ and } y_3(0) = 9.$$

Figure 13 shows the anti-synchronization of the Tigan system (27) and the Li system (28).

Figure 14 shows the time-history of the anti-synchronization errors $e_1, e_2, e_3.$

Figure 15 shows the time-history of the parameter estimates $\hat{\alpha}, \hat{\beta}, \hat{\gamma}.$

Figure 16 shows the time-history of the parameter estimation errors $e_\alpha, e_\beta, e_\gamma.$

Figure 17 shows the time-history of the applied control inputs $u_1, u_2, u_3.$

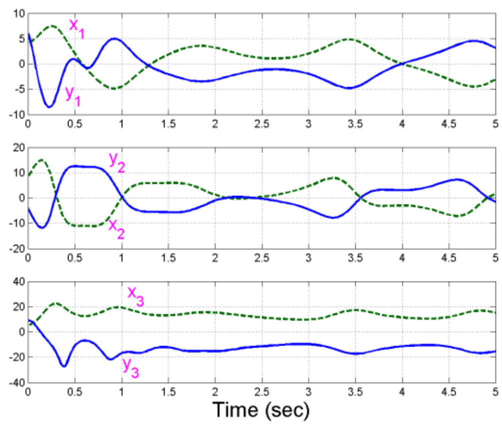

Figure 13. Anti-Synchronization of Non-Identical Tigan and Li Chaotic Systems

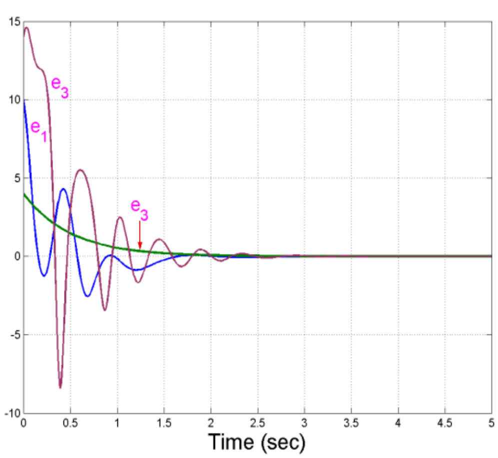

Figure 14. Time History of the Error States e_1, e_2, e_3

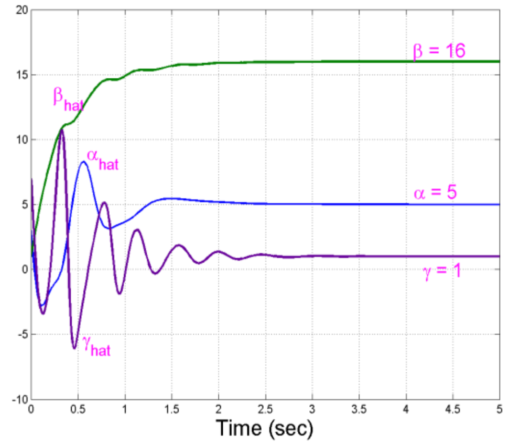

Figure 15. Time History of the Estimates $\hat{\alpha}, \hat{\beta}, \hat{\gamma}$

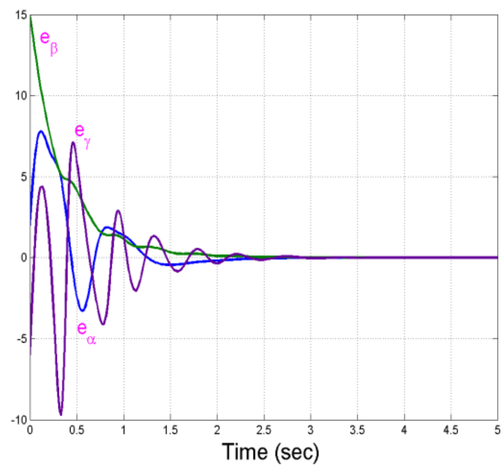

Figure 16. Time History of the Estimation Errors

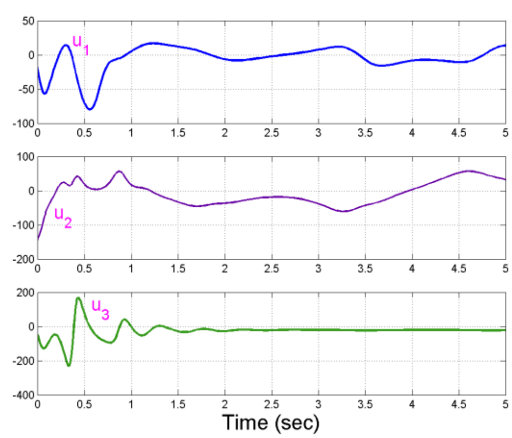

Figure 17. Time History of the Applied Control Inputs u_1, u_2, u_3

5. Conclusion

In this paper, the adaptive control method has been applied in the study of global chaos anti-synchornization of identical Tigan systems [32] identical Li systems [33] and non-identical Tigan system with known parameters and the Li system with unknown parameters. For the adaptive anti-synchronization of identical chaotic systems, it was assumed that the system parameters are unknown. For the adaptive anti-synchronization of different chaotic systems, it was assumed that the parameters of the master system are known, but the parameters of the slave system are unknown. Our therotical results have been fully established using the Lyapunov stability theory. Numerical simulations are also shown for the anti-synchronization of identical and non-identical Tigan and Li chaotic systems to demonstrate the effectiveness of the adaptive anti-synchronization schemes derived in this paper.

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