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Coordination and Optimization: The Integrated Supply Chain Analysis with Non-Linear Price-Sensitive Demand

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Abstract. In this paper, a supply chain with a coordination mechanism consisting of a single vendor and buyer is considered. Further, instead of a price sensitive linear or deterministic demand function, a price-sensitive non-linear demand function is introduced. To find the inventory cost, penalty cost and transportation cost, it is assumed that the production and shipping functions of the vendor are continuously harmonized and occur at the same rate. In this integrated supply chain, the Buyer's Linear Program (LP), vendor's Integer Program (IP) and coordinated Mixed Integer Program (MIP) models are formulated. In this research, numerical example is presented which includes the sensitivity of the key parameters to illustrate the models. The solution procedures demonstrate that the individual profit as well as joint profit could be increased by a coordination mechanism even though the demand function is non-linear. In addition, the results illustrate that Buyer's selling price, along with the consumers purchasing price, could be decreased, which may increase the demand of the end market. Finally, a conclusion is drawn in favor of the coordinated supply chain with a non-linear price sensitive demand function.

Keywords: Coordination between vendor and buyer, Inventory management mixed integer program, Non-linear demand.
AMS Classification: 90C11

1. Introduction

The classical objective of logistics is to be able to have the right products in the right quantity, at the right place, at the right moment at minimal cost. Efforts to produce an efficient supply chain are centered on managing logistical flow and inventory. In overcoming many of the new challenges of the comprehensive enterprise, the coordination of members along the supply chain is vital. Without coordination a supply chain system cannot be optimal as a whole since each party will only try to enhance his own profits. That is why to ensure the optimal system and to satisfy customer demands in today's competitive markets, significant information needs to be shared along the supply chain. Moreover, a high level of coordination between the vendor and buyer's decision making is also required. The concept of Joint Economic Lot Sizing (JELS) is introduced to filter traditional methods for independent inventory control and to find a more profitable joint production and inventory policy.

The idea of optimizing the joint total cost in a single-vendor and a single-buyer model was first introduced by Goyal [7]. Banerjee [1] further developed the model by incorporating a finite

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production rate and following a lot-for-lot policy for the vendor. By relaxing Banerjee's lot-for-lot assumption, Goyal [6] proposed a more general joint economic lot-sizing model In addition, Viswanathan and Wang [19] described the effectiveness of quantity discounts and volume discounts as a coordination mechanism in distribution channels with price sensitive demand. They concluded that the effectiveness of volume discounts as a coordination mechanism is higher when the sensitivity of demand to price changes is higher and the effectiveness of quantity discounts is higher with lower price demand. In addition, Qin et al. [13] have considered volume discounts and franchise fees as a coordination mechanism in a system of supply chain with single supplier and single buyer with price sensitive demand. Subsequently, they analyzed the problem as a Stackelberg game in which the supplier acts as the leader by announcing its pricing policy to the buyer in advance and the buyer acts as the follower by determining his unit selling price. Finally, they showed that when demand is price sensitive, channel profits achieved by employing volume discounts and franchise fees is larger than achieved by quantity discounts and franchise fees.

Further, Pourakbar et al. [11] described an integrated four-stage supply chain system, incorporating one supplier, multiple producers, multiple distributors and multiple retailers. Then they determined the optimal order quantity of each stage and shortage level of each stage to minimize the cost of the supply chain. Recently, Wu and Yen [21] have provided some patch works to enhance the volubility of the integrated single-vendor single-buyer inventory model. Zavanella and Zanoni [22] have investigated the consignment stock policy of the Vendor-Managed Inventory model and showed that consignment stock policy works better than uncoordinated optimization when implemented for an industrial case of a single-vendor and multiple-buyer production situation.

At the same time, Jokar and Sajadieh [9] have described a vendor–buyer integrated production inventory model which takes into consideration Joint Economic Lot Sizing (JELS) policy with price sensitive demand of the customer. Jokar and Sajadieh [9] detailed a JELS model where the shipment; ordering and pricing policy are all optimized. They investigated the effectiveness of customer price sensitive linear demand. Uddin and Sano [18] depicted a linear fraction model that maximizes the return on investment and finds the location for the facility. They also discussed an MIP based approach to solve linear fractional programming problem.

Qi et al. [14] described supply chain coordination with demand disruption. In their model, the market demand function is assumed to be a linear function of the retail price, $Q = D - kp$, where D is the maximum market demand, p is the retail price , k is a coefficient of price sensitivity , and Q is the real demand under retail price p. Huang et al. [8] studied the same model with non linear demand function $Q = De^{-kp}$, where D, p and Q represent the market scale ,retailer price and real demand respectively , and k>0 a coefficient of price sensitivity. Moreover, Lin and Ho [10] investigated an integrated vendor–buyer inventory system with quantity discount and price sensitive demand and proposed an iterative procedure to find the optimal solution.

For the sake of this study, LP, IP and MIP based models have been formulated by combining price sensitive demand and coordination between members of the supply chain. To control unstable consumers demand, a new exponential price sensitive demand function is introduced. This work introduces the exponential price sensitive demand function not just because it has become popular among researchers, but because the form includes an explicit term for price elasticity and is easy to manipulate mathematically. Further, it has been pointed out by many researchers, the results obtained from linear demand function may not suitable to apply directly in the case of nonlinear demand function. The goal of this work is to determine the individual and coordinated profit with the new exponential demand function. The work has three main phases. In the first phase, according to the demand market the buyer's LP model is solved to obtain the optimum order quantity and selling price. In the second phase, according to the buyer's order quantity, the vendor's IP based model is worked out to obtain the optimal shipments. Finally, in third phase, the coordinated MIP based model is figured out. Penalty/delay cost, transportation cost, ordering /

setup cost, and production cost are also considered. The sensitivity on the buyer's selling prices is discussed and hence drawn a conclusion in favor of coordination mechanism with exponential price sensitive demand function.

The reminder of this paper is organized as follows. Notations, assumptions, input parameters, decision variables and mathematical model are provided in Section 2. In Section 3, an algorithm for finding the optimal solution is proposed. In the following, a numerical example and the computational results of these models are discussed. Finally, Section 5 contains some conclusions and suggests the scopes of future research.

2. Notation, Assumption and Model Formulation Parameters

Let

s= purchasing cost of buyer (selling price of vendor)

H_B= holding cost of buyer per unit time

D= demand rate as a function of selling price

$F_{B=}$ buyers' opening cost (which included the land acquisition cost, facility construction cost, input cost and manufacturing cost)

t_B= buyer's transportation cost per unit product

F_V= vendor's opening cost (which included the land acquisition cost, facility construction cost, input cost and manufacturing cost)

H_V= holding cost of vendor year

t_V= transportation cost per shipment

V= be the size of shipment

y= Vendor's rate of production

L= Vendor's lead-time

r= is the fixed cost of per shipment from vendor to buyer

tv= travel time per shipment from vendor to buyer

g= transportation cost per unit time per shipment from vendor to buyer

w= be the penalty cost for per unit product

Decision variables:

Q= buyer's order quantity

c= selling price

N= Number of shipments

Assumptions:

1. The model deals with a single vendor-buyer for a single product

2. The buyer faces a non-linear demand function, $D(c) = ac^{-b}$, where, the slope a>0 and the constant (absolute) elasticity b>0.

3. Vendor finite production rate is greater than the demand rate.

4. The continuous production rate and shipping functions are harmonized and occur at the same rate. Products are produced for the first shipment and continue to be produced while the first shipment is being loaded and shipped

5. The inventory holding cost at the buyer is greater than that at the vendor.

2.1 Buyers' model formulation

The buyer's profit function per unit time is equal to the total revenue minus the sum of the inventory holding, opening cost, transportation and purchasing costs. Thus, the total profit of the buyer who wishes to maximize it is given the LP based Eq. (1).

Maximize,

$$P_B(c,Q) = ac^{-b} * (c-s) - ac^{-b} * F_B / Q - H_B * Q/2 - ac^{-b} t_B$$
$$= ac^{-b}(c - s - t_B) - ac^{-b} * F_B / Q - H_B * Q/2$$

The buyer's optimal Economic Order Quantity is (1)

$$Q = \sqrt{\frac{2 * ac^{-b} * F_B}{H_B}} . \tag{2}$$

Substituting Eq. (2) into Eq. (1), we have

$$P_B(c) = ac^{-b} * (c - s - t_B) - \sqrt{2 * ac^{-b} * F_B * H_B}$$

Differentiating with respect to c, we have,

$$\frac{d}{dc}(P_B(c)) = c^{-b-1}(ca - ab(c - s - t_B) + (b/2) * \sqrt{2 * a * F_B * H_B} * c^{b/2}) . \tag{3}$$

Since $\frac{d^2}{dc^2}(P_B(c))$ is negative (refer to the Appendix A), this implies that the Eq. (1) is concave in price c. Therefore, buyer LP model (1) can be restated as;

maximize,

$P_B(c,Q) = ac^{-b}(c-s-t_B) - ac^{-b} * F_B / Q - H_B * Q / 2$,

subject to:

$$ac^{-b} \geq 0$$

$$Q = \sqrt{\frac{2 * ac^{-b} * F_B}{H_B}}$$

$$ca - ab(c-s-t_B) + (b/2) * \sqrt{2 * a * F_B * H_B} * c^{b/2} = 0.$$

The first term of the objective function P_B is the total revenue obtained by multiplying the difference of Buyer's selling price and purchasing cost and transportation cost by the demand of that product. The second term is the fixed opening cost (including land acquisition costs, order processing costs, facility construction costs and manufacturing cost) and the last term is the holding cost. The first constraint ensures the non-negation of the demand function. The second constraint makes sure that the demand function should follow the EOQ policy. The last constraint is the first order optimality condition.

2.2 Vendors' model

In this section, the vendor's production and shipping functions are considered to be harmonized and occurring at identical rates, which is one of the simplest and most common cases. The vendor produces the products and continues to make them while the first shipment is being loaded and shipped to the buyer, which requires time L.

In lead time L, additional products that are produced are then added to the inventory. The first shipment of the product is shipped to the buyer after time L+V*y, where L is the lead time for the first shipment and V*y is the time required to produce V units. The total inventory held over the entire time horizon, obtained by the summation of the area under the Figure 1, determines the total inventory costs (See also [12]).

From Figure 1, *the total inventory*

$$= (L/2y * L) + (NV^2 - V^2 y)/2 + (NVL)$$
$$+ (V^2 y/2 - VL + L^2/2y) + (VL - L^2/y)$$
$$= (NV^2 y/2 + NVL)$$
$$= Q^2 y/2N + QL \quad \therefore V = \frac{Q}{N}$$

Then *the inventory holding cost is*

$$= (Q^2 y/2N + QL) * H_V$$

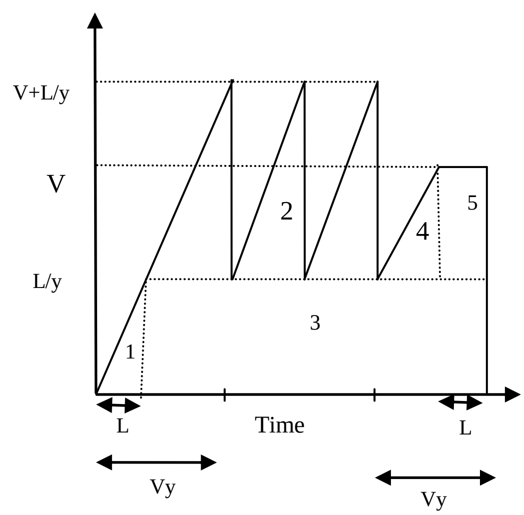

Figure 1. Vendor's inventory distribution pattern

Total transportation cost is equal to $(r + t_v * g)N$

Customer as well as buyer waiting cost (Penalty cost):

Waiting cost incurred for the first shipment is equal to

$$(V * y + L + t_v)wV \; .$$

Hence, *total waiting cost for N shipments*

$$= \sum_{i=1}^{N}(i * V * y + L + t_v)wV$$
$$= (Q * y(N+1)/2N + L + t_v)wQ$$

Vendor profit function:

The vendor's profit function per unit time is equal to the total revenue minus the sum of the inventory holding, opening, transportation and penalty costs. Thus, the total profit of the buyer who wishes to maximize it is given the LP based Eq. (4).

Maximize,

$$P_V(N) = ac^{-b} * s - ac^{-b} * F_V / QN - (Q^2 y / 2N + QL) * H_v - (Q * y(N+1)/2N + L + t_v)wQ - (r + t_v g)N \quad (4)$$

To achieve the vendor's optimal decision, it is differentiated twice with respect to N:

$$\frac{d^2}{dN^2}(P_V(N)) = -\frac{Q^2 y(w + H_v + 2ac^{-b})}{N^3} \quad (5)$$

Eq. (5) shows that the vendors' profit function is concave function in N (refer to the Appendix). Therefore, the optimum value of the profit function will occur at the point where

$$\frac{d}{dN}(P_V(N)) = 0$$

Vendors' IP Model;
maximize,

$$P_V(N) = ac^{-b} * s - ac^{-b} * F_V / NQ_B - (Q_B{}^2 y / 2N + Q_B L) * H_v - (Q_B * y(N+1)/2N + L + t_v)wQ_B - (r + t_v g)N$$

subject to;

$$ac^{-b} * F_V / N^2 Q_B + Q_B{}^2 * y * H_v / 2N^2 + Q_B^2 * yw / 2N^2 - (r + tg) = 0;$$

where N is an integer.

The first term of the objective function P_V is the total revenue obtained by multiplying buyers' selling price and demand, the second term is the fixed opening cost, the third term is the inventory holding cost and the last two terms are waiting cost (or penalty cost) and transportation cost respectively. The last constraint is the first order optimality condition.

2.3 Coordinated joint model

Suppose the situation is where both parties agreed to coordinate their production and inventory strategies and share the information with each other to obtain the best strategy for the integrated supply chain system. Then, the total profit per unit time is the sum of the individual profit function given by the MIP based Eq. (6),

maximize,

$$P_J(c,Q,N) = ac^{-b}(c - t_B) - ac^{-b} * F_B / Q - H_B Q/2 - ac^{-b} * F_V / NQ - (Q^2 y / 2N + QL)H_v - (Qy(N+1)/2N + L + t_v)wQ - (r + t_v g)N \quad (6)$$

subject to: $c, Q \geq 0$, N is an integer.

The objective function P_J represents the total revenue obtained after coordination by summing the individual profit obtained by buyer and vendor (as described in previous section).

3. Solution Procedure of the Coordinated Joint Model

According to Eq. (6), the problem is to find the consumer's purchasing price, order quantity and number of shipment such that the total joint profit will be maximized. The nonlinear optimization problem is to be solved for c, Q and N. However, Eq. (6) cannot easily be proven to be a concave function. Therefore Eq. (6) describes multiple local maxima. To solve this model, we have used the iterative algorithm described by Lin and Ho [10]. It is clear that this Eq. (6) is independent of the Buyer's purchasing price s.

First, the effect of N on the profit function for fixed (c, Q) is examined. The second order partial derivative of Eq. (6) with respect to N is:

$$\frac{\partial^2}{\partial N^2}(P_J(c,Q,N)) = -\frac{2ac^{-b}F_v}{QN^3} - \frac{Q^2 y}{N^3}(w + H_v) < 0, \forall N. \quad (7)$$

From Eq. (7), it is apparently clear that for fixed Q and c, the profit function is concave in N(refer to Appendix B). Hence, the search for option solution, N^*, is reduced to find the local optimal solution. In other words, for given N, the maximum value of $P_J(c,Q,N)$ will occur at the point which satisfies simultaneously,

$$\frac{\partial}{\partial Q}(P_J(c,Q,N)) = 0 \quad \& \quad \frac{\partial}{\partial c}(P_J(c,Q,N)) = 0.$$

Hence,

$$\frac{\partial}{\partial c}(P_J(c,Q,N)) = 0$$

$$\Rightarrow c = \frac{b(F_B / NQ + F_v / Q) + bt_B}{b - 1}, b \neq 1 \quad (8)$$

$$\frac{\partial}{\partial Q}(P_J(c,Q,N)) = 0$$

$$\Rightarrow \frac{ac^{-b}F_B}{Q^2} + \frac{ac^{-b}F_V}{NQ^2} - Qyw - \frac{Qyw}{N} - \frac{H_vQy}{N}$$

$$= \frac{H_B}{2} + lw + t_v w + lH_v$$

$$\Rightarrow A1Q^3 + A2Q^2 - A3 = 0 \quad (9)$$

where,

$$A1 = yw + \frac{yw}{N} + \frac{H_v y}{N},$$

$$A2 = \frac{H_B}{2} + lw + tvw + lH_v \quad ,$$

$$\text{And } A3 = ac^{-b}F_B + \frac{ac^{-b}F_v}{N}.$$

Therefore, in order to achieve the optimal solution (c, Q, N), we propose the following iteration algorithms:

Algorithm:

Step 1, Set N=1.

Step 2, For each j=0, 1, 2... k, perform

i. Start with c1=0 and calculate Qj from Eq. (9)
ii. For Qj and N, calculate cj from Eq. (8)
iii. Repeat steps (i) and (ii), until no change occurs in the values of Qj and cj.
iv. Rename $Qj=Q^N_j$ and $cj=c^N_j$

Step3, substitute $Qj=Q^N_j$ and $cj=c^N_j$ into Eq. (6) and calculate $P_J(c_j^N, Q_j^N, N)$

Step4, Find

$$P_J(c^N, Q^N, N) = Max_{j=0,1,2,\ldots,k} P_J(c_j^N, Q_j^N, N)$$

Step 5, Increase N by one

Step 6, If $P_J(c^N, Q^N, N) \geq P_J(c^{N-1}, Q^{N-1}, N-1)$, go to Step 5, otherwise, $P_J(c^N, Q^N, N)$ is the optimal solution with optimal point (c^*, Q^*, N^*)

4. Numerical Example

In this section, we illustrate our models by presenting an example, including the optimal solutions. Let the supply chain scenario be as follows:

Buyers' parameters: purchasing price is s=2$/unit, transportation cost is t_B=0.5$/unit, holding cost is H_B= 0.5 $/unit/unit time and opening cost is F_B = 500$.

Vendor's parameters: opening cost is F_v = 300$, production rate is y=0.5 unit time, penalty cost is w=0.001$/unit time, holding cost is H_V=0.05$/unit per unit time, fixed cost per shipment, r=10$, Transportation cost is g=5$/unit time, travel time is tv=3 unit time, lead time is L=0.5 unit time and parameters a=100000.

Using the models described in Section 2, we analyze the effect of the non-linear price sensitive demand. The effect is evaluated by the impact of the coordination mechanism as well as the impact on the decision variables. In order to achieve insight into this effect, different values of the parameter b are considered. The sensitivity of the parameter b is shown in Tables 1 and 2.

To share the coordinated profit among the members of supply chain, one researcher has proposed a method depending on the ratio of total profit and individual profit obtained before coordination. In the present study, it is assumed that share policy dependent on investments. If α is the percentage of the total amount invested by the vendor in the supply chain, then the individual profit of the vendor will be :

P_V (After coordination) $= \alpha \times P_{AC}$, and P_B (After coordination) $= (1-\alpha) \times P_{AC}$, where, P_V and P_B stand for profit of vendor and buyer, P_{AC} are the profit after coordination It is assumed that α=45%

Table 1. Different values of the parameters BC.

Before Coordination (BC)					
Cases	SP	Q	P_V	P_B	N_{BC}
1.10	28	227	4806	65150	11
1.20	15	227	7234	48345	14
1.30	11	299	8463	37488	15
1.40	8.8	308	9014	29842	15
1.50	7.5	311	7174	24187	14
1.60	6.7	308	9014	19869	15
1.70	6.1	304	8722	16491	15
1.80	5.7	297	8332	13803	15
1.90	5.3	289	7904	11632	14
2.00	5.0	281	7428	9859.1	14

Any significant findings regarding the numerical example of the proposed model are presented in Tables1 and 2. Tables 1 and 2 also offer a comparative analysis for the model before and after coordination. The percentage of change of profit after coordination is obtained by

$PI(\%) = (P_{AC} - P_{BC})*100/P_{BC}$, where, P_{BC} and P_{AC} are the profit before and after coordination between vendor and buyer.

Table 2. Different values of the parameters AC.

After Coordination (AC)				
SP	Q	P_V	P_B	PI (%)
5.63	519	33439	40870.4	6.22
3.07	515.74	28948	35380.4	15.75
2.22	511.94	26211	32035.9	26.76
1.79	508.10	24381	29799.4	39.44
1.54	504.36	23105	28239.6	63.72
1.37	500.78	22201	27134.7	70.82
1.24	497.41	21564	26356	90.06
1.15	494.31	21128	25823.3	112.1
1.08	491.48	20850	25483.6	137.2
1.02	488.97	20700	25301	166.1

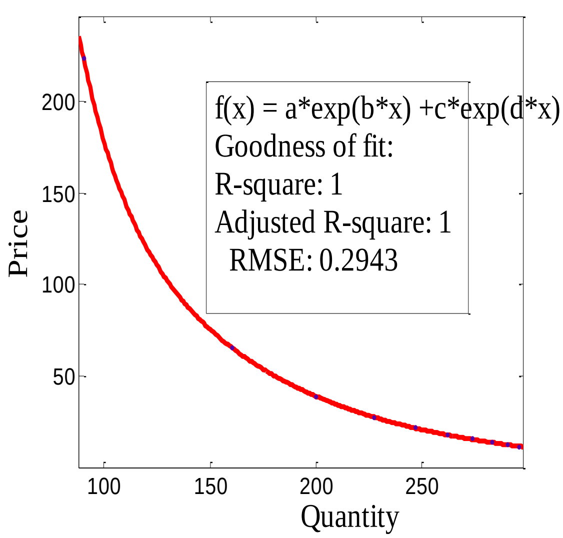

Figure 2. Quantity and price curve BC.

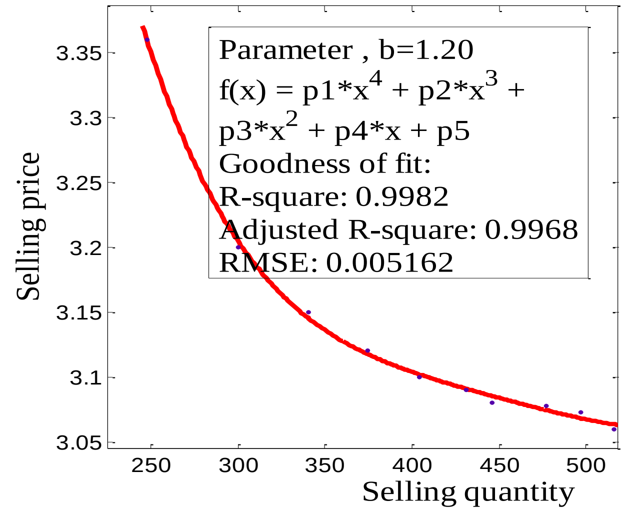

Figure 3. Quantity and price curve AC.

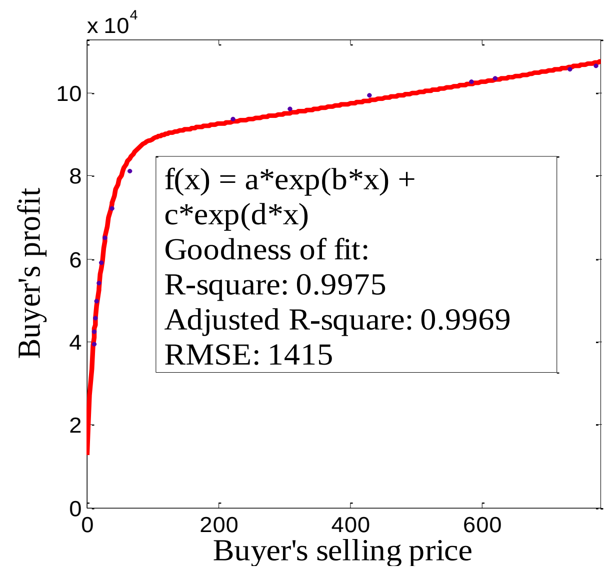

Figure 4. Relation between selling price and profit BC.

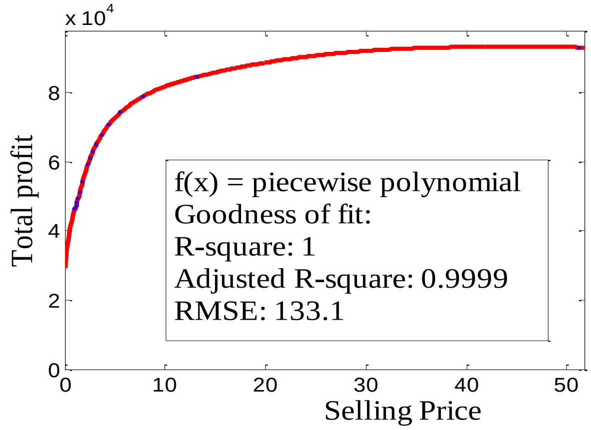

Figure 5. Relation between price and profit AC.

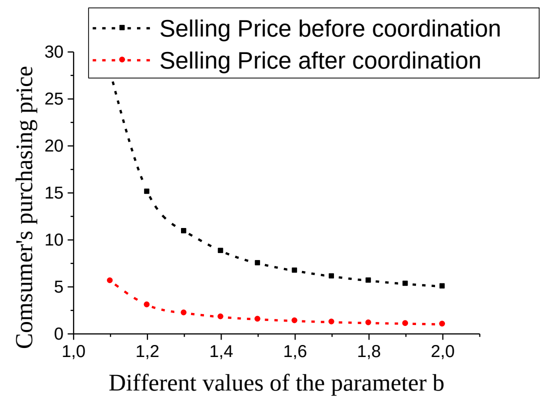

Figure 8. Consumer’s purchasing price before and after coordination

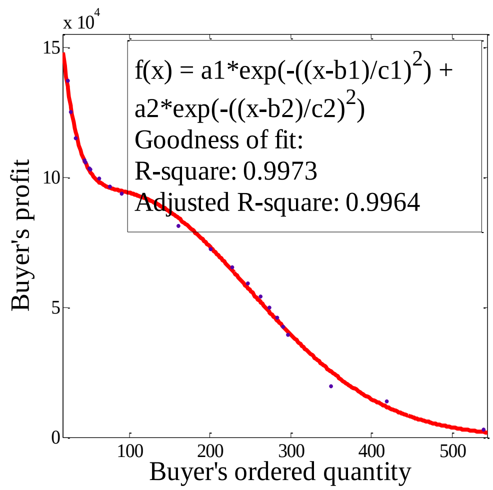

Figure 6. Relation between quantity and profit BC.

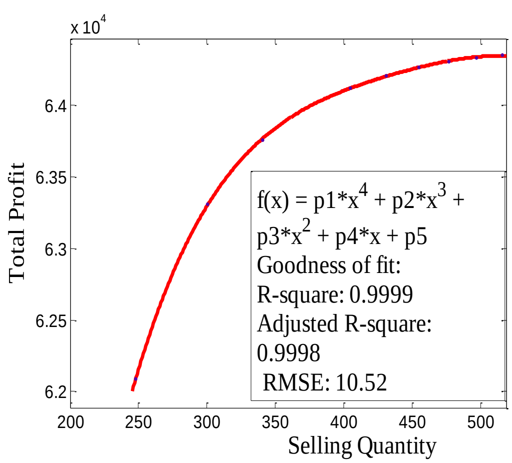

Figure 7. Changing of the profit with respect to quantity AC.

Figure 2 and 3 demonstrate the inversely proportional relationship between price and demand for both before and after coordination respectively. It is clear that if the selling quantity is gradually increased, then the profit also increases accordingly. On the other hand, Figures 4 and 5 show the proportional relationship that holds between price and profit hold until a certain level for both before and after coordination respectively. Figure 6 illustrates a non-coordinated model, where the buyer’s ordered quantity as well as the consumer’s demand increases, and then the corresponding profit decreases gradually. In contrast, the Figure 7 shows that for a coordinated model if the consumer’s demand increases slowly, then the corresponding profit also increases gradually.

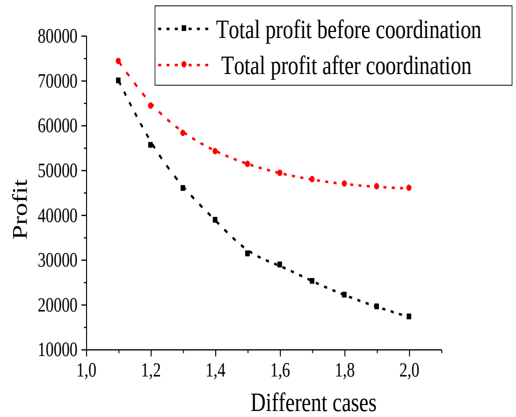

Figure 9. Profit function before and after coordination

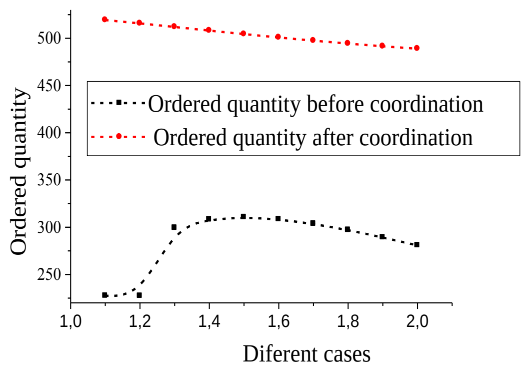

Figure 10. The changes of ordered quantity

Indeed, Figure 8, demonstrates that after coordination, the coordinated selling prices as well as the consumer's purchasing price could be reduced considerably. In most cases, the reduction of the consumer's purchasing price after coordination remains more than 70%. In addition, Figure 9 shows that the profit after coordination always remains higher than before coordination. In most cases, the percentage of profit increment increases; in particular, for this example, it lies between 6% and 166%. Further, the gradual reduction of the coordinated selling price increases the profit function dramatically. In addition, Figure 10 illustrates that for the coordinated supply chain the optimal ordered quantity always remain much higher than that of non coordinated cases. Lastly, Figure 11 displays the effect of transportation cost on the total profit. In fact, a supply chain with high transportation cost is not profitable.

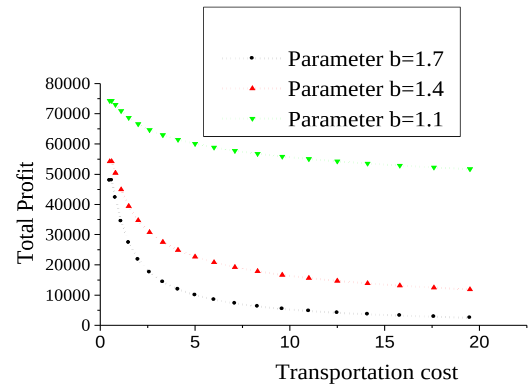

Figure 11. Effect of the transportation cost on the coordinated profit

5. Conclusion and Further Research

In this research, an exponential price sensitive demand function comprises an explicit term for price elasticity that incorporates non-linear effects of pricing and is easy to manipulate mathematically is introduced. A two echelon-supply chain consisting of a single vendor and buyer agreed on coordination principle is also considered. With the aim of demonstrating the models, we have presented a numerical example. Further, vendor's continuously harmonized production and shipping function are assumed to estimate individual profits as well as combined profit, consumer's purchasing price; vendor's selling price, optimal shipments, penalty cost, order quantity, transportation cost, and the effect of main parameters on the selling price.

By introducing the non-linear exponential demand function, we have made some significant findings: that after coordination, the end customer's demand, and consequently individual profits, could be increased without any extra investment. Moreover, such coordination among the members of an enterprise can reduce the consumer purchasing price as well as buyer selling price. But above all, it can be concluded that coordination among the members of an enterprise will be more beneficial in the current competitive environments.

Some future research topics may be of interest here. One is to apply multi vendor policy when considering bounded selling price. An investigation into the sensitivity of different inventory policies for a coordinated supply chain, and an analysis of the lead-time effects and batches of different sizes are two other subjects in the works.

Acknowledgments

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Appendix A

The appendix contains the proof of the convexity of buyer's profit function.

$$P_B(c,Q) = ac^{-b}(c-s-t_B) - ac^{-b} * F_B / Q - H_B * Q/2. \quad (10)$$

Substituting the optimal Economic order quantity,

$$P_B(c) = ac^{-b} * (c-s-t_B) - \sqrt{2 * ac^{-b} * F_B * H_B}.$$

Differentiating with respect to c, we have,

$$\frac{d}{dc}(P_B(c)) = c^{-b-1}(ca - ab(c-s-t_B) + (b/2) * \sqrt{2*a*F_B*H_B} * c^{b/2}).$$

To find the stationary point:

Let, $\frac{d}{dc}(P_B(c)) = 0$, then

$$ca - ab(c-s-t_B) + (b/2) * \sqrt{2*a*F_B*H_B} * c^{b/2} = 0, \because c^{-b-1} \neq 0.$$

Differentiating again with respect to c,

$$\frac{d^2}{dc^2}(P_B(c)) = c^{-b-1}(ab^2 - b(b+1)a(s+t_B)/c - ab - (b/2)(b/2+1) * \sqrt{2*a*F_B*H_B} * c^{b/2-1}).$$

From this expression, it is clear that $\frac{d^2}{dc^2}(P_B(c))$ is negative. This implies that the Eq. (10) is convex downward.

Appendix B

The appendix describes the concavity of the coordinated profit function
Coordinated model:

$$P_J(c,Q,N) = ac^{-b} * s - ac^{-b} * F_V / NQ - (Q^2 y / 2N + QL) * H_v - (Q * y(N+1)/2N + L + t_v)wQ - (r + t_v g)N + ac^{-b}(c-s-t_B) - ac^{-b} * F_B / Q - H_B * Q/2.$$

(11)

Differentiating with respect to N,

$$\frac{d}{dN}(P_V(N)) = ac^{-b} * F_V / N^2 Q + Q^2 * y * H_v / 2N^2 + Q^2 * yw / 2N^2 - (r + tg)$$

Differentiated again with respect to N:

$$\frac{d^2}{dN^2}(P_V(N)) = -\frac{Q^2 y(w + Hv + 2ac^{-b})}{N^3} < 0, \forall N$$

From Eq. (11), it is apparently clear that for fixed Q and c, the profit function is concave in N.

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