

RESEARCH ARTICLE

## Credibility based chance constrained programming for project scheduling with fuzzy activity durations

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ABSTRACT

This paper proposes a credibility based chance constrained programming approach for project scheduling problems with fuzzy activity durations where the objective is to minimize the fuzzy project completion time. This paper expresses the fuzzy events such as a project activity's duration or project completion time with fuzzy chance constraints and the chance of a fuzzy event is illustrated with fuzzy credibility distribution. Due to uncertainty in durations of a project, fuzzy sets and fuzzy numbers can be used in order to illustrate the uncertainty and find a solution space for the problem. Therefore, fuzzy credibility based chance constraint technique is investigated for project scheduling problems with fuzzy activity durations considering the uncertainty or chance of a fuzzy event within a closed interval. In this paper, a fuzzy mathematical model and its crisp equivalent by using credibility measure and chance-constrained programming are given for project scheduling problems with fuzzy activity durations.



### 1. Introduction

Activity durations in projects are mostly assumed as deterministic values but the real-life is full of uncertainty so determining each activity's duration in deterministic time periods may not be always possible. A decision maker or a project planner sometimes needs to express activity durations in a time interval in order to encode his/her biased judgment or experience for those activities. In order to express activity's durations in an interval or to encode the uncertainty in activity durations, fuzzy sets, and fuzzy numbers can be used. In this paper, fuzzy events such as a project activity's duration or project completion time is less than a certain real number are expressed with fuzzy chance constraints and the chance of a fuzzy event is illustrated with fuzzy credibility distribution. In order to express the uncertainty or chance of a fuzzy event within an interval, fuzzy credibility based chance constrained problem is investigated for project scheduling problems with fuzzy activity durations.

Fuzziness in project management and project scheduling has been investigated by researchers for more than 30 years. As far as findings from the literature search, the first study about fuzziness in project management was conducted by DePorter and Ellis [1]. Although they are not recent survey papers

conducted by Bonnal et al. [2] and Herroelen and Leus [3], these papers can be a good guidance for readers. Hapke et al. [4] presented a fuzzy project scheduling decision support system which is applied to software project programming. There are two performance criteria for minimizing the project makespan and the maximum delay while allocating resources among the dependent activities in their study. Hapke and Slowinski [5] proposed generalizing the prioritized heuristic method to solve resource-constrained project planning problems with ambiguous time parameters. Wang and Fu [6] investigated fuzzy project planning in inflationary conditions. In their work, they dismissed four fuzzy timing models under inflationary conditions and solved their models by the  $\alpha$ -cut method. Hapke et al. [7] investigated a multi-mode project planning problem under multi-categorized resource constraints with fuzzy activities. They presented a Simulated Annealing approach to creating a Pareto set for the problem. Özdamar and Alanya [8] investigated uncertainty modeling for software development projects and they presented a case study for their model. Wang [9] used possibility theory to encode uncertainty and he used a fuzzy beam search algorithm to minimize scheduling risk of a project for a new product development. Chanas et al. [10] analyzed the notion of necessary criticality with

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respect to the path and to activities in a network with fuzzy activity durations. Chanas and Zieliński [11] presented complexity results for problems of evaluating the criticality of activities in planar networks with duration time intervals. They also showed that problems are in classes of NP-Hard and NP-Complete. Pan and Yeh [12] studied resource-constrained project scheduling problems with uncertain activity durations and they proposed a fuzzy genetic algorithm with a tabu mechanism to find an approximate solution. In another study of Pan and Yeh [13], they proposed a fuzzy simulated annealing approach to finding a solution for resource-constrained fuzzy project scheduling problem. Wang [14] developed a robust scheduling methodology for uncertain product development projects and proposed a genetic algorithm dependent on the possibility theory. Ke and Liu [15] studied project scheduling problems with fuzzy durations and total cost. With a hybrid intelligent algorithm that includes fuzzy simulation and genetic algorithms, they proposed fuzzy chance-constrained programming approach to solve the problem. Zieliński [16] demonstrated some polynomial solvable cases for the problems of calculating the interval between the latest starting times and the activity floats in networks with uncertain durations. Agarwal et al. [17] investigated resource-constrained project scheduling problems under fuzzy activity durations aimed at minimizing the time to complete the project and proposed the artificial immune system as a solution approach to the problem. Ke and Liu [18] investigated resource allocation in project scheduling problems with the mixed uncertainty of randomness and fuzziness for fuzzy activity durations. They proposed three types of random fuzzy models and presented a genetic algorithm with random fuzzy simulations as a solution approach. Long and Ohsato [19] proposed a fuzzy critical chain method for project scheduling problems with uncertainty and resource constraints. In their method, they developed a desired deterministic schedule with resource constraints and added a project buffer at the end of the calculations with fuzzy numbers. Huang et al. [20] proposed a hybrid intelligent algorithm, including genetic algorithm and random fuzzy simulation, to solve a project scheduling problem for software development with random fuzzy activity durations. Moradi et al. [21] investigated resource constrained project scheduling problem under uncertainty of resource availability and project durations. They also proposed some game theory methods for forming coalitions among subcontractors and they coded satisfactions of subcontractors with fuzzy set theory. Hu and Li [22] proposed an improved fuzzy critical path method for trapezoidal fuzzy numbers. Zhang et al. [23] proposed a novel solution approach for fuzzy programming model by proving lower and upper bounds of fuzzy objective values at a possibility level. Then, they applied their model to a project scheduling problem. Zhang et al.

[24] investigated project selection and project scheduling problems under fuzzy environment. They proposed a multi objective nonlinear mathematical model and an algorithm for solving the problem. Some of the other recent papers in the literature are conducted by Castro-Lacouture et al. [25], Afshar and Fathi [26], Shi and Gong [27], Wang and Huang [28], Bhaskar et al. [29], Ponz-Tienda et al. [30], Maravas and Pantouvakis [31], Xu and Zhang [32], Masmoudi and Haït [33], Huang et al. [34], Chrysafis and Papadopoulos [35], Huang et al. [36], Yu et al. [37], Yousefli [38], Zohoori et al. [39], Habibi et al. [40] Alipouri et al. [41] and Birjandi and Mousavi [42].

As seen from the literature review of this paper, fuzzy project scheduling problems have been mostly investigated with fuzzy ranking methods, fuzzy simulation, and heuristic or metaheuristic. This paper investigates fuzzy scheduling problems with fuzzy chance-constrained programming approaches. For the readers, some scheduling papers (Toksari and Arik [43], Arik and Toksari [44], [45]) investigating fuzzy chance-constrained programming approach can be a good guidance for applicability and validity of the proposed method.

## 2. Basic definitions

This section presents some of basic definitions and notations of fuzzy numbers and measures of a fuzzy event such as credibility, possibility, and necessity for the readers.

If a fuzzy number  $\tilde{A}$  with a membership function  $\mu_{\tilde{A}}(x) : R \rightarrow [0, 1]$  is presented with three points on the real axis such that  $\tilde{A} = (A^L, A^C, A^R)$ , and this fuzzy number's membership function is as seen in Eq (1), then  $\tilde{A}$  is defined as a triangular fuzzy number (TFN).

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x - A^L}{A^C - A^L}, & \text{if } A^C \geq x \geq A^L \\ \frac{A^R - x}{A^R - A^C}, & \text{if } A^C \geq x \geq A^R \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

Possibility theory (Zadeh [46]) measures the chance of fuzzy event and its measure has a dual named Necessity measure. For a TFN  $\tilde{A}$  with  $\mu_{\tilde{A}}(x)$ , Possibility and Necessity measures are calculated as follows:

$$\Pi_A(A \leq r) = \sup_{x \leq r} \mu_{\tilde{A}}(x) = \begin{cases} 0, & r \leq A^L \\ \frac{r - A^L}{A^C - A^L}, & A^L \leq r \leq A^C \\ 1, & r \geq A^C \end{cases} \quad (2)$$

$$N_A(A \leq r) = 1 - \left( \sup_{x>r} \mu_{\tilde{A}}(x) \right) = \begin{cases} 0, & r \leq A^L \\ \frac{r - A^L}{A^R - A^L}, & A^L \leq r \leq A^R \\ 1, & r \geq A^R \end{cases} \quad (3)$$

where  $r$  is a real number. These fuzzy measures are dual to each other, but they are not dual by their own. Credibility measure [47] is a self-dual fuzzy measure and it is average of Possibility and Necessity measures as follows:

$$Cr_A(A \leq r) = \frac{1}{2} (\Pi_A(A \leq r) + N_A(A \leq r)) \quad (4)$$

The credibility distribution function  $\Phi: R \rightarrow [0,1]$  (see [48]) of fuzzy number  $\xi$  can be shown in Eq. (5).

$$\Phi(x) = Cr_{\xi}(\theta \in \Theta | \xi(\theta) \leq x) \quad (5)$$

The credibility distribution function  $\Phi(x)$  is a strictly increasing function on the real axis and it has an inverse function  $\Phi^{-1}(\alpha)$  that is unique for any  $\alpha$

confidence level.  $\Phi^{-1}(\alpha)$  is the inverse credibility distribution function and is a strictly increasing function as seen in Figure 1.

For a TFN  $\tilde{A}$ , the credibility distribution function and inverse credibility distribution function (see Figure 1) are calculated as follows:

$$\Phi(x) = Cr_A(A \leq r) = \begin{cases} 0, & r \leq A^L \\ \frac{r - A^L}{2(A^R - A^L)}, & A^L \leq r \leq A^C \\ \frac{A^R + r - 2A^C}{2(A^R - A^C)}, & A^C \leq r \leq A^R \\ 1, & r \geq A^R \end{cases} \quad (6)$$

$$\Phi^{-1}(\alpha) = \begin{cases} 2\alpha(A^C - A^L) + A^L, & 0 \leq \alpha < 0.5 \\ A^R - (2 - 2\alpha)(A^R - A^C), & 0.5 \leq \alpha \leq 1 \end{cases} \quad (7)$$

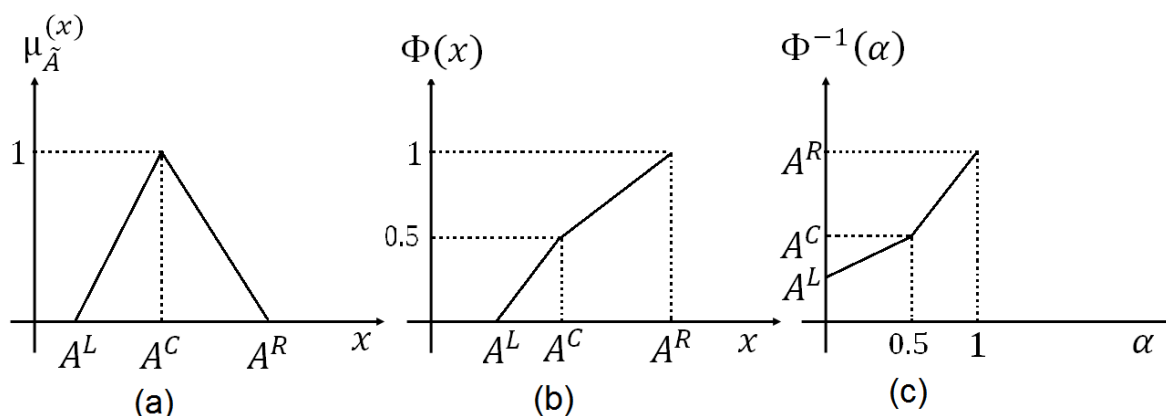


Figure 1. The membership function of a TFN (a), the credibility distribution function (b), the inverse credibility distribution function (c)

### 3. Fuzzy project scheduling mathematical model

In this chapter, the project scheduling problem with fuzzy activity durations where the objective is to minimize project completion time (makespan) is introduced. All activities are expressed with TFN because of the uncertainty in their existences. The proposed fuzzy model as follows:

Indices:

$i$ : index for activities,

Parameters:

$P_{i,j}$ : precedence relationship between activity  $i$  and activity  $j$  ( $i \neq j$ ). If activity  $i$  is a predecessor of activity  $j$ , then  $P_{i,j} = 1$ , otherwise it is zero.

$\tilde{t}_i$ : fuzzy activity duration of activity  $i$

Decision variables:

$\tilde{S}_i$ : starting time of activity  $i$

$\tilde{C}_i$ : completion time of activity  $i$

$\tilde{C}_{max}$ : completion time of the project (the makespan)

Model:

$$Min f = \tilde{C}_{max} \quad (8)$$

s.t.:

$$\tilde{C}_{max} \geq \tilde{C}_i \quad \forall i \quad (9)$$

$$\tilde{C}_i = \tilde{S}_i + \tilde{t}_i \quad \forall i \quad (10)$$

$$\tilde{S}_j \geq \tilde{C}_i P_{i,j} \quad \forall i, \forall j, P_{i,j} = 1 \text{ and } i \neq j \quad (11)$$

$$\tilde{C}_i, \tilde{S}_i \geq 0 \quad (12)$$

$$\tilde{C}_{max} \geq 0 \quad (13)$$

The objective function (8) is to minimize fuzzy makespan of the project. Constraint (9) guarantees that the fuzzy makespan is the maximum completion time of all fuzzy activities. Constraint (10) shows that completion time of an activity is equal to the sum of its start time and duration. Constraint (11) assures that the starting time of activity j must be greater than or equal to the completion time of activity i, if there is a precedence relation from activity i to activity j. Constraints (12-13) show necessary domains of all decision variables.

**4. Fuzzy chance constrained programming model**

For stochastic optimization problems, Charnes and Cooper [49] introduced the chance-constrained programming method. Liu and Iwamura [50] modified Charnes and Cooper’s [49] proposed method for the fuzzy environment and they presented an auxiliary crisp model for solving problems with fuzzy parameters and decision variables. Their auxiliary crisp model is as follows:

$$\max f(x)$$

$$s.t.: Cr\{\xi|g_i(x, \xi) \leq 0, i=1, 2, \dots, p\} \geq \alpha_i \forall i,$$

In order to convert chance constraints with credibility measures to crisp equivalents, the inverse credibility distributions (see Eq. (7)) of fuzzy events are used. For any predetermined confidence level  $\alpha_i$ , there is an unique  $\Phi^{-1}(\alpha_i | K_{\alpha_i} \leq \xi_i)$  value where  $Cr\{\xi_i | K_{\alpha_i} \leq \xi_i\} = \alpha_i$ . With help of Eq. (7), credibility based chance constrained mathematical model of project scheduling problem can be seen as follows:

$$\min f = \Phi_{C_{max}}^{-1}(\alpha) \tag{14}$$

s.t.:

$$\Phi_{C_{max}}^{-1}(\alpha) \geq \Phi_{C_i}^{-1}(\alpha) \forall i \tag{15}$$

$$\Phi_{C_i}^{-1}(\alpha) = \Phi_{S_i}^{-1}(\alpha) + \Phi_{t_i}^{-1}(\alpha) \forall i \tag{16}$$

$$\Phi_{S_j}^{-1}(\alpha) \geq \Phi_{C_i}^{-1}(\alpha) P_{i,j} \forall i, \tag{17}$$

$$\forall j P_{i,j} = 1 \text{ and } i \neq j$$

$$\Phi_{S_i}^{-1}(\alpha), \Phi_{C_i}^{-1}(\alpha), \Phi_{t_i}^{-1}(\alpha) \geq 0 \tag{18}$$

$$\Phi_{C_{max}}^{-1}(\alpha) \geq 0 \tag{19}$$

$$\Phi_{t_i}^{-1}(\alpha) = \begin{cases} 2\alpha(t^c - t^L) + t^L, & \text{if } 0 \leq \alpha < 0.5 \\ t^R - (2 - 2\alpha)(t^R - t^c), & \text{if } 0.5 \leq \alpha \leq 1 \end{cases} \forall i \tag{20}$$

The objective function (14) is to minimize the equivalent makespan value of the inverse credibility distribution function for makespan values for a given confidence level  $\alpha \in [0, 1]$ . The others constraints (15-19) have same missions with constraints (9-13). Constraint (20) transforms the fuzzy activity duration  $\tilde{t}_i = (t^L, t^c, t^R)$  to the equivalent of that activity’s duration of the inverse function of credibility distribution for a given confidence level  $\alpha$ .

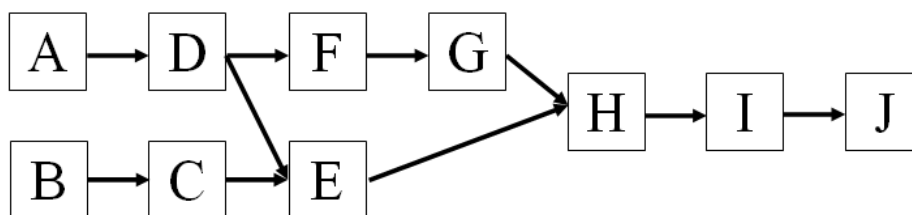
**5. Numerical example**

In this section of the paper, a numerical example is presented for the proposed solution approach. Let us have a project including ten activities. The durations of these activities are expressed with TFNs as seen in Table 1. For a given confidence level  $\alpha$ , the mathematical model in Eqs. (14-20) can be used to find the project completion time’s equivalent considering all fuzzy activity durations at the same confidence level  $\alpha$ . Table 2 shows the solution space of the problem having activities in Table 1 for different confidence levels from 0 to 1 with increment 0.05.

**Table 1.** Comparison of the mean-field predictions

| <i>i</i> | Activity Code | $\tilde{t}_i = (t_i^L, t_i^c, t_i^R)$<br>in weeks | Predecessors |
|----------|---------------|---|--------------|
| 1        | A             | (5,7,9)   | -            |
| 2        | B             | (6,8,10)  | -            |
| 3        | C             | (3,4,7)   | B            |
| 4        | D             | (13,15,19)  | A            |
| 5        | E             | (9,28,35)   | C,D          |
| 6        | F             | (2,5,6)   | D            |
| 7        | G             | (11,17,19)  | F            |
| 8        | H             | (9,13,15)   | E,G          |
| 9        | I             | (7,8,9)   | K            |
| 10       | J             | (2,3,4)   | L            |

The precedence relations of the project in Table 1 can be seen in Figure 2.

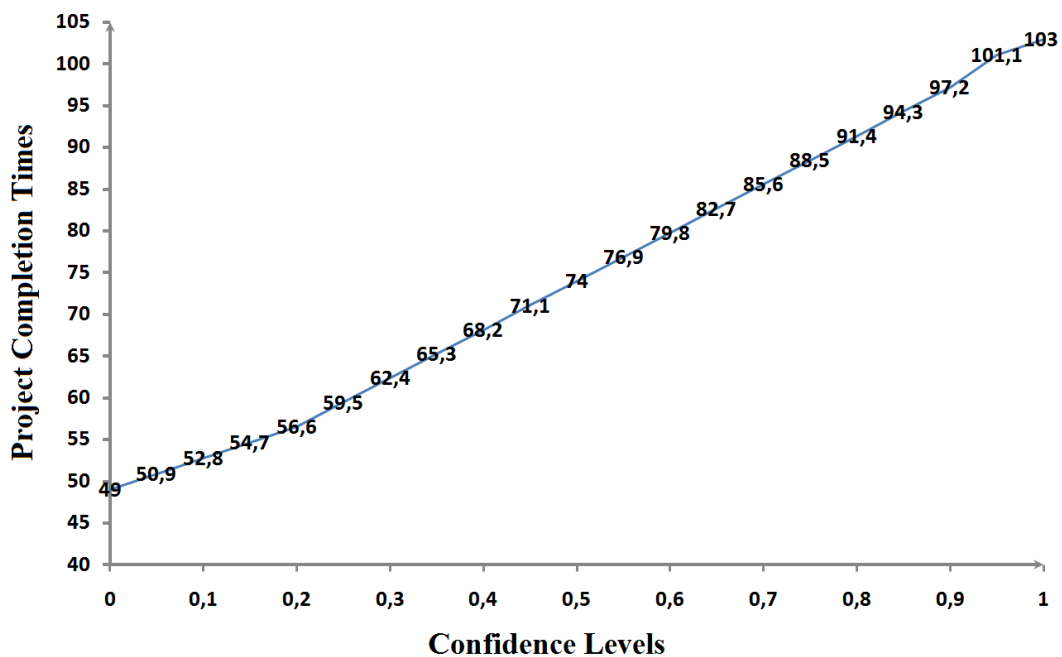


**Figure 2.** Activity-on-Arrow diagram of the project

**Table 2.** Project completion times for different confidence levels

| Confidence level $\alpha$ | Project completion time (in weeks) | Critical path    |
|---------------------------|------------------------------------|------------------|
| 0.00                      | 49.0                               | Path#1           |
| 0.05                      | 50.9                               | Path#1           |
| 0.10                      | 52.8                               | Path#1           |
| 0.15                      | 54.7                               | Path#1           |
| 0.20                      | 56.6                               | Path#1 or Path#2 |
| 0.25                      | 59.5                               | Path#2           |
| 0.30                      | 62.4                               | Path#2           |
| 0.35                      | 65.3                               | Path#2           |
| 0.40                      | 68.2                               | Path#2           |
| 0.45                      | 71.1                               | Path#2           |
| 0.50                      | 74.0                               | Path#2           |
| 0.55                      | 76.9                               | Path#2           |
| 0.60                      | 79.8                               | Path#2           |
| 0.65                      | 82.7                               | Path#2           |
| 0.70                      | 85.6                               | Path#2           |
| 0.75                      | 88.5                               | Path#2           |
| 0.80                      | 91.4                               | Path#2           |
| 0.85                      | 94.3                               | Path#2           |
| 0.90                      | 97.2                               | Path#2           |
| 0.95                      | 101.1                              | Path#2           |
| 1.00                      | 103.0                              | Path#2           |

\*Path#1 = A / D / F / G / H / I / J    \*Path#2 = A / D / E / H / I / J



**Figure 3.** The relationship between completion times and confidence levels

As seen in Figure 3 and Table 2, difference confidence levels produce different project completion times and activities on the critical path. While confidence level is increasing, the project completion time is also increasing and critical activities may change. The results in Table 2 are obtained with GAMS software. This proposed model can be considered to produce the solution space of a fuzzy problem. The solution space of project completion time is on the interval of [49].

**6. Conclusion**

This paper investigates a project scheduling problem with fuzzy activity durations where the objective is to minimize fuzzy project completion time. Fuzzy credibility based chance constraint technique is investigated for project scheduling problems with fuzzy activity durations considering the uncertainty or chance of a fuzzy event within a closed interval. In this paper, a fuzzy mathematical model and its crisp

equivalent by using credibility measure and chance-constrained programming are given for project scheduling problems with fuzzy activity durations. A numerical example is illustrated and it is shown that how the critical path of a project can be changed by using different levels of uncertainty predetermined with given confidence levels. For future researches, credibility based chance constrained programming approach can be investigated with other fuzzy project scheduling problems where other parameters such as precedence relations are expressed in fuzzy sets with resource constraints, resource allocation, resource ability constraints.

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