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RESEARCH ARTICLE

Analytical studies on waves in nonlinear transmission line media

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ABSTRACT

In this study, we introduce the lossy nonlinear transmission line equation, which is the dissipative-dispersive equation and an important problem of electrical transmission lines. For the engineers and physicist, the equation and its exact solutions are important so to obtain the exact solutions; one of the modifications of auxiliary equation method based on Chebyshev differential equation is studied. The results are discussed and given in details. Recently, the studies of lossy transmission line equation have been challenging, thus, it is believed that the proposed solutions will be key part of further studies for waves in nonlinear transmission line media, which has mixed dissipative-dispersive behavior.

1. Introduction

Solitary waves are common subject among the engineers and physicians especially for who studies on elementary particle physics and electrical engineering. Solitons surviving collisions promise to be effective tools to deliver single waves, spreading skills without small scatter, modulated data at short distances in short distances with minimal loss.

Distributed electrical transmission lines composed of many identical parts were used to experimentally investigate the propagation of solitons conforming to the Korteweg-de-Vries (KdV) equation. The lines have been used in many areas such as for the regular behavior, the nonlinear transmission lines (NLTLs) is seen in electronics.

The solutions of NLTLs are investigated via mathematical models and physical experiments since 1970s. In the literature, there are many works to analyze either analytically [1, 2, 6] or numerically [3-5].

In this work, our aim is to get the exact solutions of the model of the lossy nonlinear transmission lines composed of small circuits. The inductance and capacitance are preceded from magnetic field effects and electric filed coupling between lines, respectively. The losses in the transmission lines depend on the series and shunt resistors. Corresponds to the transmission line losses constants are r and g, the circuit parameters are c and l determined for the voltage-dependent capacitance and the linear inductance, respectively.

Kengne [2] and Rosenau [7] give the similar models, which has mixed dispersive dissipative behavior. The difference between two models is depended on the capacitor's voltages determined as $c(V)=C_0\left(1-2bV\right)$ and $c(V)=C_0\left(1-V/V_0\right)^{\alpha}$, respectively. In addition to these capacitor's voltages, $c(V)=C_0\left(1+2\alpha+3\beta V^2\right)$ is determined by Tchier et al. [3] In the view of physical laws, the lossy NLTL model is given

$$l\frac{d}{dt}\left(c(V)\frac{dV_n}{dt}\right)+r\left(c(V_n)\frac{dV_n}{dt}+gV_n\right)+gl\frac{dV_n}{dt}=V_{n-1}-2V_n+V_{n+1} \quad (1)$$

where the right-side of the equation can be given approximately with the partial derivative of the distance x, and substituting the capacitor's voltages in Eq.(1), the nonlinear models are obtained. Respect to the capacitor's voltages the nonlinear models includes inductance, resistance, conductance and capacitance terms.

In our work, we will consider the model given by Kengne [4] includes the all terms

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$$V_{tt}-\frac{1}{C_0L}V_{xx}+\frac{RG}{C_0L}V=\frac{1}{12C_0L}\delta^2V_{xxxx} \qquad (2)$$
$$+b\left(V^2\right)_{tt}-\left(\frac{R}{L}+\frac{G}{C_0}\right)V_t+\frac{Rb}{L}\left(V^2\right)_t$$

where $L=\frac{l}{\delta}, R=\frac{r}{\delta}, G=\frac{g}{\delta}$ and C_0, b are corresponding to inductance, resistance, conductance and capacitance terms, respectively. When $R=0, G=0$, the Eq. (2) corresponds to an ideal transmission line and analytical solutions are given by Afsharia in [4]. Tchier et al. [1] considered the model includes inductance, resistance and capacitance terms, they obtained the analytical solution via Riccati-Bernoulli sub-ODE method and Lie symmetry reduction method. But for Eq. (2), i.e. the lossy nonlinear transmission line equation, analytical solutions could not been find so Kengne et al. [2] try to reduce Eq. (2) into integrable partial differential equation (PDE), which is solved analytically.

As it is seen that in this study, we will also investigate the exact solutions of Eq. (2) in the manner of the variant of auxiliary equation method. The auxiliary equation is generally a first-order ordinary differential equation (ODE) that its solutions are the special functions. In the literature, many methods are known as tanh-method [14-17], Jacobi method [18], Riccati expansion method [11], sub-equation method [8, 19], Bernoulli approximation method [13, 23] and auxiliary equation method [9, 10, 12]. In this study, we consider the auxiliary equation as a second-order ordinary differential equation known as Chebyshev differential equation. Therefore, our choice is correspond to our aim, to get exact solutions of the lossy transmission line equation.

The general methodology for the auxiliary equation method, first step is reducing a nonlinear PDE to a nonlinear ODE by the transformation $u(x,t)=u(\xi)$, $\xi=\left(x-\eta t\right)$.

$$u\left(\xi\right)=\sum_{i}^{N}c_iz^i\left(\xi\right) \qquad (3)$$

the finite series expansion is considered as the exact solution of the reduced equation where c_i are unknown constants to be determined later. Also, $z\left(\xi\right)$ is the exact solution of the proposed auxiliary equation.

The determination of the parameters c_i is done in three main steps:

- First is substituting the proposed auxiliary equation into the reduced equation.
- Second is equating each coefficient of power of $z\left(\xi\right)$ to zero.
- Third, solving the corresponding algebraic system to obtain the coefficients.

Also, the integer N, which indicates the number of terms will be used in Eq. (3), is determined basically by balancing the term with the highest order derivative and the term with the highest power nonlinearity in Eq. (2) [20].

It is also known that the function $z\left(\xi\right)$ is the exact solution of proposed auxilary equation. Since nonlinear PDEs cannot be recovered by only one auxiliary ordinary differential equation, there have been many studies utilizing different exactly solvable auxiliary equations.

In this work, we consider the Chebyshev equation,

$$(1-\zeta^2)z''(\zeta)-\zeta z'(\zeta)+n^2z(\zeta)=0 \qquad (4)$$

with the transformation $\omega=\cos\zeta$ reducing Eq. (4) to

$$z''(\omega)+n^2z(\omega)=0 \qquad (5)$$

which is considered as the auxiliary equation to solve the nonlinear partial differential equation and has a solution $T_n(\omega)$ known as Chebyshev function.

It is clear that determination of the elementary function $T_n(\omega)$ by auxiliary equation is essential and plays very important role finding new travelling wave solutions of nonlinear evolution equations. This fact forces the researchers to seek a new auxiliary equation with definite solutions.

The basic idea is that if elementary function, $T_n(\omega)$, is orthogonal function, which forms complete orthogonal sets in L^2 then, the solution series will be convergent series therefore the series (3) will converge rapidly [21, 22] .

In this study, we take aim at getting the exact solutions of the lossy nonlinear transmission line equation, see Eq. (2), which has mixed dispersive-dissipative behavior. To the best of our knowledge, this is the first attempt to consider the auxiliary equation as Chebyshev equation and investigating the exact solutions of the dispersive-dissipative equation. In the literature, as we mentioned above, the analytical solutions are investigated for ideal transmission line and the lossy nonlinear transmission line equation without conductance term [1, 2]. This equation will become one of the reference equations of paper and monograph in the literature, like our previous work on scattered-Fisher-type equations. [13, 23, 24].

2. The solutions

We represent the exact solutions of the lossy nonlinear transmission line equation by the given method above. Considering the transformation $u\left(x,t\right)=u\left(\xi\right)$, $\xi=\left(x-\eta t\right)$, Eq. (2) is reduced into ODE,

$$\eta^2 V'' - \frac{1}{C_0 L}V'' + \frac{RG}{C_0 L}V = \frac{1}{12C_0 L}\delta^2 V^{(4)} + 2b\eta^2\left(V'\right)^2 + 2b\eta^2 VV'' - \left(\frac{R}{L}+\frac{G}{C_0}\right)V_t + \frac{2Rb\eta}{L}VV' \tag{6}$$

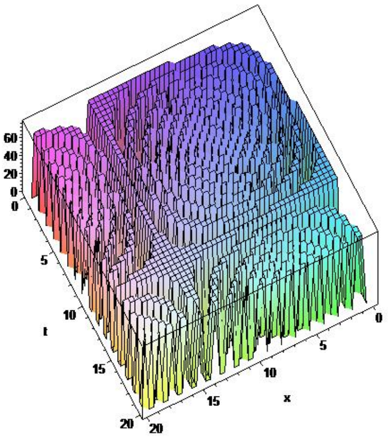
Applying the proposed steps and from the balancing principle $N=2$ is obtained. There are many solutions as a result of algebraic system of equations. But only two of them satisfy our conditions. Some of them give trivial solution, some of them give constant solution and some of them reduce the equation classical wave equation.

Case 1. The parameters for the solution is obtained as

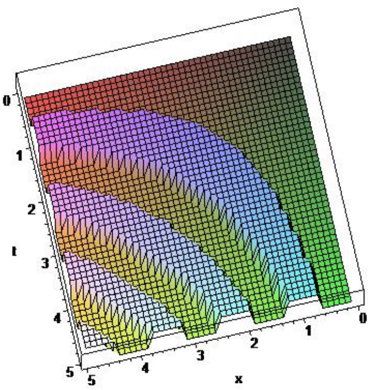
$$n=\frac{2\sqrt{21}}{7\delta}, C_2=0, L=84,$$

$$\eta=-\frac{R}{196\left(Rc_0+42G\right)}, \delta=\frac{4\sqrt{21}\cos\left(\frac{196xRc_0+8232xG-Rt}{196\left(Rc_0+42G\right)}\right)}{7\pi},$$

$$g_0=-\frac{1}{392b\eta c_0}, g_1=\pm 2Ig_2\sqrt{-\frac{g_0}{2g_2}}, g_2=\frac{I\left(Rc_0+42G\right)}{2C_1^2 bRc_0}.$$



(a) For long ranges.



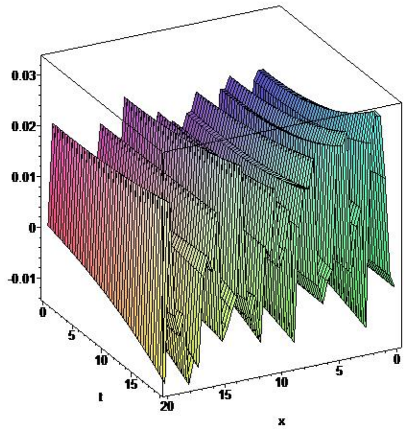
(b) For short ranges.

Figure 1. The behavior of Case 1 solution for $R=0.2, c_0=540, b=0.16, G=10^{-4}, C_1=\sin(xt)$

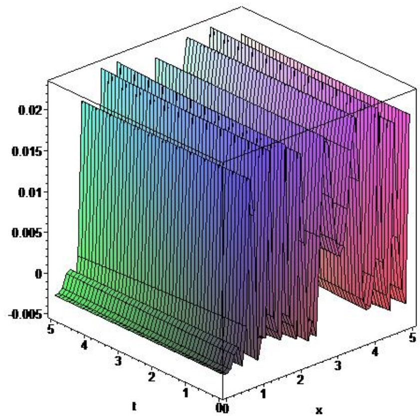
The exact solution is

$$V(x,t)=\frac{Rc_0+42G}{2bRc_0} - \frac{\left(Rc_0+42G\right)\sqrt{784\left(392IC_1^2\right)}\sin\left(\frac{\pi\cos\left(x-\frac{Rt}{196\left(Rc_0+42G\right)}\right)}{\cos\left(\frac{196xRc_0+8232xG-Rt}{196\left(Rc_0+42G\right)}\right)}\right)}{784bRc_0C_1} + \frac{I\left(Rc_0+42G\right)\sin\left(\frac{\pi\cos\left(x-\frac{Rt}{196\left(Rc_0+42G\right)}\right)}{\cos\left(\frac{196xRc_0+8232xG-Rt}{196\left(Rc_0+42G\right)}\right)}\right)^2}{2bRc_0}$$

For the special values of the parameters, the behavior of the solution is given by Figure 1.



(a) For long ranges.



(b) For short ranges

Figure 2. The solution for $R=0.1, L=63, \delta=0.1, c_0=540, b=0.16, G=10^{-4}, g_2=exp(-x)$

Case 2. The parameters for the solution is obtained as

$$n=\frac{4\sqrt{246}}{41\delta},C_2=0,g_0=-\frac{69}{574b\eta c_0\left(2\eta+49\right)},$$

$$g_1=\pm\frac{3\sqrt{42}}{7}I\sqrt{\frac{1}{82g_2b\eta^2c_0+2009g_2b\eta c_0}}g_2,g_2=g_2$$

$$\eta=\frac{1}{1722}\left(\left(\begin{array}{l}1842182685R\\-45955971432G-2781210948047Rc_0\\+861\sqrt{861}\left(\left(\begin{array}{l}1331000R^2\\-3021083975R^2c_0+1356688267340R^2c_0{}^2\\-265274306640GRc_0\\+3308829943104G^2c_0\\+400494376518768GRc_0{}^2\end{array}\right)\Big/c_0\right)^{1/2}\end{array}\right)R^2c_0{}^2\right)^{1/3}\Big/\left(R+c_0\right)$$

$$+\left(-55+\frac{689087}{6}c_0\right)R\Bigg/\left(\left(\begin{array}{l}1842182685R\\-45955971432G-2781210948047Rc_0\\+861\sqrt{861}\left(\left(\begin{array}{l}1331000R^2\\-3021083975R^2c_0+1356688267340R^2c_0{}^2\\-265274306640GRc_0\\+3308829943104G^2c_0\\+400494376518768GRc_0{}^2\end{array}\right)\Big/c_0\right)^{1/2}\end{array}\right)R^2c_0{}^2\right)^{1/3}$$

$$-\frac{49}{6}$$

For this case, the behavior of the solution is given by Figure 2.

3. Conclusion

The main idea of this study is based on obtaining the exact solutions of the lossy transmission line equation, which has mixed dispersive-dissipative behavior, by using the exact solutions of different type equations as an ansatz. To the best of our knowledge, this is the first attempt to consider the auxiliary equation as the Chebyshev equation and to investigate the exact solutions of the dispersed-dispersion equation. In the literature, as we mentioned above, the analytical solutions are only investigated for ideal transmission line and the lossy nonlinear transmission line equation without conductance term [1, 2]. Whereas to our knowledge, the analytical solutions of the considered equation, Eq. (2), is obtained for the first time in the literature.

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