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Optimization of Fleet Assignment: A Case Study in Turkey

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Abstract. Since poor fleet assignment can cause a great increase in costs for airline companies, a solution of the type 'right fleet for the right flight' would be very useful. In this paper, a fleet assignment model is set up using the data of the largest Airline Company in Turkey, Turkish Airlines. The aim of this model is to assign the most appropriate fleet type to flights while minimizing the cost and determining the optimal number of aircraft grounded overnight at each airport. We set up a model with constraints with thinking all airline operations and solve our problem using integer linear programming. Finally, we get an optimum solution which minimizes the total cost while assigning the fleet type to the flight leg. Using optimization software (Lindo 6.1), the solution to this problem generates a minimum daily cost of fleet assignment.

Keywords: Airline planning, Fleet assignment, Linear integer programming, Optimization.
AMS Classification: 90B80, 90C10

1. Introduction and Literature Review

In airline operations, schedule development involves many steps, including schedule design, fleet assignment, aircraft routing, and crew pairing [1]. In this study, we focus on fleet assignment, which is the assignment of available fleet to the scheduled flights.

The problem of fleet assignment is one of the hardest and most comprehensive problems faced in airline planning. Assigning fleet types to flight legs effectively is crucial in airline planning because the objective is to minimize cost to the airline.

Attempts to solve the fleet assignment problem have used various optimization methods. Belanger et al. [2] presented a mixed-integer linear programming formulation for the fleet assignment problem with homogeneity and showed that it is possible to produce very good quality solutions using a heuristic mixed-integer programming approach. Abara [3] formulated the solution to the fleet assignment problem as an integer linear programming model, permitting assignment of two or more fleets to a flight schedule simultaneously.

Belanger et al. [4] proposed a model for the periodic fleet assignment problem with time windows in which departure times are also determined. Anticipated profits depend on the schedule and the selection of aircraft types. A weekly fleet assignment model is presented by Kliewer and Tschöke [5]. They use a simulated annealing (SP) approach to deal with higher complexity. Chung and Chung [6] attempted to solve the fleet assignment problem using genetic algorithms.

In fleet assignment, profit is maximized by minimizing two types of costs: operational and spill costs [7]. Operational costs are those for flying the flight leg with the assigned aircraft type and usually include such things as fuel and landing fees. Spill costs represent lost opportunity costs that arise if passenger demand exceeds the aircraft capacity and, thus, potential revenue is lost [8].

In most fleet assignment models spill costs are

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leg-based. Barnhart et al. [9] developed a fleet assignment model using a branch-price approach. They proposed an Itinerary-Based fleet assignment model that is capable of capturing network effects and more accurately estimating spill and recapture of passengers.

The fleet assignment model is usually formulated for a typical day. For a regular schedule, the airline companies have to solve a more complicated weekly fleet assignment problem [10- 13]. In a daily fleet assignment, modifications for weekend flights have to be made in a separate step [14,15].

Another step of an airline scheduling process is aircraft routing. Aircraft routing and schedule models were the earliest Operations Research models of airline planning [16]. In the literature, there are several studies about aircraft routing [14,17-21]. In these studies, authors developed new models, proposed several heuristics and exact approaches, and also integrated aircraft routing with other airline planning problems.

With optimization theory, algorithms, and computational hardware, researchers were able to solve more complex problems and develop approaches to integrate sub-problems in the solution [22]. These complex problems and integrated sub-problems are the subjects of several studies. Different integrated problems were studied by several authors. Haouari et al. [23] addressed an integrated aircraft fleeting and routing problem. They developed an optimization-based two-phase heuristic algorithm that requires iteratively solving minimum-cost flow problems. Papadakos [24] introduces an integrated airline scheduling model, and its size is reduced by applying Benders decomposition combined with column generation. The integrated approach significantly reduces airline costs, and the chosen formulation proves to be better than alternative integrations attempted. Desaulniers et al. [25] integrated the fleet assignment problem with aircraft routing while giving legs' departure-time the flexibility to be within a time-window. Furthermore Barnhart et al. [26] integrated fleet assignment with maintenance routing.

Bazargan introduced operating costs in the fleet assignment model, with passenger-spill costs, recapture rate, flight cover, etc. An optimum solution is found in Bazargan's study [27].

In this paper, we discuss the fleet assignment problem, one of the most important problems with which airline companies must deal. We use the same model engaged by Bazargan but with real data of Turkish Airlines. The main contribution of this paper to the literature will be the ability it provides to see cost reduction, and to optimize fleets by using real case data.

An easier solution to this problem is ensured using optimization software, Lindo 6.1. A fleet assignment model was set up using the data of the largest Airline Company in Turkey, Turkish Airlines, with linear integer programming. Firstly, the objective function for the Turkish Airlines model was set up with selecting our binary and integer decision variables. Then operation costs and passenger spill costs were calculated for each fleet type. These values were calculated for each flight in the Turkish Airlines flight schedule according to expected demand and standard deviation for flights considering recapture rate. After the objective function was determined, the fleet assignment model was set up with respect to flight cover, aircraft balance and fleet size constraints. After all these calculations were made, our fleet assignment model was constructed. In our case, the hub selected is Istanbul Ataturk Airport and the spokes are Antalya, Izmir, Ankara, Adana, Trabzon, Erzurum, Gaziantep and Hatay, as seen on Figure 1.

Figure 1. Hub and spokes of our case

The paper is organized as follows. In Section 2, we present fleet assignment. In Section 3, we present the general mathematical model for the fleet assignment problem. In Section 4, we set up the objective function of the mathematical model for the fleet assignment problem. Indicator definitions, decision variables, operating costs, passenger-spill costs, recapture rate are also explained, while the three main sets of constraints- flight cover, aircraft balance, fleet size -are discussed in the fleet assignment model. Finally, we make an application for a fleet assignment model with the Turkish Airlines case. In Section 5, we present some conclusions.

2. Fleet Assignment

The fleet assignment is the first phase of the second step of an airline scheduling process as shown in Figure 2. The aim of fleet assignment is to match most appropriate fleet type to flights while minimizing the cost. It should be noted that this planning concerns only fleet type, not a particular aircraft.

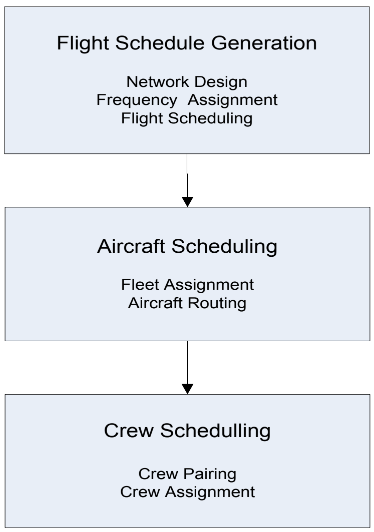

Figure 2. Airline scheduling process [16]

Figure 2 shows the process of airline schedule development and the hierarchy of planning phases of airline scheduling. The goal of fleet assignment is to assign as many flight segments as possible in a schedule to one or more fleet types, while optimizing some objective function and meeting various operational constraints [27].

3. Fleet Assignment Model (FAM)

We now present the general mathematical model for the fleet assignment problem. The following model [27], referred to as the basic fleet assignment model (FAM), is a simplified version of FAM proposed by Hane et al. 1995 [28].

Sets

F = Set of flights
K = Set of fleet types
C = Set of last-nodes, representing all nodes with aircraft grounded overnight at an airport in the network
M = Number of nodes in the network

Index

i = Flight Index
j = Index for fleet
k = Index for nodes

Parameters

$C_{i,j}$ = Cost of assigning fleet type j to flight i
N_j = Number of available aircraft in fleet ty pe j
$S_{i,k}$ = +1 if flight i is an arrival at node k, -1 if flight i is a departure from node k

Decision Variables

$x_{i,j} = \{$ 1 if flight is assigned to fleet-type j, 0 otherwise

$G_{k,j}$ = integer decision variable representing number of aircraft of fleet-type on ground at node k

The integer linear programming model is as follows:

$$\min \sum_{j\in K}\sum_{i\in F} c_{i,j}x_{i,j} \tag{1}$$

Subject to

$$\sum_{j\in K} x_{i,j} = 1 \qquad \forall i \in F \tag{2}$$

$$G_{k-1,j} + \sum_{i\in F} S_{i,k}x_{i,j} = G_{k,j} \quad \forall k \in M \text{ and } \forall j \in K \tag{3}$$

$$\sum_{k\in C} G_{k,j} \le N_j \quad \forall j \in K \tag{4}$$

$$x_{i,j} \in \{0,1\} \quad \forall i \in F \text{ and } \forall j \in K \tag{5}$$

$$G_{k,j} \in Z^+ \quad \forall k \in M \text{ and } \forall j \in K \tag{6}$$

In the above model, the objective function in (1) seeks to minimize the total cost of assigning the various fleet types to all the flights in the schedule. Constraints (2) are the flight-cover constraints to ensure that each flight is flown by one type of fleet. Constraints (3) are the aircraft balance constraints. The number of aircraft for any fleet type at any node is the number of aircraft of that fleet type just before that node (represented in the model by $G_{k-1,j}$) plus the arrivals (represented by $S_{i,k}$ taking a value +1) minus the departures (represented by $S_{i,k}$ taking a value of -1). Set of constraint (4) represents the fleet size. The number of aircraft in fleet type j, should not exceed the available number of aircraft in that fleet (N_j). Constraints (5) and (6) represent the binary and integer status of the decision variables. Z^+ is the set of positive integer numbers [27].

4. An Application in Turkish Airlines

In this paper, we study a fleet assignment problem which is set up using Turkish Airlines data with linear integer programming. In our case, there are 25 aircraft for A320 fleet type, 21 aircraft for A321 fleet type, 14 aircraft for B737 fleet type and 52 aircraft for B738.

The complete flight schedule route, incorporating the 46 flights per day, is presented in Table 1. It is assumed that demand for each flight is normally distributed with given means and standard deviations as seen on Table 1. Also Table 1 presents the demand distribution for each flight as well as distances between cities. The mean of demand and standard deviation are taken from the historical data for 2 years.

Indicator Definitions

Before addressing the mathematical model for the fleet assignment problem, some terms commonly used in the airline industry must be explained [27]:

ASM (ASK): Available Seat Miles (Kilometers) represents the annual airline capacity, or supply of seats, and refers to the number of seats available for passengers during the year multiplied by the number of miles (kilometers) that those seats are flown.

RASM (RASK): Revenue per Available Seat Mile (Kilometer), or ‘unit revenue’ represents how much an airline made across all the available seats that were supplied. RASM (RASK) is calculated by dividing the total operating revenue by available seat mile (kilometer) or ASM (ASK).

CASM (CASK): Cost per Available Seat Mile (Kilometer) or ‘unit cost’ is the average cost of flying one seat for a mile (kilometer). CASM (CASK) is calculated by dividing the total operating cost by ASM (ASK).

Operating Costs

The operating costs for a flight mainly depend on the type of the fleet assigned to that flight and are determined as follows [27]:

Operating costs of a flight = CASM of the fleet × distance × number of seats on the aircraft

We have four fleet types, namely A320, A321, B737 and B738. The seating capacities for these four fleet types are 159, 192, 142, and 165 seats, respectively. Furthermore, we have the following information for the airline under consideration:

Cost per available seat mile (CASM) for A320, A321, B737 and B738 are \$0.046 (4.6 cents), \$0.048 (4.8 cents), \$0.045 (4.5 cents) and \$0.047 (4.7 cents), respectively.

Revenue per available seat mile (RASM) is \$0.20 (20 cents).

Using the above information we can determine the operating cost for each flight in the Turkish Airlines schedule for the four fleet types. Let us take as an example flight TK2109 (Ankara-Istanbul), where the distance flown is 227 miles (see Table 1). The operating costs of this flight for the four fleet types are calculated as seen on Table 2.

Table 1. Flight Schedule, destination in miles, demand means and standard deviations for Turkish Airlines network

Flight no.	Origin	Departure time	Destination	Arrival time	Demand	Standard deviation	Distance (miles)
TK2109	Ankara	06:15	Istanbul	07:20	157	31	227
TK2839	Trabzon	07:00	Istanbul	08:50	207	42	572
TK2113	Ankara	07:30	Istanbul	08:35	147	29	227
TK2220	Istanbul	07:35	Gaziantep	09:10	113	21	542
TK2407	Antalya	08:25	Istanbul	09:40	190	38	300
TK2458	Istanbul	08:30	Adana	10:00	141	28	443
TK2123	Ankara	09:00	Istanbul	10:05	157	31	227
TK2134	Istanbul	09:00	Ankara	10:05	145	29	227
TK2127	Ankara	10:00	Istanbul	11:05	159	32	227
TK2221	Gaziantep	10:00	Istanbul	11:45	175	35	542
TK2412	Istanbul	10:00	Antalya	11:15	126	25	300
TK2845	Istanbul	10:00	Trabzon	11:50	113	21	572
TK7420	Istanbul	10:45	Erzurum	12:25	120	24	653
TK2320	Istanbul	11:00	Izmir	12:05	129	25	204
TK2317	Izmir	11:00	Istanbul	12:05	143	29	204
TK2253	Hatay	11:05	Istanbul	12:50	159	32	513
TK2414	Istanbul	11:30	Antalya	12:45	152	30	300
TK2150	Istanbul	13:00	Ankara	14:05	107	21	227
TK7421	Erzurum	13:00	Istanbul	14:40	138	27	653
TK2416	Antalya	13:20	Istanbul	14:45	172	34	300
TK2462	Istanbul	14:15	Adana	15:45	121	24	443
TK2151	Ankara	15:00	Istanbul	16:05	186	36	227
TK2158	Istanbul	15:00	Ankara	16:05	156	30	227
TK2325	Istanbul	15:00	Izmir	16:05	128	27	204
TK2254	Istanbul	15:30	Hatay	17:15	150	30	513
TK2162	Istanbul	16:00	Ankara	17:05	188	37	227
TK2463	Adana	16:45	Istanbul	18:25	150	29	443
TK2470	Istanbul	17:00	Adana	18:30	138	27	443
TK2327	Izmir	17:00	Istanbul	18:05	182	35	204
TK2224	Istanbul	17:30	Gaziantep	19:05	147	29	542
TK2255	Hatay	18:00	Istanbul	19:45	159	32	513
TK2170	Istanbul	18:00	Ankara	19:05	126	25	227
TK2467	Adana	18:25	Istanbul	20:05	124	25	443
TK2418	Istanbul	18:30	Antalya	17:45	130	25	300
TK2167	Ankara	19:00	Istanbul	20:05	136	27	227
TK7422	Istanbul	19:00	Erzurum	20:40	131	26	653
TK2422	Istanbul	19:10	Antalya	20:25	118	24	300

TK2471	Adana	19:15	Istanbul	20:55	125	25	443
TK2420	Antalya	19:20	Istanbul	20:35	117	23	300
TK2847	Trabzon	19:55	Istanbul	21:45	176	35	572
TK2182	Istanbul	21:00	Ankara	22:05	110	21	227
TK2225	Gaziantep	21:00	Istanbul	22:45	141	28	542
TK2257	Istanbul	21:00	Hatay	22:45	140	27	513
TK7423	Erzurum	21:00	Istanbul	22:40	133	26	653
TK2430	Antalya	21:10	Istanbul	22:25	131	25	300
TK2850	Istanbul	22:00	Trabzon	23:50	136	27	572

Table 2. The operating costs of flight TK2109 for the four fleet types

Fleet Type	Operating Cost
A320	$1,660.28
A321	$2,092.03
B737	$1,450.53
B738	$1,760.39

Similarly, we can determine the operating costs for all other flights for the four fleet types.

Passenger-Spill Costs

An important issue in assigning fleet types to flights is the passenger demand for each flight segment. Assigning large capacity aircraft to flights with low demand leads to low utilization and consequently low load-factor for the airline. On the other hand, assigning small aircraft to flight legs with high demand leads to passenger spills. Spill is the degree of average demand, which exceeds the capacity offered. The spill cost is therefore the revenue of lost passengers due to insufficient aircraft capacity.

The expected spill costs are determined as follows:

Expected spill cost for a fleet = expected number of passenger spill × RASM × distance

The expected number of passenger spill is calculated as follows [27]:

Expected number of passenger spill=

$$\int_c^{\infty}(x-c)\,f(x)dx\,.$$

In the above equation, c is the fleet capacity and $f(x)$ is the probability distribution function of the demand. The above integral can be obtained using mathematical software or some calculators. It is possible and perhaps easier to use a MS Excel spreadsheet to approximate the above expected number of passenger spill using simulation.

Consider flight TK2109 (Ankara-Istanbul) in our Turkish Airlines case study. Our historical data for flight TK2109 shows that the demand for this flight is normally distributed with a mean of 157 and a standard deviation of 31 passengers (see Table 1). Figure 3 shows the demand distribution for this flight.

The shaded areas show the probability of passenger spills for the four fleet types. The spill is basically the truncation of the demand distribution beyond the aircraft capacity.

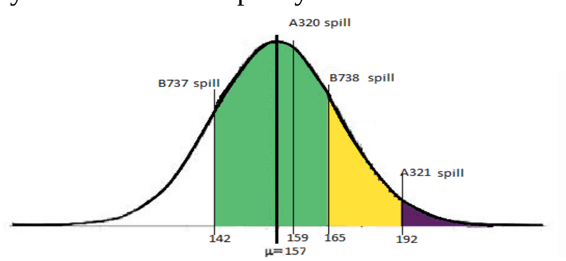

Figure 3. Demand distribution and passenger spills

By using the MS Excel functions, the expected number of spilled passengers for an A320 fleet type with 159 seats can be determined with the demand of the flight leg. If the simulated demand exceeds the capacity of the aircraft, their difference is found (i.e., passenger spill), otherwise passenger spill is zero. This simulation is repeated 1000 times, and the average is calculated as the expected number of spilled passengers.

Using this method, the expected numbers of passenger spill and spill costs for the four fleet types for flight TK2109 is calculated as seen on Table 3.

Table 3. Expected passenger spill and spill costs

Fleet Type	Seat Capacity	Expected Passenger Spill	Expected Spill Costs
A320	159	11.75	$533.45
A321	192	2.32	$105.33
B737	142	20.96	$951.58
B738	165	8.23	$373.64

We can similarly determine the expected numbers of passenger spill and the expected spill costs for all other flights for the four fleet types.

It may seem that this model attempts to assign larger capacity fleet type to all flights since expected shortages are penalized. It should be noted that the larger capacity fleet type was already penalized when we calculated the operating costs above.

Recapture Rate

A closely related topic to passenger spill is the recapture rate. The recapture rate represents the percentage of passengers that were spilled, but could be accommodated or recaptured on other flights by the same airline. That is, if a passenger cannot get a seat on a specific flight, the airline offers earlier or later flights (in some cases with bonuses) to the passenger for consideration. If the passenger accepts the offer for another flight, then this passenger is considered to be recaptured. The recapture rate among the major airlines is typically very high. This is due to high flight frequencies offered by these airlines as well as other marketing incentives such as frequent-flyer programs.

Expected spill costs for fleet types considering recapture rate is calculated as [27]:

Expected spill costs = Expected spill cost × (1-recapture rate)

In our case study, owing to low flight frequencies the recapture rate is low. Let us assume that this rate is 15% on Turkish Airlines. This rate means that 85% of passengers, who request a reservation for a flight on this airline and are denied such a request, are lost to other airlines.

The expected spill costs considering recapture rate for the four fleet types for flight TK2109 are calculated as Table 4:

Table 4. Expected spill costs

Fleet Type	**Expected Spill Costs** (considering recapture rate)
A320	$453.43
A321	$89.53
B737	$808.85
B738	$317.60

Similarly, we can determine the expected spill costs for all other flights for four fleet types.

Total Cost

We find total cost of assigning a fleet type to a flight leg by adding the operating and spill costs [27].

Total Cost = Operating Costs + Passenger-Spill Costs

Now we determine the total cost of assigning a fleet type to a flight leg by adding the operating and spill costs. The total cost for each fleet assigned to flight TK2109 is seen in Table 5:

Table 5. Total costs

Fleet Type	Total Cost
A320	$2,113.71
A321	$2,181.56
B737	$2,259.38
B738	$2,077.98

Similarly, we can determine the total costs for all other flights for four fleet types.

Objective Function

To setup the objective function for Turkish Airlines, we need to first select our decision variables in a way that addresses the assignment of the fleet type to the flight leg. The following decision variables are commonly adopted for fleet assignment models [27].

$$x_{i,j} = \begin{cases} 1, & \text{if flight is assigned to fleet} - \text{type } j \\ 0, & \text{otherwise} \end{cases}$$

$G_{k,j}$ integer decision variable representing number of aircraft of fleet type j on ground at node k

In the binary decision variable $x_{i,j}$, index i represents the flight leg ($i=2109, 2839, \ldots, 2850$), while index j represents the fleet type ($j=1, 2, 3, 4$). The fleet types for indexes are; *A320*, *A321*, *B737* and *B738*. Decision variable $G_{k,j}$ will be used to address the set of constraints for aircraft balance.

The objective function is basically to minimize the total cost by assigning the most appropriate fleet type to flights as follows:

minimize

$$2113.71x_{2109,1} + 2181.56x_{2109,2} + 2259.38x_{2109,3}$$
$$+2077.98x_{2109,4} + 9168.13x_{2839,1} + 7866.89x_{2839,2}$$
$$10139.04x_{2839,3} + 8881.67x_{2839,4} + ... + 4507.42x_{2850,1}$$
$$+5280.30x_{2850,2} + 4433.00x_{2850,3} + 4669.24x_{2850,4}$$

Constraints

There are three main sets of constraints in the fleet assignment model. They are discussed as follows [27]:

Flight Cover

The first set of constraints is what is typically known as flight cover. Flight cover implies that each flight must be flown. To cover a flight, the sum of all the decision variables that represent that flight must add up to 1.

As an example, to cover flight TK2109 in our Turkish Airlines case study, we write:

$$x_{2109,1} + x_{2109,2} + x_{2109,3} + x_{2109,4} = 1$$

This constraint ensures that flight TK2109 is covered. Furthermore, the flight will be covered by only one type of fleet since the sum of binary decision variables adds up to 1. Only one of the four binary decision variables in this constraint will take a value of 1, forcing the other variable to be zero. We write similar constraints for all other 45 flights in our case study.

Aircraft Balance

The next set of constraints concerns the aircraft balance or equipment continuity within the fleets. This set of constraints ensures that an aircraft of the right fleet type will be available at the right place at the right time. According to the concept of a time-space network in Figure5, we adopt this concept to address this set of constraints. Each node represents an arrival or departure. Recall that each node represents a specific time at a specific airport. So, the number of aircraft at any node changes with respect to an instant before that node [27].

We could phrase our formula as: *Number of aircraft of a particular fleet type on the ground at a node = Number of aircraft in that fleet on the ground an instant before that node + arrival of aircraft of the same fleet type at that node – (minus) departures of aircraft of the same fleet type from that node*.

For example, the balance constraint for the node in Figure 4 is:

Number of aircraft at this node = 2 (number of aircraft before this node) + 1 (one arrival) – 0 (no departure from this node) = 3

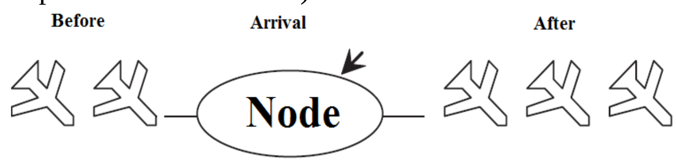

Figure 4. Example of aircraft balance [27]

Adopting this approach, we can now write the constraints for balancing each airport in our Turkish Airlines case study. Let us consider Adana. The flights in and out of Adana (extracted from our flight schedule) are as shown in Table 6.

Table 6. Arrival/departure flights for Adana

Flight no.	Origin	Dep. Time	Destination	Arrival Time
TK2458	Istanbul	08:30	Adana	10:00
TK2462	Istanbul	14:15	Adana	15:45
TK2463	Adana	16:45	Istanbul	18:25
TK2467	Adana	18:25	Istanbul	20:05
TK2470	Istanbul	17:00	Adana	18:30
TK2471	Adana	19:15	Istanbul	20:55

Figure 5 presents this table as a time-space network. We use the decision variable $G_{k,j}$ to write the constraints for balancing each fleet type. The index k, represents nodes, while index j represents the fleet type [27].

Let us first consider the A320 fleet type. Based on Figure 5, the first node at Adana is at A1. (The other nodes are represented as; Ankara as B, Antalya as C, Erzurum as M, Gaziantep as D, Hatay as E, Istanbul as F, Izmir as G and Trabzon as H). The number of A320 aircraft at this node, based on the rule for balancing, is basically the number of aircraft carried over from the previous day (wrap-around arc from node A6) plus one arrival (flight TK2458), so:

$$G_{A1,1} = G_{A6,1} + x_{2458,1} \quad (7)$$

At node A2 (see Figure 5), we have another arrival (flight TK2462) so:

$$G_{A2,1} = G_{A1,1} + x_{2462,1} \quad (8)$$

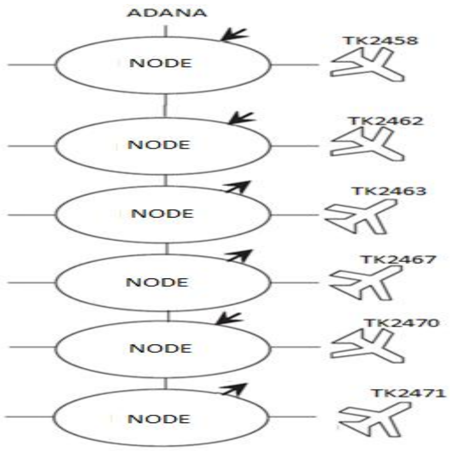

Figure 5. Time-space network for Adana

At node A3, we have a departure (flight TK2463), therefore:

$$G_{A3,1} = G_{A2,1} - x_{2463,1} \tag{9}$$

Similarly, we write the other three constraints for this fleet type as follows:

$$G_{A4,1} = G_{A3,1} - x_{2467,1} \tag{10}$$

$$G_{A5,1} = G_{A4,1} + x_{2470,1} \tag{11}$$

$$G_{A6,1} = G_{A5,1} - x_{2471,1} \tag{12}$$

We can also write the balance constraints for all other airports in the schedule. There are 46 flights in our Turkish Airlines case study where each flight has a departure and an arrival. Therefore, the total number of constraints for aircraft balance is 352.

For Istanbul, there are some flights that depart or arrive at the same time. For example, there are 2 departures at 10:00, and $G_{F8,1}$ represents the number of aircraft after both of 2 departures from Istanbul. There are 2 arrivals at 20:05 and G_{F341}, represents the number of aircraft after both of 2 arrivals to Istanbul. Therefore the number of aircraft balance constraints that have a different arrival/departure time slot for Istanbul will be 42, instead of 46.

Fleet Size

This set of constraints is adopted to ensure that the number of aircraft within each fleet does not exceed the available fleet size. To address this, we must count the number of aircraft that are grounded overnight for that fleet type at different airports [27].

Referring to Figure 5, the last node, A6 (originating node for wraparound arc), represents the total number of aircraft in Adana at the end of the day. For this airport, $G_{A6,1}$ represents the total number of grounded A320 aircraft in Adana overnight. The total number of A320 aircraft in our network (for Turkey) is therefore:

$$G_{A6,1} + G_{B12,1} + G_{C8,1} + G_{M4,1} + G_{D4,1} + G_{E4,1} + G_{F42,1} + G_{G4,1} + G_{H4,1}$$

In the above expression, the integer variables represent the number of aircraft at the last nodes at Adana, Ankara, Antalya, Erzurum, Gaziantep, Hatay, Istanbul, Izmir, and Trabzon respectively. Note that at Istanbul, we have 46 daily flights arriving at or departing from this airport. Therefore, the last node is represented as $F42$. Similarly, the total number of A321 aircraft in our network is:

$$G_{A6,2} + G_{B12,2} + G_{C8,2} + G_{M4,2} + G_{D4,2} + G_{E4,2} + G_{F42,2} + G_{G4,2} + G_{H4,2}$$

We write similar constraints for all other fleet type in our case study.

$$G_{A6,3} + G_{B12,3} + G_{C8,3} + G_{M4,3} + G_{D4,3} + G_{E4,3} + G_{F42,3} + G_{G4,3} + G_{H4,3}$$

$$G_{A6,4} + G_{B12,4} + G_{C8,4} + G_{M4,4} + G_{D4,4} + G_{E4,4} + G_{F42,4} + G_{G4,4} + G_{H4,4}$$

In our case study, we have 25, 21, 14 and 52 aircraft in our A320, A321, B737 and B738 fleets, respectively. We can now add these constraints into our model as follows:

$$G_{A6,1} + G_{B12,1} + G_{C8,1} + G_{M4,1} + G_{D4,1} + G_{E4,1} + G_{F42,1} + G_{G4,1} + G_{H4,1} \leq 25 \tag{13}$$

$$G_{A6,2} + G_{B12,2} + G_{C8,2} + G_{M4,2} + G_{D4,2} + G_{E4,2} + G_{F42,2} + G_{G4,2} + G_{H4,2} \leq 21 \tag{14}$$

$$G_{A6,3} + G_{B12,3} + G_{C8,3} + G_{M4,3} + G_{D4,3} + G_{E4,3} + G_{F42,3} + G_{G4,3} + G_{H4,3} \leq 14 \tag{15}$$

$$G_{A6,4} + G_{B12,4} + G_{C8,4} + G_{M4,4} + G_{D4,4} + G_{E4,4} + G_{F42,4} + G_{G4,4} + G_{H4,4} \leq 52 \tag{16}$$

Solution to Fleet Assignment Problem

The integer linear program for fleet assignment for Turkish Airlines has 536 (184 binary and 352 integer) variables and 402 constraints. By using optimization software, the solution to this problem generates a minimum daily cost of fleet assignment of $151,311.8. However, prices are not certain because of firm politics; the firm does not share the certain prices. The following table shows the number of aircraft for each fleet type staying overnight at each airport. Other aircraft which are not shown in Table 7 will be located in Istanbul during the night, because of parking, nightly maintenance, and the probability that they will be required at other destinations. These numbers represent the right number of aircraft for each fleet type at the right airport at the right time.

Table 7. Optimal number of aircraft grounded overnight at each airport

Airports	A320 Fleet	A321 Fleet	B737 Fleet	B738 Fleet
Adana	-	-	-	-
Ankara	1	1	2	1
Antalya	1	1	-	-
Erzurum	-	-	-	-
Gaziantep	-	1	-	-
Hatay	-	-	-	1
Istanbul	2	-	6	2
Izmir	-	-	1	-
Trabzon	-	1	-	-

Table 8 presents the assignment of each flight to either one of the four fleet types. Note that the above solution only shows the assignment of flights to fleet type.

Table 8. Fleet assignment for Turkish Airlines

Flight no.	Origin	Destination	Fleet type	Flight no.	Origin	Destination	Fleet type
TK2109	Ankara	Istanbul	B738	TK2325	Istanbul	Izmir	B737
TK2839	Trabzon	Istanbul	A321	TK2254	Istanbul	Hatay	B738
TK2113	Ankara	Istanbul	B737	TK2162	Istanbul	Ankara	A321
TK2220	Istanbul	Gaziantep	B737	TK2463	Adana	Istanbul	A320
TK2407	Antalya	Istanbul	A321	TK2470	Istanbul	Adana	B737
TK2458	Istanbul	Adana	A320	TK2327	Izmir	Istanbul	B738
TK2123	Ankara	Istanbul	A320	TK2224	Istanbul	Gaziantep	A321
TK2134	Istanbul	Ankara	B738	TK2255	Hatay	Istanbul	B738
TK2127	Ankara	Istanbul	B737	TK2170	Istanbul	Ankara	B737
TK2221	Gaziantep	Istanbul	A321	TK2467	Adana	Istanbul	B737
TK2412	Istanbul	Antalya	B737	TK2418	Istanbul	Antalya	A320
TK2845	Istanbul	Trabzon	A320	TK2167	Ankara	Istanbul	B737
TK7420	Istanbul	Erzurum	B737	TK7422	Istanbul	Erzurum	B737
TK2320	Istanbul	Izmir	B738	TK2422	Istanbul	Antalya	B737
TK2317	Izmir	Istanbul	B737	TK2471	Adana	Istanbul	B737
TK2253	Hatay	Istanbul	A320	TK2420	Antalya	Istanbul	B737
TK2414	Istanbul	Antalya	A321	TK2847	Trabzon	Istanbul	A320
TK2150	Istanbul	Ankara	B737	TK2182	Istanbul	Ankara	B737
TK7421	Erzurum	Istanbul	B737	TK2225	Gaziantep	Istanbul	B737
TK2416	Antalya	Istanbul	A320	TK2257	Istanbul	Hatay	A320
TK2462	Istanbul	Adana	B737	TK7423	Erzurum	Istanbul	B737
TK2151	Ankara	Istanbul	A321	TK2430	Antalya	Istanbul	B737
TK2158	Istanbul	Ankara	A320	TK2850	Istanbul	Trabzon	A321

5. Conclusion and Further Researches

Air transportation plays a supplementary role in our life. It represents the fastest way to ship over long distances, and people prefer air transportation for vacations, business trips, and almost any travelling needs. Consequently, airline planning has become very important.

Airlines companies face with hard and comprehensive problems as, fleet assignment, airline scheduling, crew scheduling, etc. Operating costs and passenger-spill costs are the highest costs for airline companies. Assigning fleet types to flight legs effectively is crucial in airline planning.

In this paper we set up a model for efficient fleet assignment, and studied a real-world case study for the largest Airline Company in Turkey, Turkish Airlines. This model was coded and solved with optimization software, using linear integer programming. The aim was minimizing the cost, and the benefits of this study are explained below:

- The determination of a Fleet Type Assigned to a Specific Flight. The importance of this is to minimize the assignment cost while assigning the right aircraft to the right flight. The solution to this problem generates a minimum daily cost of fleet assignment of $151,311.8.
- Optimizing the Number of Aircraft that will be Grounded Overnight at each Airport. To impede any negative situation while assigning fleets during a time horizon, it is very important to know at least how many aircraft must be on the ground during the night in all of the airports.

As a further research project, we plan to work with a weekly or monthly schedule in the same way we studied daily assignment in this study. We also plan to define route planning and crew scheduling problems and plan to integrate it with this model. Using this model and integrating variables from other problems with it, we can develop new solution algorithms; ones that will be more appropriate for real world cases and help solve airline problems more easily and quickly.

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