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RESEARCH ARTICLE

An application of the new function method to the Zhiber–Shabat equation

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ABSTRACT

This paper applies a new approach including the trial equation $u' = f\left(e^u\right)$ based on the exponential function in order to find new traveling wave solutions to Zhiber-Shabat equation. By the using of this method, we obtain a new elliptic integral function solution. Also, this solution can be converted into Jacobi elliptic functions solution by a simple transformation .

1. Introduction

Achieving solutions of nonlinear partial differential equations has great importance. Because these solutions are related to real life problems. Thus, many methods are constructed and applied for these problems. Some of them can be respectively given as the trial equation method [1], the extended trial equation method [2], the new function methods [3-5], Kudryashovs method [6], and so on. In this study, we apply New function method to solve a nonlinear physical problem and find new interactions among traveling wave solutions. Also, the advantages of this approach are listed by this study. In Section 2, the stages of New Function method and also the functional traveling wave transformation $\mu = x - ct$ where μ is a particular solution of the known nonlinear differential equation. In Section 3 New Function method is applied to the Zhiber-Shabat equation and the new exact wave solution to this problem is obtained. The two and three dimensional graphics of the solutions were created with the help of the package program. There is a short discussion and conclusion in last section. Finally, we give some basic properties of eliptic integral function and several Jacobi elliptic functions such as sn and ns. We applies a new function approach based on the exponential function e^u for solving the Zhiber-Shabat equation [7,8]

$$u_{xt} + pe^{u} + qe^{-u} + re^{-2u} = 0 \tag{1}$$

The Zhiber-Shabat equations have importance in the fields of integrable quantum field theory, kink dynamics, and fluid dynamics.

2. New function method

The new function methods have been developed by using the exponential function. In this study, we apply a new version of new function method by depending on the exponential function. Firstly, we take the general form of the Zhiber-Shabat equation[7,8]

$$P\left(u, e^{u}, e^{-u}, e^{-2u}, u_{xt}\right) = 0, \tag{2}$$

and then use the wave transformation $u = u(x,t) = u(\mu) = u(x - ct)$, $\mu = x - ct$ where $c \neq 0$. Thus, we get a nonlinear ordinary differential equation

$$N\left(u, u', u'', \ldots\right) = 0. \tag{3}$$

This method assumes that the function u provides the following relation

$$F\left(u''\right) = G\left(g\left(u\right)\right), \tag{4}$$

where F, G and g are any functions. Also we use the following equations, respectively

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$$u' = f(g(u)), \tag{5}$$

$$u'' = f(g(u))g'(u)f'(g(u)). \tag{6}$$

Substituting Eq. (5)-(6) into Eq. (4), we have

$$F\left(f(g(u))g'(u)f'(g(u))\right) = G(g(u)). \tag{7}$$

If we take $\psi = g(u)$, then we can write

$$F\left(\psi' f(\psi) f'(\psi)\right) = G(\psi). \tag{8}$$

Solving Eq. (8), is sometimes a variable separated ordinary differential equation, yields the function f. By integration, we can find the solutions as follows:

$$\frac{du}{f(g(u))} = d\mu \Rightarrow \int \frac{du}{f(g(u))} = \int d\mu = \mu + P, \tag{9}$$

where P is an integration constant. The explicit solutions can be derived by the inverse function. Otherwise, the implicit solutions can be retrieved if the above integration is much complex.

3. Solutions for the generalized zhiber-shabat equation

By the travelling wave transformation to Eq. (1), we have

$$u_{xt} + pe^{u} + qe^{-u} + re^{-2u} = 0. \tag{10}$$

We assume that the equation

$$u' = f(e^{u}), \tag{11}$$

defined by $u'(\mu)$ and e^{u} satisfies Eq. (10). From Eq. (11), we can compute

$$u'' = e^{u} f(e^{u}) f'(e^{u}). \tag{12}$$

By substituting Eq. (12) into Eq. (10), we derive

$$cf(e^{u}) f'(e^{u}) = p + qe^{-2u} + re^{-3u}. \tag{13}$$

Let u' is a function of e^{u} and $e^{u} = \psi$, then f is a function of ψ. Therefore we can easily write

$$u = \ln(\psi) \Rightarrow u' = \frac{\psi'}{\psi} = f(e^{u}) = f(\psi). \tag{14}$$

Now, we can try to have the form of the function f:

$$cf(\psi) f'(\psi) = p + q\psi^{-2} + r\psi^{-3}. \tag{15}$$

Eq. (15) is an ordinary differential equation of variable separated:

$$\frac{c}{2} f^{2}(\psi) = p\psi - q\psi^{-1} - \frac{r}{2}\psi^{-2} + k, \tag{16}$$

where k is a constant of integration. From Eq. (16), we can easily compute

$$f(\psi) = \pm\sqrt{\frac{1}{c}\left(2p\psi - \frac{2q}{\psi} - \frac{r}{\psi^{2}} + 2k\right)}. \tag{17}$$

Using the equation $\frac{\psi'}{\psi} = f(\psi)$, we have

$$\psi' = \frac{d\psi}{d\mu} = \pm f(\psi) = \pm\sqrt{\frac{1}{c}\left(2p\psi^{3} + 2k\psi^{2} - 2q\psi - r\right)}. \tag{18}$$

By using of the symbolic computation software program Mathematica, Eq. (18) that is a variable separated ordinary differential equation is solved. So, the following elliptic integral function F solution to Eq. (1) is obtained as

$$\eta + Q = -\frac{\sqrt{2}EllipticF[ArcSin[\frac{\sqrt{\psi_1+\psi_2}}{\sqrt{\psi-\psi_1}}], \frac{\psi_1-\psi_3}{\psi_1-\psi_2}]}{\sqrt{-\psi_1+\psi_2}\,cp}, \tag{19}$$

where Q is a constant of integration. $\psi_i \ (i=1,\cdots,3)$ are roots of equation $\left(2p\psi^{3} + 2k\psi^{2} - 2q\psi - r\right) = 0$

$$\psi_1 = \frac{8(2)^{1/3}k^2 + 24(2)^{1/3}pq - 4kd + (2)^{2/3}d^2}{12pd},$$

$$\psi_2 = -\left(\frac{8(-2)^{1/3}k^2 + 24(-2)^{1/3}pq + 4kd -}{12pd} - \frac{(-2)^{2/3}d^2}{12pd}\right),$$

$$\psi_3 = -\left(\frac{8k + \frac{8(2)^{1/3}(1-i\sqrt{3})(k^2+3pq)}{d}}{24p} + \frac{(2)^{2/3}(1+i\sqrt{3})d}{24p}\right), \tag{20}$$

where $\left(d = (-16k^3 - 72kpq + 108p^2r + \sqrt{4(-4k^2-12pq)^3 + (-16k^3 - 72kpq + 108p^2r)^2})^{1/3}\right)$.

Then, we find

$$\psi = JacobiNS[\varphi, \frac{\psi_1-\psi_3}{\psi_1-\psi_2}]^2 (-\psi_1 + JacobiSN[\varphi, \frac{\psi_1-\psi_3}{\psi_1-\psi_2}]^2\psi_1 + \psi_2). \tag{21}$$

Replace ψ with e^{u}, η with $\eta = x - ct$ in (21), and then the explicit solutions for Eq. (1) can be obtained as follows:

$$u = \ln\left[ns^2[\varphi,\ell^2](-\psi_1 + sn^2[\varphi,\ell^2]\psi_1 + \psi_2)\right], \tag{22}$$

Where $\varphi=\sqrt{\frac{cp}{2}}\sqrt{-\psi_1+\psi_2}\left(x-ct+Q\right)$ and $\ell^2=\frac{\psi_1-\psi_3}{\psi_1-\psi_2}$.

4. 2D and 3D graphics of solution

4.1. 2D graphic of solution

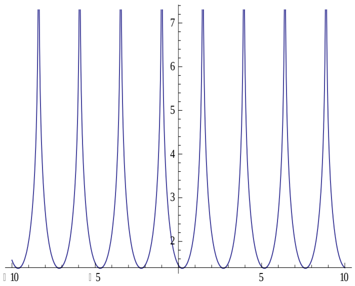

Figure 1. 2D graphic represents the solution (22) at $t=1$.

4.2. 3D graphic of solution

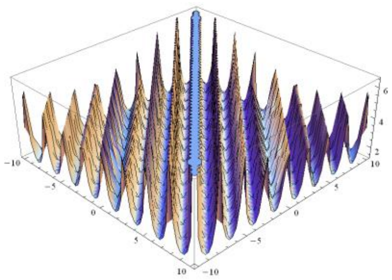

Figure 2. The solution (21) is shown at, $p=-1$, $r=0$, $d=\sqrt[6]{6912}$, $Q=0$, $c=-1$, and $k=0$.

5. Conclusion

We consider the Zhiber-Shabat equation to obtain new exact traveling wave solutions by using a new function method. Thus, we have a new elliptic integral function solution. Using simple mathematical operations, we can transform the elliptic integral function solution to the Jacobi elliptic function *sn and ns*. The obtained results show that the new function technique is very effective and suitable mathematical tool for solving the nonlinear partial differential equations.

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