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Sales plan generation problem on TV broadcasting

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Abstract. Major advertisers and/or advertisement agencies purchase hundreds of slots during a given broadcast period. Deterministic optimization approaches have been well developed for the problem of meeting client requests. The challenging task for the academic research currently is to address optimization problem under uncertainty. This paper is concerned with the sales plan generation problem when the audience levels of advertisement slots are random variables with known probability distributions. There are several constraints the TV networks must meet including client budget, product category and demographic information, plan weighting by week, program mix requirements, and the lengths of advertisement slots desired by the client. We formulate the problem as a chance constrained goal program and we demonstrate that it provides a robust solution with a user specified level of reliability.

Keywords: Chance constrained goal programming; media planning; scheduling.

AMS Classification: 90C29, 90C90, 90B36

1. Introduction

The Broadcasting companies make most of their revenues from selling impressions through advertisement space during various programs or shows. U.S. ad spending was USD 168.8 billion in 2008, where it is approximately USD 500 billion worldwide. Of this, television accounts for approximately 50%, and TV advertising is still the biggest player followed by the internet which has a 30% share of total spend. Advertisers are ready to pay up to approximately half a million dollars for a 20 or 30 second advertisement in a popular show [1].

In North America and several European countries, most of the advertising space is sold before the broadcast season which is also called "upfront market" --occurring for a couple of weeks in May--, following the announcement of program schedules and prices for the following year. During upfront market period, major advertisers and companies request from the TV networks to purchase time for the entire season. A typical request consists of the dollar amount, the demographic in which the client is interested, the program mix, weekly weighting, unit-length distribution, and a negotiated cost per 1,000 viewers. Broadcast Companies must develop a detailed sales plan consisting of the schedule of advertisements to be aired to meet the requirements and they have to pay a penalty when it is unable to meet its commitment for client requests. This happens when a broadcast company sells a large amount of rating points during the upfront market and its shows turn out to be misses or when the broadcast company cannot meet the weekly demand or demand per show. In addition, the plan should also meet the objectives of the company, whose goal is to minimize the penalties or to maximize the revenues for the available fixed amount of inventory.

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The objective of this paper is to prepare a sales plan that meets all the client requirements after clients' sales requests have been received. This plan is modeled to include a complete schedule of advertisements that will be aired for each client and target audience levels requested by these clients. Audience uncertainty has not been taken into account in this problem of media planning, to the best of our knowledge. Due to penalty rates that Broadcast Companies accept to pay to the clients in case of not meeting their specific constraints, we try to model sales plan generation problem using chance constrained programming to minimize the penalty costs under audience uncertainty.

This paper is organized as follows. The next section provides a review of the related literature. We give a brief description of goal programming, chance constrained programming and represent our model in Section 3. In Section 4, we present computational studies that complement our analytical findings. We then describe some managerial implications and give some future directions in Section 5.

2. Literature review

There are several studies dealing with scheduling programs for the television networks to optimize some specified criteria which are usually audience ratings. The placement strategy called "lead in" is widely used to schedule programs which use the strength of the preceding program to boost the ratings of a newly introduced program. A comprehensive review of these models was provided by Rust [2]. Models and heuristic methods for scheduling programs were developed by Horen [3], Rust and Echambadi [4], and Reddy et al. [5]. Simon [6], Mahajan and Muller [7] have studied on strategies for scheduling advertisements. These studies are concerned with whether advertising programs should be steady or turned on and off. A review of these models is collected by Lilien et al. [8].

There are also several studies about advertising allocation problem concerning the distribution of available budget to different media channels. Mihiotis and Tsakiris [9] considered advertisement allocation problem by using mathematical programming. In their problem the best possible combination of placements of an advertisement is asked including the channel, time, and frequency with the objective of the highest rating. The clients have a budget limitation for advertising. They used integer programming to solve the model. Cetin and Esen [10] have studied media allocation problem giving a good example of military operations research models that can be adapted to contemporary business world applications. They modelled the problem as a weapon-target model and solved it using integer nonlinear programming. Saha et al. [11] have also studied media allocation problem. They applied a linear time algorithm that finds a solution to the 'maximum weight 1 colouring' problem for an interval graph with interval weight. To solve the problem that involves selecting different program slots they telecast on different television channels in a day so as to reach the maximum number of viewers.

Despite its richness and complexity, the problem of scheduling advertisements on broadcast television or sales plan generation problem has received very little attention in the literature. A math-programming-based algorithm to rapidly generate near-optimal sales plans that meet advertiser requirements have been developed by Bollapragada et al. [12] where a sales plan consists of a complete schedule of advertisements to be aired for an advertiser during a broadcast year to meet its requirements. Bollapragada et al. [13] have also developed an algorithm to schedule client videotapes in the advertisement slots they purchased to meet certain client specific objectives. Araman and Popescu [14] developed stylized stochastic optimization models of airtime inventory planning and allocation across multiple clients under audience uncertainty. They devised a simple procedure for accepting upfront client contracts and estimating their overall inventory requirements.

In this paper, we formulate the problem as a chance constrained goal program to satisfy the client needs by a specified service level and we demonstrate that it provides a strong solution with a user specified level of reliability. The problem that is discussed in this paper has not been addressed in the literature to the best of our knowledge.

3. Problem definition

The problem of Broadcasting Companies is to prepare sales plans to meet clients' requirements. During upfront market, clients declare their special requests by using standard forms supplied by Broadcasting Companies (Figure 1). There are three most important requests of clients to be satisfied. The first one is the target audience level. Clients generally appraise the audience level in terms of number of people that they want to reach. But audience levels per show per week are unknown. Broadcasting Companies have to use estimated data derived from past data by the media rating agencies to prepare a future plan for client specific requests. Second one is the weekly distribution. Because of seasonal factors, special days of the year such as Mother's Day or St. Valentine's day or the special weeks, clients want their advertisements to be aired in these specific weeks because of the demographic features of their target customers. The request form includes the distribution of the advertisements over various weeks. These distribution criteria may be specified as a fraction of the total number of equivalent length advertisements or sometimes as a percentage of the total amount of expenditure or number of people achieved in the plan. The third important request is to satisfy the distribution of advertisements by shows. Clients generally specify the shows in the program schedule that they want their advertisements to be aired in. This is why their target customers are known to be potential viewers of these shows. Clients normally specify these requirements as fractions of the total number of equivalent length advertisements.

Prime Time Plan Request

Client		Date	
Agency		Contact	
Product		Phone	
Demographic		Email	
Date Range			

Week	%	Week	%	Week	%	Week	%
1		14	3.75	27	3.75	40	
2		15		28	1.88	41	
3		16	3.75	29	1.88	42	1.88
4		17	0.94	30	3.75	43	1.88
5	3.75	18	0.94	31	3.75	44	3.75
6	1.88	19	1.88	32	1.88	45	3.75
7	1.88	20		33	1.88	46	
8	3.75	21		34	3.75	47	3.75
9	0.94	22	1.88	35	2.5	48	
10	0.94	23	3.75	36		49	
11	1.88	24	1.88	37	1.88	50	1.88
12	3.75	25	1.88	38	3.75	51	3.75
13	1.88	26		39	3.75	52	3.75

Show	Number	Show	Number	Show	Number	Show	Number
1	3	5		7		11	4
2	7	6		8	5	12	
3		7	10	9	5	13	12
4		8		10		14	10

Comments:

Figure 1. Sample client request form

For instance, a client prepares a request form as in Figure 1. They request that 3.75% of all their advertisements to be aired in week 5 and 1.88% of all advertisements in week 6, etc.

In addition to this, 3 of their advertisements should be aired in show 1 and 7 of them should be aired in show 2, etc. They also define their target audiences in demographic part of the form. Then the broadcasting company tries to optimize the clients' requests.

To use minimum advertisement inventory to meet the client requests is also important for Broadcasting Companies. The inventory that is not used during upfront market period will be sold during the following broadcast period which is also called scatter market. The scatter market sales prices per second of advertisement inventory are generally higher than the upfront market sale prices. Broadcasting Companies have also an offset option where they may use scatter market inventory rather than to pay penalty in terms of dollars.

The problem of sales departments is to prepare a sales plan that meets all the client requirements after clients' sales requests are received. Broadcasting Companies legally accept to pay penalties in case of not meeting the client requests before a client request an advertisement slot. A sales plan generally includes a complete schedule of advertisements to be aired, target audience levels, and legal terms and options to cut back or expand the plan. We try to model a sales plan to satisfy client requirements, to minimize the penalty costs incurred not meeting the client requests, and to minimize the amount of premium inventory assigned to a plan.

4. Model

We use chance constrained goal programming in our model. The goal programming (GP) model is one of the well-known multi-objective mathematical programming models. This model allows to take into account simultaneously several objectives in a problem for choosing the most satisfactory solution within a set of feasible solutions. More precisely, the GP designed to find a solution that minimizes the deviations between the achievement level of the objectives and the goals set for them. In the case where the goal is surpassed, the deviation will be positive and in the case of the underachievement of the goal, the deviation will be negative. First developed by Charnes and Cooper [17] and Charnes et al. [18] then applied by Lee [19] and Lee and Clayton [20], the GP model gained a great deal of popularity and its use has spread in diversified field such as management of water basins, management of solid waste, accounting and financial aspect of stock management, marketing, quality control, human resources, production, transportation and site selection, space studies, telecommunications, agriculture and forestry and aviation. The goal or aspiration levels assigned to the various objectives can be probabilistic where the decision maker does not know its value with complete certainty. Several techniques have been proposed to solve the Stochastic GP model. But the most popular technique is a chance constrained programming developed by Charnes and Cooper [15, 16, 22, 23]. Belaid et al. [21] have exploited the concept of the satisfaction function to explicitly integrate the decision maker's preferences in the stochastic goal programming model.

In our study, we have also many objectives to be satisfied and one of the constraints related to meeting target audience level is defined as probabilistically. That is, a client may say that their target audience level is satisfied by 95% for example. Therefore, we use chance constrained goal programming that the model is given in the next section.

4.1. Model formulation

Our problem under audience uncertainty is described as a chance constrained goal programming model. We assume that the lengths of all advertisements for all clients are same.

The parameters used in the model are as follows:

A_{swk} : number of audiences of break *k* in show *s* that is aired in week *w*; which has a probability distribution function.

σ_{swk} : standard deviation of audiences of break k in show s that is aired in week w.

β_{si} : rate of ads demanded per show per week for client i.

α_{wi} : rate of ads demanded per week for client i.

S : set of slots in week w.

Z : set of slots in show s.

C : set of competitor companies conflicted.

g_i : target number of audience to be reached for client i.

n_i : total slots purchased by client i.

U_k : number of available slots in break *k*.

γ_i : service level of meeting target audience demand of client i.

π_1^+, π_1^-: penalty rate of not meeting and exceeding target audience demand, respectively.

π_2^+, π_2^-: penalty rate of not meeting and exceeding audience per week demand.

π_3^+, π_3^-: penalty rate of not meeting and exceeding audience per show demand.

Decision variables of the model are:

Y_{ijswk} : Binary variable (1, if *j*th advertisement of client *i* is aired in the *k*th break of show *s* in week *w*; 0, otherwise).

NG_i, PG_i : negative and positive deviation from target number of audience of client *i*.

NW_i, PW_i : negative and positive deviation from target number of weekly aired ads of client *i*.

NS_i, PS_i: negative and positive deviation from target number of ads per show of client *i*.

The mathematical formulation of the problem is as follows:

min $$\pi_1^+ \sum_i PG_i + \pi_1^- \sum_i NG_i + \pi_2^+ \sum_i PW_i + \pi_2^- \sum_i NW_i + \pi_3^+ \sum_i PS_i + \pi_3^- \sum_i NS_i \qquad (13)$$

subject to $$P(g_i \geq \sum_j \sum_s \sum_w \sum_k Y_{ijswk}.A_{swk}) \geq \gamma_i \qquad \forall i \qquad (14)^1$$

$$\alpha_{wi}.n_i - \sum_j \sum_s \sum_k Y_{ijswk} = PW_i - NW_i \qquad \forall i, w \qquad (15)$$

$$\beta_{si}.n_i - \sum_j \sum_w \sum_k Y_{ijswk} = PS_i - NS_i \qquad \forall i, s \qquad (16)$$

$$\sum_j Y_{ijswk} \leq 1 \qquad \forall i, s, w, k \qquad (17)$$

$$\sum_i \sum_j Y_{ijswk} \leq U_k \qquad \forall s, w, k \qquad (18)$$

$$\sum_j \sum_s \sum_w \sum_k Y_{ijswk} = n_i \qquad \forall i \qquad (19)$$

$$\sum_j \sum_i Y_{ijswk} \leq 1 \qquad \forall s, w, k \qquad (20)$$

$$Y_{ijswk} = 0,1 \qquad \forall i, j, s, w, k \qquad (21)$$

$$NG_i, PG_i, NW_i, PW_i, NS_i, PS_i \geq 0 \qquad \forall i \qquad (22)$$

The objective function of our model (13) is to minimize the total penalty incurred in meeting requirements. The first two terms in the objective function is the penalty incurred in not meeting and exceeding the total audience demand respectively. Third and fourth terms are the penalty incurred in not meeting and exceeding the number of ads per week demand, respectively, and the last two terms represent the penalty arising from not meeting and exceeding the number of ads per show demand. Since these objectives are wanted to be satisfied definitely, the negative and positive deviations corresponding to each constraint are available in the objective function.

Constraints (14), (15) and (16) are the goal constraints in the model. Moreover constraint (14) is a chance goal constraint and ensures that the target number of audience should be satisfied by a pre-specified service level and therefore the probability that the difference between the target number of audience to be reached for company *i* (g_i) and the total expected number of audience reached by scheduling the advertisements $(Y(.) \times A(.))$ will be greater than the probability level γ_1 specified by the client. To define constraint (14) as a goal constraint, it should be written in equality form. An important pre-requisite for this model is to understand the structure of past audience metrics. Audience means the gross sum of the number of impressions watching a given show [13]. Media rating agencies have realized data to be used by Broadcasting Companies. In our model, A_{swk}, the number of audience in break *k* of show *s* that is aired in week *w*, is a random variable and normally distributed. Constraint (14) was given as below:

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[1] This inequality will be replaced by (25)

$$P(g_i \geq \sum\sum\sum\sum Y_{ijswk} A_{swk}) \geq \gamma_i \qquad (14)$$

When we arrange (14), we get the following inequalities. $\Phi(.)$ denotes the cumulative distribution function of a standard normal random variable.

$$\Phi\left(\frac{g_i - \sum_j\sum_s\sum_w\sum_k Y_{ijswk} A_{swk}}{\sum_j\sum_s\sum_w\sum_k Y_{ijswk}\sigma_{swk}}\right) \leq 1-\gamma_i \qquad (23)$$

$$\frac{g_i - \sum_j\sum_s\sum_w\sum_k Y_{ijswk} A_{swk}}{\sum_j\sum_s\sum_w\sum_k Y_{ijswk}\sigma_{swk}} \leq \Phi^{-1}(1-\gamma_i) \qquad (24)$$

Then (24) can be converted to the goal constraint easily and (25) is obtained. $\Phi^{-1}(1-\gamma_i)$ denotes the standard value of $(1-\gamma_i)$. Therefore, (25) should be replaced with (14) to solve the model. Furthermore, deviation terms PG_i and NG_i were added to the objective function (13) because of this reason.

$$g_i - \sum_j\sum_s\sum_w\sum_k Y_{ijswk} A_{swk} - \Phi^{-1}(1-\gamma_i)\sum_j\sum_s\sum_w\sum_k Y_{ijswk}\sigma_{swk} = PG_i - NG_i \qquad \forall i, \qquad (25)$$

Constraint (15) ensures that the number of advertisements of a company that are planned to be aired in a specific week should be equal to the total scheduled number of advertisements that will be aired in aforementioned week for each client *i*. Constraint (16) ensures that there is no difference between the number of advertisements of a company that are planned to be aired in a specific show and the total scheduled number of advertisements that will be aired in aforementioned show for each client *i*.

The remaining constraints are the functional constraints in the model. Constraint (17) ensures that an advertisement can be placed only one which means that it can only be assigned to one position in one break. Constraint (18) ensures that the number of advertisements scheduled cannot exceed the available number of slots in a break. Constraint (19) ensures that the total number of advertisements assigned to a break must be equal to the total number of advertisements bought by all clients. Constraint (20) ensures that the number of advertisements assigned to a break from clients that has conflicts must be less than or equal to 1, so conflicting companies' advertisements will not be aired in the same break. Decision variable Y is a binary variable and all other variables are greater than or equal to zero.

5. Computational study

We define two hypothetical examples in which we try to observe if the model is relevant in meeting client requirements. We assume that penalties for all clients are constant. Penalty cost of not meeting the audience demand is 2.50 TRY, and it is 2000 TRY per show. Moreover penalty cost of not meeting weekly demand is 1500 TRY, penalty of exceeding the audience demand is 2.45 TRY, it is 1900 TRY per show and the penalty of exceeding weekly demand is 1400 TRY. We used GAMS high-level modeling system for mathematical programming and optimization problems to solve these hypothetical examples.

5.1. 2 clients, 2 weeks, 7 shows, 3 breaks case

In this case we suppose there are 2 clients. The broadcast period is 2 weeks and there are 7 shows to be aired in each week. Only one show is aired in a day and there are 3 advertisement breaks in each show. We observe two subcases in this case: one of them is that the clients are non-conflicting and the other is they are conflicting.

2 non-conflicting clients, 2 weeks, 7 shows, breaks case

We assume that clients are non-conflicting with each other. Therefore, constraint (20) is removed from the model. Client 1 requests 15,000 audiences by using 15 advertisements, and wants 8 of its ads to be aired in week 1 and 4 of its ads to be aired in show 1. Client 2 request 13,500 audiences by using 10 advertisements, and 8 of its ads to be aired in week 2 and 7 of its ads to be aired in show 6. These 2 clients are not conflicting so any 2 advertisements of these clients can be aired in same break.

We suppose that Broadcast Company have 2000 potential viewers, and we know that the number of these potential viewers who see the advertisements on each break is normally distributed. The expected numbers of viewers which watch the advertisements in break *k* of show *s* in week *w* and variances of them are known.

The clients wanted their target audience level is satisfied by 99%. According to the GAMS results, the advertisements of each company were placed in ad slots given in Table 1.

Table 1. Advertisement plan for each company

Company	Ad	Show	Week	Break	Company	Ad	Show	Week	Break
1	1	6	2	1	**2**	1	1	2	3
1	2	2	2	3	**2**	2	6	1	3
1	3	3	2	2	**2**	3	6	2	3
1	4	3	2	1	**2**	4	6	2	1
1	5	3	1	2	**2**	5	5	2	3
1	6	7	1	2	**2**	6	3	2	3
1	7	2	1	1	**2**	7	6	1	2
1	8	6	1	1	**2**	8	4	2	3
1	9	1	2	3	**2**	9	6	2	2
1	10	5	2	3	**2**	10	7	2	3
1	11	7	1	3					
1	12	4	1	1					
1	13	1	1	3					
1	14	1	1	2					
1	15	1	2	1					

Due to the restrictions the number of viewers that watch all advertisements of client 1 is 14,790 although 15,000 viewers were contracted, and the number of viewers that watch all advertisements of client 2 is 14,661 although 13,500 viewers were contracted.

On the other hand, the week and show constraints are met for client 1. For client 1, 8 advertisements are aired in week 1 where 8 of 15 advertisements were contracted and 4 advertisements are aired in show 1 where 4 of 15 advertisements were contracted. Week constraint is also met for client 2. 8 advertisements are aired in week 1 where 8 of 10 advertisements were contracted for client 2. Show constraint is not exactly met for second client because 5 advertisements are aired in show 6 where 7 of 15 advertisements were contracted.

Because the 2 clients are non-conflicting for this example, the advertisements of client 1 and client 2 are both aired in the slots (6, 2, 1), (1, 2, 3), (5, 2, 3) corresponding to the (show, week, break). Then the minimized cost of the deviations from goals is calculated as 9323.384 TRY.

2 conflicting clients, 2 weeks, 7 shows, 3 breaks case

Suppose the clients in this example are conflicting clients. They don't want that any of their advertisements to be aired in the same break of any show in any week. All other constraints are same with the first case. According to the GAMS results, the advertisements of each company were placed in ad slots given in Table

Table 2. Placement of the advertisements for each company

Company	Ad	Show	Week	Break	Company	Ad	Show	Week	Break
1	1	1	1	2	2	1	5	2	3
1	2	4	1	3	2	2	4	2	3
1	3	7	2	2	2	3	7	2	3
1	4	5	1	3	2	4	6	1	3
1	5	2	2	2	2	5	6	2	3
1	6	1	1	3	2	6	6	2	2
1	7	4	2	2	2	7	1	2	3
1	8	7	1	3	2	8	6	2	1
1	9	2	2	3	2	9	6	1	2
1	10	2	1	3	2	10	3	2	3
1	11	1	2	2					
1	12	1	1	1					
1	13	2	2	1					
1	14	3	1	3					
1	15	5	2	2					

Due to the restrictions the number of viewers that watch all advertisements of client 1 is 15,080 although 15,000 viewers were contracted, and the number of viewers that watch all advertisements of client 2 is 14,661 although 13,500 viewers were contracted.

The week and show constraints are satisfied for client 1. 8 advertisements are aired in week 1. Week constraint is met for client 2 although show constraint is not exactly met for second client because 5 advertisements are aired in show 6 where 7 of 15 advertisements were contracted. There are also no conflicting advertisements according to the results so the conflicting constraint is also satisfied in this example. Then the minimized cost of the deviations from goals is calculated as 9102.972 TRY.

5.2. 10 clients, 13 weeks, 10 shows, 7 breaks case

In this case, we try to model a problem much closer to the real life. In real life, there are also many conflicting companies and to satisfy their constraints together is a very hard and complex issue. Suppose we have 10 clients with some conflict constraints. Client 1 and Client 2 do not want their advertisements to be aired in the same break. Client 5 and Client 6 are also conflicting and want their advertisements not to be aired in the same breaks. Client 7 is conflicting with Client 1 and Client 2, and Client 5 and Client 6, respectively.

Client 1 requests 10 advertisements to be aired in week 1 while Client requests 12 advertisements. Both Client 5 and Client 6 want 12 of their advertisements to be aired separately in week 7 and week 10. The demand of Client 7 for week 1, week 7 and week 10 are 8, 12 and 12, respectively. Client 3 also requests 6 advertisements to be aired in week 4.

Furthermore, Client 1 requests 8 of its advertisements to be aired in show 1 while Client 2 wants 10 of its advertisements. In addition to this, Client 5 and Client 6 informs that the number of advertisements that will be aired in show 5 as 10 and 12 respectively. The demand of Client 7 is 12 for show 1 and 12 for show 5. Client 7 also restricted its plan by informing that none of its advertisements to be aired in show 6. Finally, Client 10 demands for 10 advertisements to be aired in show 2.

The target and realized data for number of advertisements and audience levels per client are given in Table 3.

Table 3. Results for number of advertisements and audience level

Client	Target		Realized	
	No.of Ads	Audience Level	No.of Ads	Audience Level
1	20	20,000	20	19,901
2	25	25,000	25	25,005
3	6	8000	6	7284
4	10	13,000	10	14,303
5	35	40,000	35	39,296
6	35	40,000	35	38,773
7	40	50,000	40	49,388
8	15	15,000	15	21,591
9	15	15,000	15	21,280
10	10	8000	10	9715

The total number of viewers who watch the Client 1 advertisements are 19,901 although 20,000 viewers were contracted, 25,005 viewers for Client 2 although 25,000 viewers were contracted, and 49,388 viewers for Client 7 although 50,000 viewers were contracted. They are not much different from the target values as seen in Table 3. All conflict constraints are met, so that there are no breaks that are used for 2 different clients. The week and show constraints related with Client 1 and Client 2 are exactly satisfied. Only two show constraints of Client 7 are met. In show 5, 12 advertisements are aired and in show 6 no ads are aired. The other show and week constraints related to Client 7are not met. For example, 5 advertisements are aired in show 1 but it should have been 12. Moreover, only one advertisement is aired in week 1 but it should have been 8.

The parameters in parenthesis shown in Table 4 are related to company, advertisement, show, week and break indexes, respectively. For example, the first row for Client 1 shows that advertisement 1 of Client 1 was planned to be aired in break 5 of show 3 in week 1.

Table 4. Advertisement plans of the Client 1, Client 2, and Client 7

Client 1	Client 2	Client 7
(1 , 1 , 3 , 1 , 5)	(2 , 1 , 1 , 1 , 6)	(7 , 1 , 8 , 7 , 5)
(1 , 2 , 1 , 1 , 2)	(2 , 2 , 1 , 3 , 4)	(7 , 2 , 1 , 12 , 5)
(1 , 3 , 1 , 1 , 1)	(2 , 3 , 6 , 1 , 5)	(7 , 3 , 4 , 7 , 5)
(1 , 4 , 1 , 13 , 6)	(2 , 4 , 8 , 1 , 5)	(7 , 4 , 5 , 12 , 5)
(1 , 5 , 5 , 1 , 2)	(2 , 5 , 7 , 1 , 2)	(7 , 5 , 7 , 7 , 5)
(1 , 6 , 9 , 10 , 2)	(2 , 6 , 3 , 1 , 2)	(7 , 6 , 7 , 13 , 5)
(1 , 7 , 5 , 1 , 5)	(2 , 7 , 1 , 13 , 5)	(7 , 7 , 10 , 1 , 5)
(1 , 8 , 10 , 3 , 5)	(2 , 8 , 3 , 13 , 5)	(7 , 8 , 8 , 4 , 5)
(1 , 9 , 3 , 1 , 3)	(2 , 9 , 6 , 1 , 2)	(7 , 9 , 10 , 7 , 5)
(1 , 10 , 4 , 1 , 2)	(2 , 10 , 1 , 1 , 5)	(7 , 10 , 5 , 10 , 3)
(1 , 11 , 1 , 10 , 6)	(2 , 11 , 4 , 9 , 5)	(7 , 11 , 7 , 2 , 5)
(1 , 12 , 1 , 1 , 4)	(2 , 12 , 7 , 3 , 2)	(7 , 12 , 1 , 9 , 5)
(1 , 13 , 1 , 5 , 4)	(2 , 13 , 3 , 7 , 5)	(7 , 13 , 2 , 2 , 5)
(1 , 14 , 1 , 1 , 3)	(2 , 14 , 10 , 1 , 2)	(7 , 14 , 3 , 8 , 5)
(1 , 15 , 5 , 7 , 5)	(2 , 15 , 1 , 13 , 4)	(7 , 15 , 5 , 13 , 5)
(1 , 16 , 6 , 9 , 5)	(2 , 16 , 1 , 9 , 2)	(7 , 16 , 5 , 3 , 6)
(1 , 17 , 8 , 1 , 2)	(2 , 17 , 4 , 1 , 5)	(7 , 17 , 1 , 6 , 5)
(1 , 18 , 1 , 7 , 3)	(2 , 18 , 1 , 1 , 7)	(7 , 18 , 8 , 13 , 5)
(1 , 19 , 4 , 13 , 5)	(2 , 19 , 1 , 3 , 2)	(7 , 19 , 3 , 3 , 5)
(1 , 20 , 2 , 8 , 5)	(2 , 20 , 10 , 1 , 6)	(7 , 20 , 8 , 2 , 5)
	(2 , 21 , 8 , 6 , 5)	(7 , 21 , 9 , 5 , 5)
	(2 , 22 , 1 , 2 , 6)	(7 , 22 , 1 , 5 , 5)
	(2 , 23 , 9 , 1 , 2)	(7 , 23 , 10 , 8 , 5)
	(2 , 24 , 1 , 12 , 6)	(7 , 24 , 4 , 4 , 5)
	(2 , 25 , 5 , 2 , 5)	(7 , 25 , 5 , 10 , 6)
		(7 , 26 , 5 , 5 , 5)
		(7 , 27 , 5 , 4 , 2)
		(7 , 28 , 1 , 10 , 2)
		(7 , 29 , 7 , 3 , 5)
		(7 , 30 , 5 , 11 , 5)
		(7 , 31 , 1 , 3 , 5)
		(7 , 32 , 5 , 3 , 5)
		(7 , 33 , 4 , 10 , 5)
		(7 , 34 , 9 , 11 , 5)
		(7 , 35 , 7 , 11 , 5)
		(7 , 36 , 5 , 5 , 2)
		(7 , 37 , 5 , 6 , 5)
		(7 , 38 , 5 , 4 , 5)
		(7 , 39 , 10 , 10 , 5)
		(7 , 40 , 2 , 7 , 4)

Table 3 and Table 5 give the realized data and scheduled slots for Client 5, Client 6, and Client 7. The total number of viewers related to Client 5, Client 6, and Client 7 are 39,296, 38,773 and 49,388 respectively. No single breaks are used for 2 different clients, so all conflict constraints are met. Client 5 have 8 ads and 8 of its advertisements been aired in week 7 and week 10 respectively when 12 of their advertisements to be separately aired. 8 advertisements in week 7 and 8 advertisements in week 10 of Client 6 are aired, when Client 6 requests 12 of their advertisements to be separately aired. The demands of Client 5 and Client 6 for show 5 are 10 and 12, respectively. Show constraints are met for both clients.

In Table 6, the scheduled slots for Client 3, Client 4, Client 8, Client 9, and Client 10 are given. For Client 3, all the advertisements are contracted to be aired in week 4, therefore week request is met. Client 10 has a show constraint. They request that all of their advertisements to be aired in show 2 and this constraint is also met. Remaining constraints are not related to show or week constraints. The realized number of advertisements aired and the audience level reached are given in Table 3.

Table 5. Advertisement plans of the Client 5, Client 6, and Client 7

Client 5	Client 6	Client 7
(5, 1, 5, 1, 5)	(6, 1, 5, 1, 2)	(7, 1, 8, 7, 5)
(5, 2, 10, 12, 5)	(6, 2, 6, 3, 5)	(7, 2, 1, 12, 5)
(5, 3, 5, 8, 5)	(6, 3, 4, 7, 2)	(7, 3, 4, 7, 5)
(5, 4, 5, 2, 2)	(6, 4, 10, 7, 2)	(7, 4, 5, 12, 5)
(5, 5, 5, 7, 4)	(6, 5, 10, 9, 5)	(7, 5, 7, 7, 5)
(5, 6, 5, 10, 1)	(6, 6, 5, 7, 5)	(7, 6, 7, 13, 5)
(5, 7, 4, 9, 5)	(6, 7, 2, 7, 5)	(7, 7, 10, 1, 5)
(5, 8, 6, 9, 5)	(6, 8, 6, 10, 5)	(7, 8, 8, 4, 5)
(5, 9, 6, 10, 2)	(6, 9, 6, 7, 5)	(7, 9, 10, 7, 5)
(5, 10, 9, 10, 5)	(6, 10, 8, 11, 5)	(7, 10, 5, 10, 3)
(5, 11, 2, 5, 5)	(6, 11, 1, 13, 5)	(7, 11, 7, 2, 5)
(5, 12, 8, 3, 5)	(6, 12, 6, 8, 5)	(7, 12, 1, 9, 5)
(5, 13, 6, 12, 5)	(6, 13, 6, 11, 5)	(7, 13, 2, 2, 5)
(5, 14, 10, 13, 5)	(6, 14, 6, 2, 5)	(7, 14, 3, 8, 5)
(5, 15, 7, 8, 5)	(6, 15, 5, 13, 2)	(7, 15, 5, 13, 5)
(5, 16, 2, 10, 5)	(6, 16, 5, 9, 4)	(7, 16, 5, 3, 6)
(5, 17, 9, 7, 2)	(6, 17, 4, 3, 5)	(7, 17, 1, 6, 5)
(5, 18, 7, 9, 5)	(6, 18, 4, 13, 5)	(7, 18, 8, 13, 5)
(5, 19, 4, 11, 5)	(6, 19, 8, 6, 5)	(7, 19, 3, 3, 5)
(5, 20, 6, 7, 2)	(6, 20, 6, 6, 5)	(7, 20, 8, 2, 5)
(5, 21, 5, 7, 6)	(6, 21, 7, 7, 6)	(7, 21, 9, 5, 5)
(5, 22, 3, 7, 5)	(6, 22, 4, 8, 5)	(7, 22, 1, 5, 5)
(5, 23, 6, 5, 5)	(6, 23, 5, 7, 2)	(7, 23, 10, 8, 5)
(5, 24, 10, 3, 5)	(6, 24, 8, 10, 5)	(7, 24, 4, 4, 5)
(5, 25, 2, 8, 5)	(6, 25, 5, 5, 6)	(7, 25, 5, 10, 6)
(5, 26, 7, 10, 2)	(6, 26, 9, 7, 5)	(7, 26, 5, 5, 5)
(5, 27, 8, 12, 5)	(6, 27, 5, 2, 5)	(7, 27, 5, 4, 2)
(5, 28, 3, 10, 5)	(6, 28, 3, 10, 2)	(7, 28, 1, 10, 2)
(5, 29, 5, 10, 2)	(6, 29, 5, 10, 4)	(7, 29, 7, 3, 5)
(5, 30, 3, 13, 5)	(6, 30, 4, 10, 2)	(7, 30, 5, 11, 5)
(5, 31, 1, 7, 5)	(6, 31, 5, 10, 5)	(7, 31, 1, 3, 5)
(5, 32, 5, 7, 3)	(6, 32, 5, 9, 5)	(7, 32, 5, 3, 5)
(5, 33, 5, 7, 1)	(6, 33, 7, 10, 5)	(7, 33, 4, 10, 5)
(5, 34, 5, 12, 2)	(6, 34, 5, 8, 6)	(7, 34, 9, 11, 5)
(5, 35, 1, 10, 5)	(6, 35, 5, 10, 7)	(7, 35, 7, 11, 5)
		(7, 36, 5, 5, 2)
		(7, 37, 5, 6, 5)
		(7, 38, 5, 4, 5)
		(7, 39, 10, 10, 5)
		(7, 40, 2, 7, 4)

Table 6. Advertisement plan of Client 3, Client 4, Client 8, Client 9, and Client 10

Client 3	Client 4	Client 8	Client 9	Client 10
(3, 1, 8, 4, 5)	(4, 1, 10, 1, 5)	(8, 1, 8, 6, 5)	(9, 1, 10, 1, 5)	(10, 1, 2, 13, 3)
(3, 2, 4, 4, 5)	(4, 2, 5, 7, 5)	(8, 2, 3, 7, 5)	(9, 2, 8, 2, 5)	(10, 2, 2, 12, 6)
(3, 3, 9, 4, 5)	(4, 3, 5, 11, 5)	(8, 3, 4, 9, 5)	(9, 3, 8, 6, 5)	(10, 3, 2, 7, 1)
(3, 4, 5, 4, 5)	(4, 4, 6, 9, 5)	(8, 4, 10, 8, 5)	(9, 4, 5, 11, 5)	(10, 4, 2, 8, 2)
(3, 5, 10, 4, 5)	(4, 5, 7, 8, 5)	(8, 5, 2, 8, 5)	(9, 5, 3, 8, 5)	(10, 5, 2, 4, 4)
(3, 6, 3, 4, 5)	(4, 6, 4, 10, 5)	(8, 6, 5, 11, 5)	(9, 6, 3, 3, 5)	(10, 6, 2, 11, 1)
	(4, 7, 5, 12, 5)	(8, 7, 3, 3, 5)	(9, 7, 7, 13, 5)	(10, 7, 2, 1, 5)
	(4, 8, 8, 6, 5)	(8, 8, 3, 13, 5)	(9, 8, 5, 12, 5)	(10, 8, 2, 9, 5)
	(4, 9, 3, 3, 5)	(8, 9, 5, 7, 5)	(9, 9, 7, 8, 5)	(10, 9, 2, 3, 5)
	(4, 10, 3, 8, 5)	(8, 10, 5, 3, 5)	(9, 10, 2, 1, 5)	(10, 10, 2, 8, 5)
		(8, 11, 8, 2, 5)	(9, 11, 8, 13, 5)	
		(8, 12, 10, 1, 5)	(9, 12, 6, 9, 5)	
		(8, 13, 2, 1, 5)	(9, 13, 5, 13, 5)	
		(8, 14, 10, 3, 5)	(9, 14, 5, 7, 5)	
		(8, 15, 8, 7, 5)	(9, 15, 8, 7, 5)	

6. Conclusion and future directions

Media planning is a complex problem and sales plan generation is a subproblem of this area. We studied the sales plan generation problem as a stochastic decision problem because the number of audience that is wanted to be reached is uncertain for any show in any week.

In our examples, we give a relatively high penalty rate of not meeting and/or exceeding total audience demand than the penalty rates of not meeting and/or exceeding audience per week and show demands. Therefore, some of the week and show constraints are not met for all companies. These results and unsatisfied constraints will be changed if the penalty rates are changed. By using results derived from the model, company should increase its revenue from advertisement sales, so these results can be used in stochastic revenue management models.

In this study, we plan an optimal schedule for advertisement slots bought at upfront market. By generating these kinds of optimal plans, broadcast companies can save millions of dollars. These plans also increase revenues because of a better use of advertisement inventory. The time needed to produce a sales plan and the time needed to rework on created plans will decrease by using optimization models. Broadcasting companies will also be able to respond more quickly to their clients, hence they will have a stronger position against their competitors.

We formulate the problem from the broadcast companies' point of view. The penalties, weekly rates and show rates are assumed to be constant for all clients. In future, same problem can be reformulated from the clients' point of view or an integrated approach can be formulated. In addition to this, the problem modeled in this paper is a minimization problem where it can be reformulated as a profit maximization problem with available budget data of the clients. Some other constraints from real world can be added to these models such that different penalty rates for more influential clients which will be more realistic especially for countries in which the business environment is more of a relationship-based type.

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