

Portfolio selection problem: a comparison of fuzzy goal programming and linear physical programming

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(Received November 11, 2015; in final form April 21, 2016)

Abstract. Investors have limited budget and they try to maximize their return with minimum risk. Therefore this study aims to deal with the portfolio selection problem. In the study two criteria are considered which are expected return, and risk. In this respect, linear physical programming (LPP) technique is applied on Bist 100 stocks to be able to find out the optimum portfolio. The analysis covers the period from April 2009 to March 2015. This period is divided into two; April 2009-March 2014 and April 2014 – March 2015. April 2009-March 2014 period is used as data to find an optimal solution. April 2014-March 2015 period is used to test the real performance of portfolios. The performance of the obtained portfolio is compared with that obtained from fuzzy goal programming (FGP). Then the performances of both method, LPP and FGP, are compared with BIST 100 in terms of their Sharpe Indexes. The findings reveal that LPP for portfolio selection problem is a good alternative to FGP.

Keywords: Portfolio selection problem; linear physical programming; fuzzy goal programming.

AMS Classification: 90C70, 90C05

1. Introduction

The purpose of investors is to maximize the total return of their investments while considering the risk factor. In order to minimize the risk, a portfolio concept has arisen. Investing funds into a portfolio instead of one asset may be less risky because poor performance of one investment instrument can be easily balanced with good performance of another investment instrument.

In order to maximize the return of assets in their portfolio, investors need to manage the portfolio efficiently [1]. Portfolio management can be defined as the allocation of the funds between the securities for ensuring the maximum return and minimum risk [2]. In real world, the ambiguity exists because of uncertainty and the lack of efficient information. Therefore, portfolio

selection problem is a challenging problem for researchers. And various studies have been done so far about portfolio selection problem.

Modern portfolio optimization studies began with the work of Markowitz in the 1950's. Markowitz suggested a mean-variance model. Markowitz studied how to ensure a portfolio that includes stocks with maximum return at a given level of risk [3]. Markowitz portfolio optimization model was the source of inspiration to many studies and was theoretically mostly known model, but the model was criticized for the need of gathering accurate information and the large number of calculations [4, 5].

Several authors have tried to develop Markowitz' modern portfolio theory. Sharpe [6] proposed to estimate the total risk of market

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instead of estimating the risk of each stock with simple regression model. Later on modern portfolio theory was elaborated with Sharpe [7], Lintler [8], Ross [9], Huberman [10] by proposing capital asset pricing and the multifactor arbitrage pricing models. Sharpe [11] and Lintler [12] developed Capital Asset Pricing Model (CAPM). In this model different from modern portfolio theory investors have the opportunity to invest in risk-free assets. And this theory was evolved with arbitrage pricing theory which was proposed by Ross [13] and extended by Huberman [14]

In later years, there were studies that try to transform the quadratic problems into linear problems such as [15], [16], [17] and [18]. One of the most popular of them was mean absolute deviation model. Konno and Yamazaki [19] proposed an alternative model to quadratic models called as mean absolute deviation model. In this model, they accepted absolute deviation as the risk measure instead of the standard deviation. Many researchers try to extend portfolio selection problem by using linear models such as maximum model [20] minimax model [21, 22].

In real world, uncertainty exists for determining the expected return and expected risk of stocks, therefore some researchers have devoted considerable efforts to deal with the vague aspirations of a decision maker using fuzzy set theory. When the information about the objectives is naturally vague, Fuzzy goal programming (FGP) approach lets the involvement of decision makers (DMs) to the determination process of imprecise aspiration levels for the goals. FGP have already been applied to the portfolio selection problem by Parra et al. [23] and Alinezhad et al. [24].

In order to obtain the optimum stocks for portfolio, this study proposes to use linear physical programming (LPP) approach. In LPP approach, DM can take in account different goals and determine these goals in different desirability levels such as ideal desirable, tolerable, undesirable, highly desirable and unacceptable. The major advantage of linear physical programming is its capability of taking into account of numerous constraints, numerous goals and considering the preference range for the goals [25]. To show the effectiveness of the use of LPP in portfolio selection problem, FGP was also applied to the problem and the results of the both methods are compared.

The remainder of this paper is organized as follows. The second section briefly explains the linear physical programming method.

Mathematical modelling of portfolio selection problem will be presented in the third section. In the fourth part, a real life portfolio selection problem will be solved under two conflicting objectives: maximum return and minimum risk possible. Finally conclusion and further research will be discussed..

2. Linear physical programming

LPP is a multi-objective optimization method that proposes specific algorithms for obtaining the weights of multiple objectives and use these weights in the optimization process to obtain optimal results [26].

Different from goal programming and fuzzy goal programming techniques that have already been applied to portfolio optimization problem, LPP uses the satisfaction levels (such as desirable, tolerable, undesirable, highly undesirable, or unacceptable) at which a particular goal (i.e. expected return) to obtain the weights and reach optimal results. Mainly, LPP distinguishes itself from the other techniques by removing the decision maker from the weight determination process [27].

Weight determination is one of the essential steps of multi-objective optimization which has inherently the challenge in determining the correct weights. A traditionally preprocessing constant weight determination may lead to bias in some cases [28]. In the LPP, it is not needed to set the weights of objectives in priori. Differently, LPP determines the weights in a systematic approach with the integration of the solution phase to find optimal results. Weight determination procedure uses the one versus others criterion rule (OVO rule) and contains a little complicated arithmetic. Therefore it needs to use a computer program to obtain the weights. The details of weight determination procedure can be found in [28]

In the LPP, DMs use four different classes named as soft classes to express their preferences according to each objective function. The most frequently used class functions, Class-1S (Smaller is Better) and Class-2S (Larger is Better), can be shown in figure 1. The decision variables, the q th objective function, the class function that will be minimized for the q th objective function are denoted by x and $g_q(x)$, Z_q respectively. In the figure 1, $g_q(x)$ is on the horizontal axis, Z_q is on the vertical axis. As it can be deduced from the figure, the smaller value of a class function improves the satisfaction level of the goal. Therefore, it is desired to obtain the value of the class function as zero. Besides soft classes, the

constraints that must be satisfied without any deviation is defined as *Hard Classes*. Each soft class function is a part of the weighted aggregated objective function of LPP that is wanted to be minimized. The weights of Soft Class functions are determined by LPP weight algorithm [28].

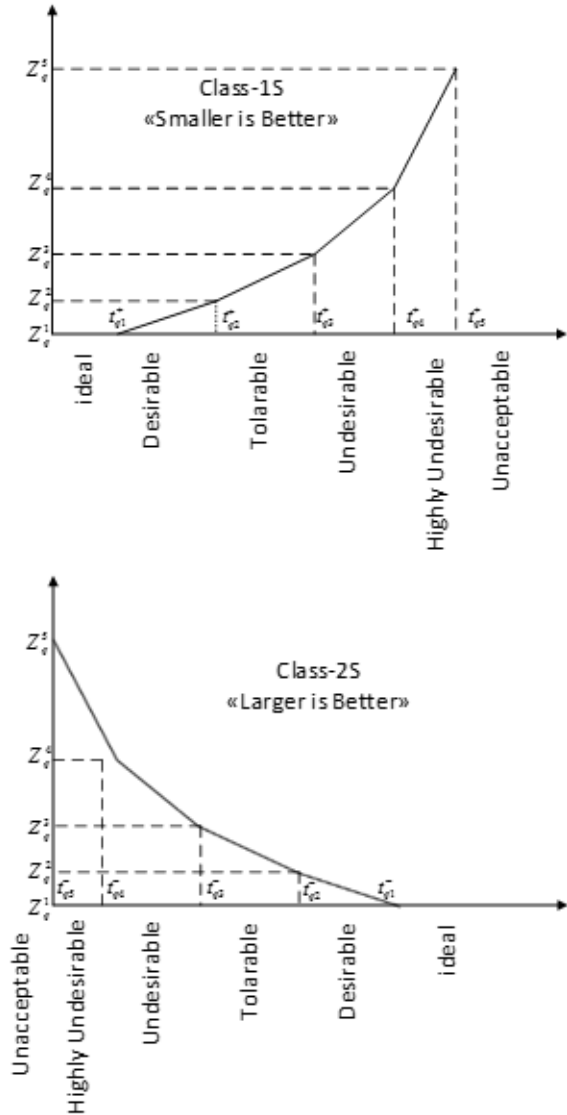


Figure 1. Smaller is Better and Larger is Better soft class functions [27]

- Step 1. Selection of the appropriate soft and hard classes for each criterion,
- Step 2. Determination of the target values that can be defined as the limits of the ranges of different degrees of desirability (i.e. t_{qs}^-, t_{qs}^+).
- Step 3. Determination of the weights for each criterion by using the weight algorithm.
- Step 4. Solving the following LP problem:

$$\text{Min} = \sum_{q=1}^{n_{sc}} \sum_{s=2}^5 (w_{qs}^- d_{qs}^- + w_{qs}^+ d_{qs}^+) \quad (1)$$

$$g_q - d_{qs}^+ \leq t_{q(s-1)}^+; \quad (2)$$

$$d_{qs}^+ \geq 0; g_q \leq t_{q5}^+, s = 2, \dots, 5 \forall q \text{ in } 1S$$

$$g_q + d_{qs}^- \geq t_{q(s-1)}^-;$$

$$d_{qs}^- \geq 0; g_q \geq t_{q5}^-, s = 2, \dots, 5 \forall q \text{ in } 2S \quad (3)$$

and *hard constraints*

where, s denotes a range, d_{qs}^+ and d_{qs}^- are deviational variables, n_{sc} denotes the number of soft classes, t_{qs} is the limit of different ranges, and, w_{qs} denotes the weight of range s in goal q . As it can be seen from Figure 1, there are five ranges that differs six degrees of desirability form ideal to unacceptable.

3. Model construction for portfolio selection problem

In a portfolio selection problem, it is assumed that there are N stocks from M sectors and K indexes to be selected for satisfying decision maker's objectives. The selected objectives are as follows;

Expected Rate of Return: The expected rate of return measures the return of each stock. The price of the stock x at time t is subtracted from the price of stock x at time $(t-1)$ then divided by the price of stock x at time $(t-1)$

Risk: The standard deviation of the expected rate of return of each stock

The system parameters and assumptions are given in below.

- i stock type $i = 1, 2, \dots, n$.
- j sector type $j = 1, 2, \dots, m$.
- k Stock indexes $k = 1, 2, \dots, k$.
- X_i the ratio of stock i .

The mathematical representation of the objective functions are shown as below:

Objective 1: Maximization of the Expected Rate of Return

$$Z_1 = \sum_{i=1}^n (r_i * X_i) \quad (4)$$

Where r_i denotes the expected rate of return of the Stock i over the planning period.

Objective 2: Minimization of the Risk

$$Z_2 = \sum_{i=1}^n (\sigma_i * X_i) \quad (5)$$

Where σ_i represents the standard deviation of the expected rate of return of each stock over the planning period.

The constraints of the portfolio selection problem are represented below;

Constraint 1: The following formula ensures that the total weights of the stocks must equal to 1.

$$\sum_{i=1}^n X_i = 1 \tag{6}$$

Constraint 2: Beyond the objective of minimizing expected risk of portfolio, it is important to avoid allocating all resources to the small number of stocks which operates in the same sector. In order to diversify the portfolio, at least four different sectors must be included in the portfolio selected. In other words, the weights of each sector must be at most 25 %.

$$\sum_{i \in IE_j} X_i \leq 0.25, \forall j \tag{7}$$

Where SE_j represents the set of stocks which belong to the j^{th} sector.

Constraint 3: In order to ensure the long-term profitability and to maximize the possibility of success in the long run, the model proposes to invest at least 50 % or more on the firms in Bist 50 index.

$$\sum_{i \in IE_1} X_i \geq 0.5(\sum_{i \in IE_1} X_i + \dots + \sum_{i \in IE_k} X_i) \tag{8}$$

Where IE_k represent the set of stocks which belong to the kth Bist index.

Moreover in the model lower and upper bound for each stock was decided as $0 \leq x_j \leq 0.1$ in order to ensure diversity.

$$0 \leq X_i \leq 0,1 \tag{9}$$

3.1. Linear physical programming model for portfolio selection problem

To maximize the expected rate of return of the selected portfolio, the first goal is defined as Class-2S type (i.e. “Larger is Better”).

$$\sum_{i=1}^n (r_i * X_i) + d_{1S}^- \geq t_{1(s-1)}^-; d_{1S}^- \geq 0; \sum_{i=1}^n (r_i * X_i) \geq t_{15}^-; s = 2, \dots, 5 \tag{10}$$

The second goal is for portfolio risk measurement which is represented by Class-1S type (i.e. “Smaller is Better”).

$$\sum_{i=1}^n (\sigma_i * X_i) - d_{2S}^+ \leq t_{2(s-1)}^+; d_{2S}^+ \geq 0; \sum_{i=1}^n (\sigma_i * X_i) \leq t_{25}^+; s = 2, \dots, 5 \tag{11}$$

Then, the LPP model can be constructed as follows:

$$Min = \sum_{q=1}^2 \sum_{s=2}^5 (w_{qs}^- d_{qs}^- + w_{qs}^+ d_{qs}^+) \tag{12}$$

Subject to (6) - (11).

3.2. Fuzzy goal programming model for portfolio selection problem

Since there are two conflicting objectives which force decision maker to accept trade-off values in the final decision, the problem can also be modelled by using fuzzy goal programming which can handle the ambiguity of the decision making process as follows:

Objective 1: The expected rate of return

$$\sum_{i=1}^n (r_i * X_i) > \tilde{Z}_1 \tag{13}$$

Where \tilde{Z}_1 represents the desirable achievement value for the expected rate of return objective. The symbol “>” denotes the statement of “approximately greater than or equal to”. The fuzzy goal can be expressed as a triangular membership function $\mu(Z_1)$ with tolerance limits for the goal (Z_1^L, Z_1^U) as follows:

$$\mu(Z_1) = \begin{cases} 1 & \text{if } Z_1^U \leq Z_1 \\ \frac{Z_1 - Z_1^L}{Z_1^U - Z_1^L} & \text{if } Z_1^L \leq Z_1 \leq Z_1^U \\ 0 & Z_1 \leq Z_1^L \end{cases} \tag{14}$$

The membership function for fuzzy expected rate of return goal is shown as in the Figure 2.

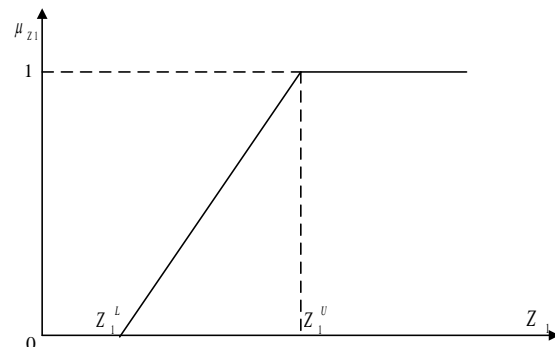


Figure 2. Membership function of fuzzy expected rate of return goal

Objective 2: The Risk

$$\sum_{i=1}^n (\sigma_i * X_i) < \tilde{Z}_2 \tag{15}$$

Where \tilde{Z}_2 represents the desirable achievement value for the risk. The symbol “<” means that the objective function should be “approximately less than or equal to” the predefined limits. The fuzzy goal can be expressed as a triangular membership function $\mu(Z_2)$ with two parameters (Z_2^L, Z_2^U) as follows:

$$\mu(Z_2) = \begin{cases} 1 & \text{if } Z_2 \leq Z_2^L \\ \frac{Z_2^U - Z_2}{Z_2^U - Z_2^L} & \text{if } Z_2^L \leq Z_2 \leq Z_2^U \\ 0 & Z_2 \geq Z_2^U \end{cases} \quad (16)$$

The membership function for fuzzy risk goal is shown as in the Figure 3.

The lower and upper tolerance limits (i.e. Z^L, Z^U aspiration levels) are determined by constructing a pay-off table which contains the solutions of two single objective problems. In the solution methodology, the problem is solved separately with expected rate of return and risk objectives, respectively. Then the best and worst values are determined and used as the aspiration levels for the fuzzy goals.

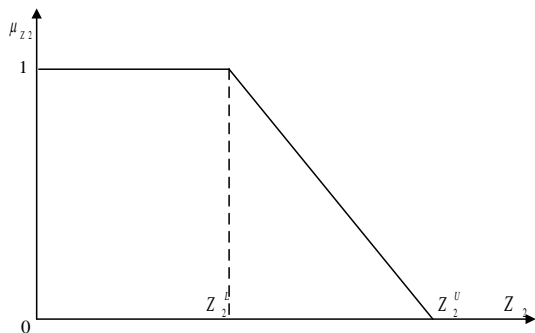


Figure 3. Membership function of fuzzy risk goal

After constructing fuzzy membership functions for the goals, fuzzy goal programming model can be presented as follows [29]:

$$\begin{aligned} \text{Max} &= \lambda & (17) \\ \mu(Z_1) &\geq \lambda \\ \mu(Z_2) &\geq \lambda \\ \text{Subject to} & (6) - (9) \end{aligned}$$

Where λ denotes overall achievement level of fuzzy goals.

4. A portfolio selection model with the help of linear physical programming

In order to show the effectiveness of LPP on portfolio optimization problem, a real-life experimental study was performed by selecting stocks operating in Borsa İstanbul. The performance of the obtained portfolio is compared with that obtained from fuzzy goal programming (FGP). Then the performances of both methods, LPP and FGP are compared with BIST 100 in terms of their Sharpe Indexes. The details about numerical analysis can be found in the following subsections.

4.1. Results for linear physical programming model

In the model we consider two criteria: the expected return of stocks and risk. The sample consists of 89 companies that traded continuously at BIST 100 between April 2009 - March 2014. The observation period is April 2009- March 2015. This period is divided into two; April 2009-March 2014 and April 2014 – March 2015. April 2009-March 2014 period is used as data to find an optimal portfolio. April 2014-March 2015 period is used to test the real performance of selected portfolios. The expected rate of return values are calculated by using the closing prices at the beginning of each month for each stock. The data are gathered on monthly basis for April 2009 – March 2014 period. The number of observations gathered was 60.

The physical programming represents different desirability degrees for each criteria. These desirability degrees are expressed by using six types of ranges which are ideal, desirable, tolerable, undesirable, highly undesirable and unacceptable [30]. Table 2 represents the target values for expected rate of return and risk. Generally, decision makers estimate the target values based on their knowledge and experience. In the paper, the interval target values are also estimated, however, a payoff table (see Table 1) which contains the solutions of 2-single objective problem is constructed to estimate the max. and min. limits of these target values which are also used in constructing the FGP membership functions.

Table 1. Corresponding pay-off table

| Objectives | Expected Rate of Return | Risk |
|----------------------------------|-------------------------|--------|
| Maximize Expected Rate of Return | 4.059 | 16.326 |
| Minimize Risk | 2.148 | 10.347 |

Table 2. Target values for criteria's

| | Expected Rate Of Return | Risk |
|--------------------|-------------------------|---------------|
| Ideal | >4.0590 | <10.340 |
| Desirable | 4.059-3.247 | 10.340-12.408 |
| Tolerable | 3.247-2.760 | 12.408-13.648 |
| Undesirable | 2.760-2.354 | 13.648-14.688 |
| Highly Undesirable | 2.354-2.140 | 14.680-16.320 |
| Unacceptable | <2.14 | >16.320 |

Once the class function is defined according to the target values, the LPP weight algorithm was used to calculate the weights presented below.

$$\begin{aligned}
 w_{12}^+ &= 0.1232, w_{13}^+ = 0.2258, \\
 w_{14}^+ &= 0.3626, w_{15}^+ = 1.5842 \\
 w_{22}^- &= 0.0484, w_{23}^- = 0.0887, \\
 w_{24}^- &= 0.1411, w_{25}^- = 0.0229
 \end{aligned}$$

By solving the LLP mathematical model, 12 stocks were selected for our portfolio. Table 3 presents the stocks notations, their expected rate of returns, the risks for April 2009 – March 2014 period, Bist index classification and their sectors. And the last column shows the proportions of each stock in the portfolio for optimal solution.

Table 3. Selected stocks for portfolio with the help of linear physical programming

| Notation | Expected Return | Risk | Bist Index | Proportions |
|----------|-----------------|---------|------------|-------------|
| DOAS E | 3.5010 | 15.3098 | 50 | 0.0814 |
| NTTUR E | 1.4591 | 9.0580 | 100 | 0.0686 |
| TCELL E | 0.6816 | 6.3395 | 30 | 0.1000 |
| CCOLA E | 3.4262 | 8.5122 | 50 | 0.1000 |
| TTRAK E | 5.2838 | 11.6482 | 100 | 0.1000 |
| ULKER E | 3.9302 | 10.8275 | 30 | 0.0500 |
| NTHOL E | 3.8334 | 10.8504 | 100 | 0.1000 |
| TAVHL E | 3.3310 | 9.8978 | 30 | 0.1000 |
| YAZIC E | 2.4507 | 8.8134 | 100 | 0.0500 |
| ASELS E | 3.3459 | 16.3101 | 30 | 0.1000 |
| LOGO E | 4.1332 | 13.4042 | 100 | 0.1000 |
| NETAS E | 2.7919 | 23.2095 | 100 | 0.0500 |

4.2. Results for fuzzy goal programming model

In order to compare the performance of LPP, we have also solved the problem with FGP. The lower and upper tolerance limits are determined as in Table 1 by constructing a pay-off table which contains the solutions of 2-single objective problem. These max-min limits guarantee the feasibility of each fuzzy goal in the solution. Figure 4 and 5 shows the membership functions for satisfaction levels of the expected return and risk goals, respectively.

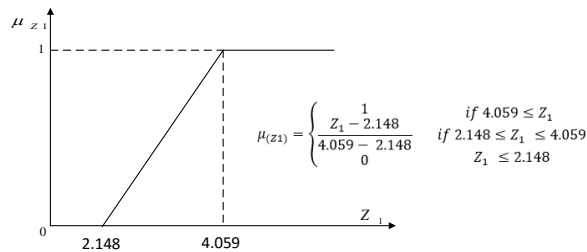


Figure 4. The membership function of expected return goal

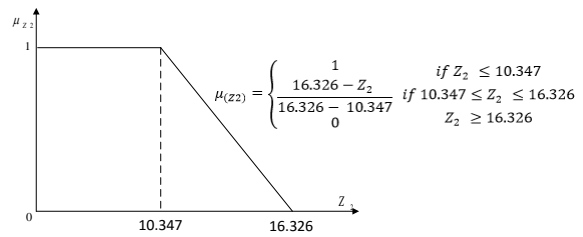


Figure 5. The membership function of risk goal

After applying FGP, the following results were obtained.

Table 4. Selected stocks for portfolio with the help of fuzzy goal programming

| Notation | Expected Return | Risk | Bist Index | Proportions |
|----------|-----------------|---------|------------|-------------|
| DOAS E | 3.5010 | 15.3098 | 50 | 0.1000 |
| NTTUR E | 1.4591 | 9.0580 | 100 | 0.0500 |
| TCELL E | 0.6816 | 6.3395 | 30 | 0.1000 |
| CCOLA E | 3.4262 | 8.5122 | 50 | 0.1000 |
| TTRAK E | 5.2838 | 11.6482 | 100 | 0.1000 |
| ULKER E | 3.9302 | 10.8275 | 30 | 0.0500 |
| NTHOL E | 3.8334 | 10.8504 | 100 | 0.1000 |
| TAVHL E | 3.3310 | 9.8978 | 30 | 0.1000 |
| VKGYO E | 5.7724 | 19.8805 | 100 | 0.0434 |
| YAZIC E | 2.4507 | 8.8134 | 100 | 0.0066 |
| ASELS E | 3.3459 | 16.3101 | 50 | 0.1000 |
| LOGO E | 4.1332 | 13.4042 | 100 | 0.1000 |
| NETAS E | 2.7919 | 23.2095 | 100 | 0.0500 |

Table 4 presents the stocks, their notation, their expected rate of return and the risk for April 2009 – March 2014 period when the problem is solved with the help of fuzzy goal programming. And the last column shows the weights of each stock in the portfolio for optimal solution.

The overall results of both LPP and FGP models are provided in Table 5. The results show that the portfolio returns obtained from LPP model is fewer than those obtained from the FGP model. However, the risk obtained from LPP is fewer than the one obtained from FGP model. Although both FGP and LPP models provide compromise solutions, the piecewise linear goal functions and multiple target values of LPP model allow to generate the different sets of Pareto optimal solutions.

Table 5. Results for fuzzy goal programming and linear physical programming

| Objective | FGP | LPP |
|-------------------------|---------|--------|
| Expected Rate of Return | 3.4292 | 3.247 |
| Risk | 12.3025 | 11.706 |

4.3. The performance of portfolio for control period

In order to test the performance of our portfolio, we use the control period. The control period is April 2014 – March 2015. The performance of our portfolio performance will be compared with the present market. We assume that the investor invests his/her fund in the selected portfolio determined by linear physical programming in April 2014 and hold this portfolio for 12 months till March 2015.

BIST 100 index is selected to represent the market. Firstly the return and risk are calculated for both BIST 100 and the selected portfolio on monthly basis. In order to compare the performance of selected portfolio and the index more vigorously, Sharpe index [31] was used. The success of the portfolio will be evaluated by comparing the Sharpe index of market and Sharpe index of selected portfolio. Higher Sharpe Index is better. So if the Sharpe Index value of the portfolio is higher than the Sharpe Index value of the market, the performance of portfolio will be better than the market. Sharpe index considers both risk and return at the same time. Sharpe index is calculated as follows;

$$S = (rp - rf) / \partial rp \quad (17)$$

rp = The average return of portfolio for a given period,

rf = The average risk free interest rate (usually state bond or treasury bond interest rates are accepted) for a given period,

∂rp = the standard deviation of portfolio for a given period (represents the risk criteria).

Table 6. Sharpe index value of Bist 100 and the portfolios selected with the help of LPP and FGP

| | BIST 100 Index | LPP | FGP |
|-------------------------|----------------|--------|-------|
| Expected Rate of Return | % 2.4 | % 4.33 | %3.90 |
| Risk | % 6.48 | % 8.41 | %8.96 |
| Sharpe Index | 0.26 | 0.43 | 0.36 |

Table 6 represents the expected rate of return, risk and Sharpe Index value of both BIST 100 and the portfolio obtained by LPP and FGP. Sharpe Index of BIST 100 and the portfolio are calculated for April 2014 and March 2015. The risk-free interest rate is taken as the average Treasury bond interest rate for April 2014 – March 2015 period

(%0,7220). The findings reveal that the performance of the portfolio obtained by LPP was better than both the performance of BIST 100 Index and the portfolio obtained by FGP in terms of sharpe index.

5. Conclusion

The success of portfolio selection problem can only be ensured by successful selection of stocks. In this paper, a new technique for portfolio optimization problem with the aid of LPP is presented. The main purpose of the study is to find an optimum portfolio that maximizes the return which at the same time minimizes the risk. We compared the performance of the portfolios obtained by LPP and FGP approaches with the present market (BIST 100 Index) for the control period within April 2014 – March 2015 in terms of Sharpe index. The results revealed that LPP has potential to help the investors to find the efficient portfolio as much as possible. Finally, this study is thought to make contribution to literature by introducing the LPP for portfolio selection problems. Further research may consider more criteria and more constraints.

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