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Fuzzy multi objective linear programming problem with imprecise aspiration level and parameters

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Abstract. This paper considers the multi-objective linear programming problems with fuzzy goal for each of the objective functions and constraints. Most existing works deal with linear membership functions for fuzzy goals. Our method finds an efficient solution to more general case. The ranking function used in this paper can be each linear ranking function. In this paper, exponential membership function has been used.

Keywords: Fuzzy efficient solution; fuzzy multi-objective linear programming; Pareto optimal solution.

AMS Classification: 90C70,90C29.

1. Introduction

A lot of real world decision problems are described by multi objective linear programming models and sometimes it is necessary to formulate them with uncertainty elements. In these cases, the fuzzy theory might be more helpful. The fuzzy programming approach to multi objective linear programming problems was first introduced by Zimmerman [14]. Some fuzzy optimal solution concepts of multi objective linear programming problems have been introduced by some researchers [9,12]. Different works of fuzzy multi-objective linear programming, such as weighted coefficients in two-phase approach and pareto optimal solution, have been introduced by researchers [2,5,7].

In this paper, we propose an extension of Guu and Wu and Dubois and Fortemps approaches [3,5,6]. Most existing works deal with linear membership functions for fuzzy goals. Our method finds an efficient solution to more general case. The ranking function used in this paper can be each linear ranking function, for example the function used in [7]. In this paper, we use exponential membership function to define the decision maker's level of desirability for the objective functions and constraints.

In Section 2, we give some necessary concepts of fuzzy sets theory. In Section 3, first we review the multi objective linear programming model, then we consider the fuzzy multi-objective linear programming problem (FMOLP) with fuzzy parameters and fuzzy constraints, in which vague aspiration levels are represented by exponential membership functions. Solving FMOLP is given in Section 4 and finally in Section 5, we explain it by an illustrative example.

2. Preliminaries

In this section, we review some preliminaries which are needed in the next sections. For more details see [10,11].

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Let X be a given set of possible alternatives which contains the solution of a decision making problem under considerations. A fuzzy goal $\tilde{G}$ and a fuzzy constraint $\tilde{C}$ are fuzzy sets on X which is characterized by membership functions $\mu_{\tilde{G}}(x) : X \to [0,1]$ and $\mu_{\tilde{C}}(x) : X \to [0,1]$ respectively. Bellman and Zadeh [1] defined the fuzzy decision $\tilde{D}$ resulting from the fuzzy goal $\tilde{G}$ and fuzzy constraint $\tilde{C}$ as the intersection of $\tilde{G}$ and $\tilde{C}$. To be more explicit, the fuzzy decision of Bellman and Zadeh is the fuzzy set $\tilde{D}$ on X defined as $\tilde{D} = \tilde{G} \bigcap \tilde{C}$ where its membership function is as $\mu_{\tilde{D}}(x) = min(\mu_{\tilde{G}}(x), \mu_{\tilde{C}}(x))$. A optimal decision is defined as

$$\max \mu_{\tilde{D}}(x) = \max \min(\mu_{\tilde{G}}(x), \mu_{\tilde{C}}(x)).$$

More generally, the fuzzy decision $\tilde{D}$ resulted from k fuzzy goals $\tilde{G}_1, \cdots, \tilde{G}_k$ and m fuzzy constraints $\tilde{C}_1, \cdots, \tilde{C}_m$ is defined by $\tilde{D} = \tilde{G}_1 \bigcap ... \bigcap \tilde{G}_k \bigcap \tilde{C}_1 \bigcap ... \bigcap \tilde{C}_m$.

Among many applications of fuzziness in real world applications and mathematics, we consider fuzzy multi-objective linear programming (FMOLP) in which the objectives and parameters are fuzzy.

Definition 1. *A fuzzy number $\tilde{A} = (a, b, c)$ is said to be a triangular fuzzy number if its membership function is given by*

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a}{b-a} & a \le x \le b, \\ \frac{x-c}{b-c} & b \le x \le c, \\ 0 & otherwise. \end{cases}$$

A triangular fuzzy number (a, b, c) is said to be non-negative fuzzy number if $a \ge 0$. Two triangular fuzzy numbers $\tilde{A} = (a, b, c)$ and $\tilde{B} = (e, f, g)$ are said to be equal if and only if $a = e$, $b = f$, $c = g$. Let $\tilde{A} = (a, b, c)$ and $\tilde{B} = (e, f, g)$ be two triangular fuzzy numbers. Based on Extension principle some arithmetic operations are as follow:

$$\begin{aligned} &\tilde{A} + \tilde{B} = (a + e,\, b + f,\, c + g), \\ &-\tilde{A} = (-c,\, -b,\, -a), \\ &\tilde{A} - \tilde{B} = (a - g,\, b - f,\, c - e). \end{aligned}$$

A simple method for ordering the elements of $F(R)$, the set of all of the fuzzy numbers, consists of defining a ranking function $F : F(R) \to \mathbb{R}$ which maps each fuzzy number into a number of the real numbers set and it define as follows [10]:

$$\begin{aligned} &\tilde{a} \underset{F}{\ge} \tilde{b} \text{ iff } F(\tilde{a}) \ge F(\tilde{b}) \\ &\tilde{a} \underset{F}{>} \tilde{b} \text{ iff } F(\tilde{a}) > F(\tilde{b}) \\ &\tilde{a} \underset{F}{=} \tilde{b} \text{ iff } F(\tilde{a}) = F(\tilde{b}) \end{aligned}$$

where $\tilde{a}, \tilde{b} \in F(R)$. Also $\tilde{a} \underset{F}{\le} \tilde{b}$ iff $F(\tilde{a}) \le F(\tilde{b})$.

We restrict our attention to linear ranking functions, that is, a ranking function F such that $F(k\tilde{a} + \tilde{b}) = kF(\tilde{a}) + F(\tilde{b})$ where $\tilde{a}, \tilde{b} \in F(R)$ and $k \in \mathbb{R}$.

3. Fuzzy multi objective linear programming

In this section, we introduce the fuzzy multi objective linear programming problems with fuzzy goals.

A multi-objective linear programming problem can be formulated as follows:

$$\begin{array}{ll} \min & \mathbf{z}(\mathbf{x}) = (\mathbf{z_1}(\mathbf{x}),\, \mathbf{z_2}(\mathbf{x}),\, ...,\, \mathbf{z_k}(\mathbf{x})) \\ s.t. & \mathbf{x} \in \mathbf{X}, \end{array} \tag{1}$$

where $X = \{\mathbf{x} \in \mathbb{R}^n | A\mathbf{x} \ge \mathbf{b}, \mathbf{x} \ge 0\}$, $z_i(\mathbf{x}) = \mathbf{c}_i\mathbf{x}$, $\mathbf{c}_i = (c_{i1}, \cdots, c_{in}) \in \mathbb{R}^n$, for $i = 1, \cdots, k$, $\mathbf{b} = (b_1, \cdots, b_m) \in \mathbb{R}^m$ and A is a m by n matrix.

In general, there is not a complete optimal solution for problem (1) which simultaneously minimizes all of the objective functions. Instead of a complete optimal solution, a new optimal solution concept, called Pareto optimal solution is introduced.

Definition 2. *A feasible solution $\mathbf{x}^0 \in X$ is said to be a Pareto optimal solution for the MOLP (1) if there is no $\mathbf{y} \in X$ such that $z_i(\mathbf{x}^0) \ge z_i(\mathbf{y})$ for all i and $z_j(\mathbf{x}^0) > z_j(\mathbf{y})$ for at least one j.*

There are several methods for finding the Pareto optimal solutions of MOLP in text book [5]. We introduce the fuzzy multi objective linear programming problem with imprecise aspiration levels and parameters.

The fuzzy multi objective linear programming with fuzzy aspiration levels for objective functions and constraints and fuzzy parameters can be formulated as

$$\begin{array}{lll} Find & \mathbf{x} & \\ s.t. & \tilde{z}_i(\mathbf{x}) \preceq_{\tilde{p}_i} \tilde{g}_i & i = 1, ..., k, \\ & \tilde{\mathbf{a}}_r\mathbf{x} \succeq_{\tilde{q}_r} \tilde{b}_r & r = 1, ..., m, \\ & \mathbf{x} \ge 0, & \end{array} \tag{2}$$

where $\tilde{g}_i$ for $i = 1, \cdots, k$ are fuzzy quantities which represent the aspiration levels of the objective functions and $\tilde{p}_i$ for $i = 1, \cdots, k$ measures the adequacy between the objective functions $\tilde{z}_i(\mathbf{x}) = \tilde{c}_i\mathbf{x}$ and the aspiration level $\tilde{g}_i$, and $\tilde{q}_r$ for $r = 1, \cdots, k$ the r^{th} component of the fuzzy vector $\tilde{q}$ measures the adequacy between the fuzzy number $\tilde{\mathbf{a}}_r\mathbf{x}$ and $\tilde{b}_r$, which are the $r-th$ components of the fuzzy vectors $\tilde{A}\mathbf{x}$ and $\tilde{b}$ respectively. Here, "$\succeq$" and "$\preceq$" indicate that the inequalities are flexible and may be described by a fuzzy set whose membership function tells whether or not the decision maker's degree of satisfaction is fulfilled; these inequalities can be interpreted as "essentially greater than" and "essentially less than" , as defined by Zimmermann [15].

Now, by considering an arbitrary linear ranking function F, we can easily derive the following fuzzy multi objective linear programming problem with only imprecise aspiration levels [10]:

$$\begin{array}{lll} Find & \mathbf{x} & \\ s.t. & F(\mathbf{c}_i\mathbf{x}) \leq_{F(\tilde{p}_i)} F(\tilde{g}_i) & i = 1, ..., k, \\ & F(\tilde{\mathbf{a}}_r\mathbf{x}) \leq_{F(\tilde{q}_r)} F(\tilde{b}_r) & r = 1, ..., m, \\ & \mathbf{x} \geq 0. & \end{array} \tag{3}$$

We consider an exponential membership function to define the decision maker's level of desirability for the objective functions and constraints of problem (3) as follows:

$$\mu_i(\mathbf{x}) = \begin{cases} 1 & F(\tilde{\mathbf{c}}_i\mathbf{x}) \leq F(\tilde{g}_i) \\ \frac{e^{-\alpha_i(\frac{F(\tilde{g}_i)-F(\tilde{\mathbf{c}}_i\mathbf{x})}{-F(\tilde{p}_i)})} - e^{-\alpha_i}}{1-e^{-\alpha_i}} & F(\tilde{g}_i) < F(\tilde{\mathbf{c}}_i\mathbf{x}) < F(\tilde{g}_i) + F(\tilde{p}_i) \\ 0 & F(\tilde{\mathbf{c}}_i\mathbf{x}) \geq F(\tilde{g}_i) + F(\tilde{p}_i) \end{cases}$$

for $i = 1, 2, \cdots, k$, and

$$\mu_r(\mathbf{x}) = \begin{cases} 1 & F(\tilde{\mathbf{a}}_r\mathbf{x}) \geq F(\tilde{b}_r) \\ \frac{e^{-\alpha_r(\frac{F(\tilde{\mathbf{a}}_r\mathbf{x})-F(\tilde{b}_r)}{-F(\tilde{q}_r)})} - e^{-\alpha_r}}{1-e^{-\alpha_r}} & F(\tilde{b}_r) - F(\tilde{q}_r) < F(\tilde{\mathbf{a}}_r\mathbf{x}) < F(\tilde{b}_r) \\ 0 & F(\tilde{\mathbf{a}}_r\mathbf{x}) \leq F(\tilde{b}_r) - F(\tilde{q}_r) \end{cases}$$

for $r = 1, 2, \cdots, m$, where $0 < \alpha_i, \alpha_r < \infty$ are fuzzy parameters which measure the degree of vagueness and are called shape parameters. When the parameters α_i, α_r are increased, their vagueness decreases. The exponential membership function may change shape according to the parameters α_i and α_r. By giving values to these parameters, the aspiration levels of the objective functions and the system constraints may be described more accurately.

Definition 3. *A feasible solution* $\mathbf{x}^0 \in X$ *is said to be a fuzzy Pareto optimal solution to the FMOLP problem (2) if there is no* $\mathbf{y} \in X$ *such that* $\mu_i(z_i(\mathbf{x}^0)) \leq \mu_i(z_i(\mathbf{y}))$ *for all* i *and* $\mu_j(z_j(\mathbf{x}^0)) < \mu_j(z_j(\mathbf{y}))$ *for at least one* j.

Now, by considering Bellman-Zadeh's fuzzy decision and the above fuzzy membership functions, we can easily derive the following crisp linear programming problem from (3) as

$$\begin{array}{lll} \max & \lambda & \\ s.t. & \lambda \leq \frac{e^{-\alpha_i(\frac{F(\tilde{g}_i)-F(\tilde{\mathbf{c}}_i\mathbf{x})}{-F(\tilde{p}_i)})} - e^{-\alpha_i}}{1-e^{-\alpha_i}}, & i = 1, ..., k \\ & \lambda \leq \frac{e^{-\alpha_r(\frac{F(\tilde{\mathbf{a}}_r\mathbf{x})-F(\tilde{b}_r)}{-F(\tilde{q}_r)})} - e^{-\alpha_r}}{1-e^{-\alpha_r}}, & r = 1, ..., m \\ & \lambda \in [0, 1], \ \mathbf{x} \geq 0. & \end{array} \tag{4}$$

The problem (4) can be rewritten as

$$\begin{array}{ll} \max & \lambda \\ s.t. & \\ \multicolumn{2}{l}{F(\tilde{p}_i) \ \ln\left(\lambda(1-e^{-\alpha_i}) + e^{-\alpha_i}\right) \leq \alpha_i(F(\tilde{g}_i) - F(\tilde{\mathbf{c}}_i\mathbf{x})), \quad i = 1, ..., k} \\ \multicolumn{2}{l}{F(\tilde{q}_r) \ \ln\left(\lambda(1-e^{-\alpha_r}) + e^{-\alpha_r}\right) \leq \alpha_r(F(\tilde{\mathbf{a}}_r\mathbf{x}) - F(\tilde{b}_r)), \quad r = 1, ..., m} \\ \multicolumn{2}{l}{\lambda \in [0, \ 1], \ \mathbf{x} \geq 0.} \end{array} \tag{5}$$

Remark 1. Let $\mathbf{x}^0$ be a fuzzy Pareto optimal solution to the FMOLP problem (2) such that $\mu_j(z_j(\mathbf{x}^0)) = 1$ for some j, that is $z_j(\mathbf{x}^o) \leq g_j$, then it could be the case that $\mathbf{x}^0$ is not a Pareto optimal solution. This is due to the fact that on the left of g_j the membership function μ_j is constantly equal to 1. Suppose that for example $\mu_1(z_1(\mathbf{x}^0)) = 1$, then it could be some $\mathbf{y} \in X$ such that $\mu_i(z_i(\mathbf{y})) = \mu_i(z_i(\mathbf{x}^0))$for all i, where $z_1(\mathbf{y}) < z_1(\mathbf{x}^0)$.

Theorem 1. *If there exists a unique optimal solution of (5), then it is a fuzzy Pareto optimal solution to the FMOLP problem (2).*

Proof. *Its Proof is straightforward and we omit it.*

4. Solution algorithm

In this section we give a solution algorithm for finding fuzzy pareto optimal solutions in FMOLP problem with exponential membership functions:

Step 1: Solve the max-min model (5):
1-1-If the optimal solution is unique:

(a) If all of the satisfaction degrees are strictly less than 1, then the solution is fuzzy pareto optimal. In this case the algorithm is finished.

(b) If some of the satisfaction degrees are equal to 1, that is at least one of the targets is fully achieved, so the solution may not to be pareto optimal. In this case go to Step 3.

1-2- If the optimal solution is not unique, then go to step 2.

Step 2: Solve the two phase model, that is maximize sum of the satisfaction degrees without making the achievement degrees obtained in the previous step worse, see the following model:

$$\begin{aligned} max \quad & \sum_{q=1}^{k+m} \lambda_q \\ s.t. \quad & 1 \geq \mu_q(\mathbf{x}) \geq \lambda_q \geq \mu_q(\mathbf{x}^*),\ q = 1, ..., k+m, \\ & \mathbf{x} \in X. \end{aligned}$$

2-1- If all of the satisfaction degrees are strictly less than 1, then the solution is fuzzy Pareto optimal. In this case the algorithm is finished.
2-2- If some of the satisfaction degrees are equal to 1, that is at least one of the targets is fully achieved, so the solution may not be Pareto optimal. In this case go to step 3.

Step 3: Maximize sum of the negative deviations, for targets that are fully achieved, without making the values obtained in the previous step worse, see the following model. The solution is fuzzy Pareto optimal. In this case the algorithm is finished.

$$\begin{aligned} max \quad & \sum_{s=1}^{p} \eta_s \\ s.t. \quad & z_s(\mathbf{x}) + \eta_s = z_s(\mathbf{x}^{**}), \qquad s = 1, ..., p, \\ & \mu_z(\mathbf{x}) = \mu_z(\mathbf{x}^{**}), \qquad z = p+1, ..., k+m, \\ & \mathbf{x} \in X. \end{aligned}$$

5. Illustrative example

In this section, we apply the algorithm for obtaining fuzzy pareto optimal solution.

Example 1. Consider a problem as

$$\begin{aligned} & Find \quad (x_1, x_2, x_3) \\ & s.t. \\ & (2,3,4)x_1 + (2,3,4)x_2 + (2,3,4)x_3 \preceq_{24} (20,21,22) \\ & (1,2,3)x_1 + (0,1,2)x_2 + (1,2,3)x_3 \preceq_{10} (7,8,9) \\ & (3,4,5)x_1 + (3,4,5)x_2 + (1,2,3)x_3 \preceq_{15} (12,13,14) \\ & (3,4,5)x_1 + (1,2,3)x_2 + (3,4,5)x_3 \succeq_{24} (17,18,19) \\ & x_1 \geq 1, x_2 \geq 1, \quad 0 \leq x_3 \leq 3. \end{aligned}$$

Step 1: We solve the max-min problem.

$$\begin{aligned} Max \quad & \lambda \\ s.t. \quad & \lambda \leq \frac{e^{-1.23(\frac{21-(3x_1+3x_2+3x_3)}{-24})} - e^{-1.23}}{1-e^{-1.23}} \\ & \lambda \leq \frac{e^{-1.23(\frac{8-(2x_1+x_2+2x_3)}{-10})} - e^{-1.23}}{1-e^{-1.23}} \\ & \lambda \leq \frac{e^{-1.23(\frac{13-(4x_1+4x_2+2x_3)}{-15})} - e^{-1.23}}{1-e^{-1.23}} \\ & \lambda \leq \frac{e^{-1(\frac{(4x_1+2x_2+4x_3)-18}{-16})} - e^{-1}}{1-e^{-1}} \\ & x_1 \geq 1,\ x_2 \geq 0, \quad 0 \leq x_3 \leq 3, \\ & \lambda \in [0,\, 1]\,. \end{aligned}$$

The optimal solutions are as ${x_1}^* = 1, {x_2}^* = 1, {x_3}^* = 3$, $\lambda^* = 0.5$ and $z_1(\mathbf{x}^*) = 15, z_2(\mathbf{x}^*) = 9, z_3(\mathbf{x}^*) = 14$. Also $\mu_1(x^*) = 1, \mu_2(x^*) = \mu_3(x^*) = 0.5, \mu_4(x^*) = 0.81$.

Step 2: A fuzzy Pareto optimal solution is obtained by solving the following problem:

$$\begin{aligned} max \quad & \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 \\ s.t. \quad & \lambda_1 \leq \frac{e^{-1.23(\frac{21-(3x_1+3x_2+3x_3)}{-24})} - e^{-1.23}}{1-e^{-1.23}} \\ & \lambda_2 \leq \frac{e^{-1.23(\frac{8-(2x_1+x_2+2x_3)}{-10})} - e^{-1.23}}{1-e^{-1.23}} \\ & \lambda_3 \leq \frac{e^{-1.23(\frac{13-(4x_1+4x_2+2x_3)}{-15})} - e^{-1.23}}{1-e^{-1.23}} \\ & \lambda_4 \leq \frac{e^{-1(\frac{(4x_1+2x_2+4x_3)-18}{-16})} - e^{-1}}{1-e^{-1}} \\ & x_1 \geq 1,\ x_2 \geq 0, \quad 0 \leq x_3 \leq 3, \\ & \lambda_1 = 1,\ 0.5 \leq \lambda_2 \leq 1,\ 0.5 \leq \lambda_3 \leq 1, \\ & 0.81 \leq \lambda_4 \leq 1. \end{aligned}$$

We have ${x_1}^{**} = 1.25, {x_2}^{**} = 0.5, {x_3}^{**} = 3$, where ${\lambda_1}^{**} = 1, {\lambda_2}^{**} = 1, {\lambda_3}^{**} = 0.5, {\lambda_4}^{**} = 0.81$ and $z_1(\mathbf{x}^{**}) = 14.25, z_2(\mathbf{x}^{**}) = 9, z_3(\mathbf{x}^{**}) = 13$. We observe that the decision $\mathbf{x}^{**}$ improves $\mathbf{x}^*$ obtained in the previous step because $z_1(\mathbf{x}^{**}) < z_1(\mathbf{x}^*), z_2(\mathbf{x}^{**}) = z_2(\mathbf{x}^*), z_3(\mathbf{x}^{**}) < z_3(\mathbf{x}^*)$. But there is at least one target that is fully achieved ($\mu_1(\mathbf{x}^{**}) = \mu_3(\mathbf{x}^{**}) = 1$) therefore $\mathbf{x}^{**}$ may not to be Pareto optimal. So we go to the step 3.

Step3: A Pareto optimal solution by solving the following problem:

$$\begin{aligned} max \quad & \eta_1 + \eta_3 + \eta_4 \\ s.t. \quad & 3x_1 + 3x_2 + 3x_3 + \eta_1 = 14.25, \\ & 4x_1 + 4x_2 + 2x_3 + \eta_3 = 13, \\ & 4x_1 + 2x_2 + 4x_3 + \eta_4 = 18, \\ & \frac{e^{-1.23(\frac{8-(2x_1+x_2+2x_3)}{-10})} - e^{-1.23}}{1-e^{-1.23}} = 0.5, \\ & x_1 \geq 1,\ x_2 \geq 0,\ 0 \leq x_3 \leq 3, \\ & \eta_1, \eta_3, \eta_4 \geq 0. \end{aligned}$$

We have $x_1^0 = 1.5, x_2^0 = 0, x_3^0 = 3$ and $z_1(\mathbf{x}^0) = 13.5, z_2(\mathbf{x}^0) = 9, z_3(\mathbf{x}^0) = 12$. The decision $\mathbf{x}^0$ improves $\mathbf{x}^{**}$, because $z_1(\mathbf{x}^0) < z_1(\mathbf{x}^{**}), z_2(\mathbf{x}^0) = z_2(\mathbf{x}^{**}), z_3(\mathbf{x}^0) < z_3(\mathbf{x}^{**})$. So $\mathbf{x}^0 = (1.5, 0, 3)$ is a Pareto optimal solution.

6. Conclusion

In this paper, we studied fuzzified versions of conventional Multi-objective linear programming by considering fuzziness in both the parameters and the constraints. We have shown that, in a MOLP problem with fuzzy goals, a fuzzy-efficient solution in which one of the goals is fully achieved may not be Pareto optimal. We use exponential membership functions. One of the advantages of using exponential membership functions is the flexibility in changing the shape of the parameters. By changing the shape of the parameters, we can explore the different fuzzy utilities of the decision maker.

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