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On constrained reliability maximization using active redundancy in coherent systems with non-overlapping subsystems

Debasis Bhattacharya[a] and Soma Roychowdhury[b]

[a] Visva-Bharati University, Santiniketan, India
Email: debasis_us@yahoo.com

[b] Indian Institute of Social Welfare and Business Management, Calcutta, India
Email: srcdb@yahoo.com

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Abstract. The paper investigates the problem of constrained reliability maximization by allocating redundancy and proposes how to solve it for a broad group of complex coherent systems. Redundancy is an effective engineering tool to enhance system reliability to make a system fail-safe. Since adding redundancy increases the cost and complexity of a system design, it should be used wisely. The work considers an exact solution to the problem under resource constraints and finds optimal redundancy numbers. The proposed method can accommodate any number of constraints. Numerical examples have been included. A sensitivity analysis has been carried out to show how sensitive the optimal allocation of redundant components and the gain in system reliability are to the budget allocation.

Keywords: coherent system; redundancy; reliability optimization; system reliability
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1. Introduction

System reliability enhancement is a very important issue to the reliability practitioners. There are various ways to increase system reliability, such as using high-quality components, providing better maintenance, rearranging components according to some component importance measures, etc. Another effective way to increase system reliability is using redundancy. But the amount of increase varies with the number of redundant components and the choice of system component to which they are added. Problem becomes complex when the optimal number of redundant components is to be decided, which maximizes the system reliability under some constraints, such as the constraints of cost, weight, volume, capacity etc. There is large amount of literature that studied the redundancy allocation problem for enhancing the system reliability for a target time for specific system designs. Morrison [1] considered optimal allocation of spares in systems with two subsystems in the problem of maximizing system life with the main focus on exponential component lives. Misra [2] used the least square concept to find an approximate solution. Shaked and Shanthikumar [3] studied the problem of allocating m active redundancies to an n-component series system where the lifetimes of the original components and redundant components are identically and independently distributed. Boland et al. [4] solved a redundancy allocation problem for series and parallel systems. Hsieh [5] solved the redundancy allocation problem using a linear programming approach to approximate the non-linear reliability function, where bounds to the reliability function were used. Liu [6] proposed a combination

Corresponding Author. Email: srcdb@yahoo.com

method to obtain the optimal allocations of component redundancy.

It is to be noted that the system reliability can be enhanced whenever redundant components are added to the system components, but that will result in a costlier, heavier system. Therefore, while maximizing system reliability using redundancy, a trade-off is necessary. Prasad et al. [7] maximized the percentile life of a series system subject to the resource constraints. In a recent work, Bhattacharya and Roychowdhury [8] solved a cost minimizing redundancy allocation problem where total cost of adding redundancy is minimized subject to a constraint on system reliability. But there may be some situations where achieving a high reliability is of more concern. There redundancy should be added in order to maximize system reliability under budget constraints. The present work solved a reliability maximizing redundancy allocation problem subject to cost and other constraints such as weight, volume, space or capacity etc.

Here a constrained redundancy allocation problem is considered with a broader class of coherent systems, which can be decomposed into a number of non-overlapping subsystems, in such a way that the system fails with the failure of any of the subsystems, while a subsystem fails when all of its constituent components fail. This class of systems is commonly referred to as having the series-parallel structures, and very much used in various fields of important applications. One can mention an office sprinkler system used for fire-extinguishing purposes (smoke detectors in parallel, a standby battery and main supply in parallel, a sprinkler in series), river water supply system (a duty raw water pump and a standby raw water pump in parallel, treatment plant in series, trunk mains in parallel, service reservoir, emergency supply in series), an air conditioning system (number of air conditioning machines in parallel, connected to the power source in series), an uninterrupted power supply (UPS) in an alternate current power supply system (AC power supply and battery in parallel, rectifier, inverter, in series), a coal transportation system from bin to boiler in coal mines (primary feeders in parallel, a reclaimer in series, secondary feeders in parallel) and many more.

The novelty of the present work is that the method developed here is capable of accommodating any number of subsystems and any number of constraints. Moreover, no fixed form of component life distribution has been assumed. Here the redundancy number for each subsystem that maximizes the super system reliability is the decision variable. The proposed method is simple to apply and produces an explicit expression for getting an optimal solution. Here active redundancy is considered, in which the original and the redundant component, both function simultaneously. When one fails, other continues to work so that the system continues to function without interruption. The redundant components are connected in parallel to the original components of the system. Active redundancy may be used in case it is difficult or not possible to replace the failed components when the system is in operation.

The structure of the article is as follows: Section 2 discusses the preliminaries necessary to develop the allocation policy. The method of finding an optimal solution is discussed in Section 3. A numerical example is included in Section 4 to illustrate the application of the rule with a sensitivity analysis. Section 5 concludes the article.

2. Preliminaries

Let us consider a complex coherent system with components whose lifetimes are independently distributed among themselves and independent of the lifetimes of the redundant components. An n-component system is said to be coherent if its every component is relevant and the system is monotone [9].

Let Y_1, Y_2, ..., Y_n be the independently distributed random lives of the components of an n-component coherent system decomposed into k non-overlapping subsystems, M_1, M_2, ..., M_k, of sizes n_1, n_2, ..., n_k, respectively. The components of i^{th} subsystem, M_i, of an n-component system can be written as $\{i_1, i_2, \ldots, i_{n_i}\}$, which is a permutation of n_i components from the index set $I = \{1, 2, \ldots, n\}$. Here i_j denotes the j^{th} component of i^{th} subsystem, M_i, $j = 1, 2, \ldots, n_i$, $i = 1, 2, \ldots, k$. The reliability of the system is given by

$$R(t) = P(T > t) = E[\prod_{i=1}^{k}\{\coprod_{j\in M_i} \nu_j(t)\}], \qquad (1)$$

where T is the system life, $v_j(t)$ is the state variable, which takes the value 0, if j^{th} component is in failing state at time t, and 1, if j^{th} component is in functioning state at time t with $E(v_j(t)) = \rho_j(t)$. From (1) it is clear that the system reliability can be expressed in terms of its subsystem reliabilities involving $\rho_j(t)$'s.

The present work finds out a general rule as to how many redundant components should be added at the design stage to each subsystem in order to maximize the super system reliability under various constraints. The results have been derived under the following set up: The coherent system considered here is binary (having two states, viz., functioning and failing), and all components belonging to the same subsystem have the same reliability. Sometimes it may also be reasonable to assume the reliability of the redundant components to be same as that of the components belonging to the respective subsystem, to which the redundant components are to be added.

3. Optimal Allocation of Redundancy

Let us consider an n-component binary coherent system, with p_i as the reliability of the components belonging to the i^{th} subsystem, $i = 1, 2, \ldots, k$, η_i as the number of active redundant components to be attached to the components of i^{th} subsystem, $i = 1, 2, \ldots, k$, in order to optimize the system reliability under cost constraint, and r_i as the respective redundant component reliability. Suppose C to be the budget limit and c_i, the cost of a redundant component to be attached to the components of i^{th} subsystem, $i = 1, 2, \ldots, k$, with $\sum_{i=1}^{k} c_i\eta_i \le C$.

Our objective is to determine the optimal values of decision variables, $\eta_1, \eta_2 \ldots, \eta_k$, that maximize the non-linear objective function, the system reliability, which, by (1), is as follows:

$$R = R(\eta_1, \eta_2 \ldots, \eta_k) = \prod_{i=1}^{k} [1 - (1-p_i)^{n_i} \times (1-r_i)^{\eta_i}].$$

The problem is to maximize $R(\eta_1, \eta_2 \ldots, \eta_k)$ or, equivalently, maximize:

$$L(\eta_1, \eta_2 \ldots, \eta_k) = \log_e R(\eta_1, \eta_2 \ldots, \eta_k)$$

subject to

$$\sum_{i=1}^{k} c_i \eta_i \le C, \qquad (2)$$

where $\eta_i \ge 0$, $c_i \ge 0$, for all $i = 1, 2, \ldots, k$.

Now we prove the following propositions, which are required to obtain the optimal solution $\eta^* = (\eta_1^*, \eta_2^*, \ldots, \eta_k^*)$ that maximizes $L(\eta_1, \eta_2 \ldots, \eta_k)$ or $R(\eta_1, \eta_2 \ldots, \eta_k)$.

Proposition 1. *$L(\eta_1, \eta_2 \ldots, \eta_k)$ is a concave function.*

Proof. The $k \times k$ Hessian matrix of the function $L = L(\eta_1, \eta_2 \ldots, \eta_k)$ is given by

$$D^2 L(\eta_1, \eta_2 \ldots, \eta_k) = \begin{pmatrix} \partial^2 L/\partial\eta_1^2 & \partial^2 L/\partial\eta_2\partial\eta_1 & \cdots & \partial^2 L/\partial\eta_k\partial\eta_1 \\ \partial^2 L/\partial\eta_1\partial\eta_2 & \partial^2 L/\partial\eta_2^2 & \cdots & \partial^2 L/\partial\eta_k\partial\eta_2 \\ \vdots & \vdots & \vdots & \vdots \\ \partial^2 L/\partial\eta_1\partial\eta_k & \partial^2 L/\partial\eta_2\partial\eta_k & \cdots & \partial^2 L/\partial\eta_k^2 \end{pmatrix}.$$

Here

$$\frac{\partial L(\eta_1, \eta_2 \ldots, \eta_k)}{\partial \eta_i} = \frac{\partial \log_e [\prod_{i=1}^{k} \{1 - (1-p_i)^{n_i} (1-r_i)^{\eta_i}\}]}{\partial \eta_i}$$

$$= \frac{\partial}{\partial \eta_i} [\sum_{i=1}^{k} \log_e \{1 - (1-p_i)^{n_i} \times (1-r_i)^{\eta_i}\}],$$

which reduces to

$$\frac{\partial L(\eta_1, \eta_2 \ldots, \eta_k)}{\partial \eta_i} = \frac{\{q_F^{(i)} \times (1-r_i)^{\eta_i}\} \times \log_e(\frac{1}{1-r_i})}{1 - \{q_F^{(i)} \times (1-r_i)^{\eta_i}\}}, \qquad (3)$$

where $q_F^{(i)} = \prod_{j \in M_i} (1-p_j) = (1-p_i)^{n_i} =$ unreliability of i^{th} subsystem, M_i.

From (3),

$$\frac{\partial^2 L(\eta_1, \eta_2 \ldots, \eta_k)}{\partial \eta_i^2} = \frac{\partial}{\partial \eta_i} [\frac{q_F^{(i)} \times (1-r_i)^{\eta_i} \times \log_e(\frac{1}{1-r_i})}{1 - \{q_F^{(i)} \times (1-r_i)^{\eta_i}\}}]$$

$$= \log_e(\tfrac{1}{1-r_i}) \times \frac{q_F^{(i)} \times (1-r_i)^{\eta_i} \times \log_e(1-r_i)}{1 - \{q_F^{(i)} \times (1-r_i)^{\eta_i}\}}$$

$$\times [1 + \frac{q_F^{(i)} \times (1-r_i)^{\eta_i}}{1 - \{q_F^{(i)} \times (1-r_i)^{\eta_i}\}}], \text{ for all } i = 1, 2, \ldots, k,$$

which is negative, since $\log_e(1-r_i) < 0$ and other factors are positive, and

$$\frac{\partial^2 L(\eta_1, \eta_2 \ldots, \eta_k)}{\partial \eta_i \partial \eta_j} = \frac{\partial}{\partial \eta_j} [\frac{q_F^{(i)} \times (1-r_i)^{\eta_i} \times \log_e(\frac{1}{1-r_i})}{1 - \{q_F^{(i)} \times (1-r_i)^{\eta_i}\}}] = 0,$$

for all $i, j = 1, 2, \ldots, k$, $j \ne i$.

Here all of n leading principal minors of the Hessian matrix $D^2 L(\eta_1, \eta_2 \ldots, \eta_k)$ alternate in sign so that the odd-ordered minors are negative and even-ordered minors are positive, and hence the matrix $D^2L(\eta_1,\eta_2...,\eta_k)$ is negative definite, indicating concavity of the function $L(\eta_1,\eta_2...,\eta_k)$. □

Let us now define a non-linear function H as follows:

$$H(\eta_1,\eta_2...,\eta_k)$$
$$=\log_e R(\eta_1,\eta_2...,\eta_k)-\lambda(\sum_{i=1}^{k} c_i\eta_i - C), \tag{4}$$

where the constant λ (> 0) is a positive real number, which is known as Lagrangian multiplier. Here $H(\eta_1,\eta_2...,\eta_k)$ is a concave function, being a linear combination of two concave functions, $\log_e R(\eta_1,\eta_2...,\eta_k)$ (by Proposition 1) and $\sum_{i=1}^{k} c_i\eta_i$ (which is convex as well, being a linear function).

The next proposition finds the stationary point of $H(\eta_1,\eta_2...,\eta_k)$ at which $DH(\eta_1,\eta_2...,\eta_k)=\mathbf{0}$, where $DH(\eta_1,\eta_2...,\eta_k)$, the gradient or Jacobian of the real-valued function $H=H(\eta_1,\eta_2...,\eta_k)$, is given by

$$DH(\eta_1,\eta_2...,\eta_k)=\begin{pmatrix}\partial H/\partial\eta_1\\ \partial H/\partial\eta_2\\ \vdots\\ \partial H/\partial\eta_k\end{pmatrix}.$$

Proposition 2. *The stationary point of* $H(\eta_1,\eta_2...,\eta_k)$, *at which* $DH(\eta_1,\eta_2...,\eta_k)=\mathbf{0}$, *is obtained by solving the following equations*:

$$\frac{\{q_F^{(i)}\times(1-r_i)^{\eta_i}\}}{1-\{q_F^{(i)}\times(1-r_i)^{\eta_i}\}}=\frac{\lambda c_i}{\log_e(\frac{1}{1-r_i})},\ i=1,2,\ldots,k,\ \lambda>0.$$

Proof. Here, from (4), we get

$$\frac{\partial H(\eta_1,\eta_2...,\eta_k)}{\partial\eta_i}=\frac{\{q_F^{(i)}\times(1-r_i)^{\eta_i}\}\times\log_e(\frac{1}{1-r_i})}{1-\{q_F^{(i)}\times(1-r_i)^{\eta_i}\}}-\lambda c_i$$

and hence

$$DH(\eta_1,\eta_2...,\eta_k)=\frac{\partial H(\eta_1,\eta_2...,\eta_k)}{\partial\eta_i}=0 \text{ gives}$$

$$\frac{\{q_F^{(i)}\times(1-r_i)^{\eta_i}\}}{1-\{q_F^{(i)}\times(1-r_i)^{\eta_i}\}}=\frac{\lambda c_i}{\log_e(\frac{1}{1-r_i})}, i=1,2,\ldots,k. \tag{5}$$

Hence the result. □

Let $\boldsymbol{\eta}^*=(\eta_1^*,\eta_2^*,\ldots,\eta_k^*)$ and λ^* be the solution of the k equations, as given by (5), and the cost constraint $\sum_{i=1}^{k} c_i\eta_i = C$. Since $H(\eta_1,\eta_2...,\eta_k)$ is concave, ($\boldsymbol{\eta}^*$, λ^*) maximizes $H(\eta_1,\eta_2...,\eta_k)$.

The sufficient condition for global maximum in the problem of maximizing a function $f(x_1,x_2,\ldots,x_k)$ subject to $h_l(x_1,x_2,\ldots,x_k)=b_l$, l = 1, 2, …, m, is given below, which will be required to prove Proposition 3.

If $\mathbf{x}^*=(x_1^*,x_2^*,\ldots,x_k^*)$ is a maximizer, then there exists a $\boldsymbol{\lambda}^*=(\lambda_1^*,\lambda_2^*,\ldots,\lambda_m^*)$ such that $(\mathbf{x}^*,\boldsymbol{\lambda}^*)$ satisfies Lagrange conditions

$$\left.\begin{array}{l} Df(\mathbf{x}^*)-\boldsymbol{\lambda}^*\times Dh(\mathbf{x}^*)=0 \text{ and}\\ h_l(\mathbf{x}^*)=b_l, l=1,2,\ldots,m.\end{array}\right\} \tag{6}$$

Then the sufficient condition for global maximum is:

If $f(x_1,x_2,\ldots,x_k)$ *is a concave function*, *each* $h_l(x_1,x_2,\ldots,x_k)$ *is convex*, $(\mathbf{x}^*,\boldsymbol{\lambda}^*)$ *satisfies Lagrange conditions* (6), *and* $\lambda_l \geq 0$, l = 1, 2, …, m, *then* $\mathbf{x}^*$ *is a global maximizer*.

The following proposition shows that ($\boldsymbol{\eta}^*$, λ^*) is the global maximizer of $L=L(\eta_1,\eta_2...,\eta_k)$, and hence gives the optimal solution that maximizes system reliability.

Proposition 3. ($\boldsymbol{\eta}^*$, λ^*), that maximizes $H(\eta_1,\eta_2...,\eta_k)$, *is a global maximizer of* $L=L(\eta_1,\eta_2...,\eta_k)$.

Proof. From (4), as given by

$$H(\eta_1,\eta_2...,\eta_k)=\log_e R(\eta_1,\eta_2...,\eta_k)-\lambda(\sum_{i=1}^{k}c_i\eta_i-C),$$

we find

$$DH(\eta_1,\eta_2...,\eta_k)=\frac{\partial H}{\partial\eta_i}=\frac{\partial L}{\partial\eta_i}-\lambda\times\{\frac{\partial}{\partial\eta_i}(\sum_{i=1}^{k}c_i\eta_i-C)\}.$$

Thus ($\boldsymbol{\eta}^*$, λ^*), the stationary point of $H(\eta_1,\eta_2,\ldots,\eta_k)$, which is the solution of $DH(\eta_1,\eta_2...,\eta_k)=\mathbf{0}$, satisfies the following Lagrange conditions:

$$\frac{\partial L}{\partial \eta_i} - \lambda \times \{\frac{\partial}{\partial \eta_i}(\sum_{i=1}^{k} c_i\eta_i - C)\} = 0 \text{ , at } (\boldsymbol{\eta}^*, \lambda^*), \text{ i.e.,}$$

$$\left[\frac{\partial L}{\partial \eta_i}\right]_{\eta^*} - \lambda^* \times \left[\frac{\partial}{\partial \eta_i}(\sum_{i=1}^{k} c_i\eta_i - C)\right]_{\eta^*} = 0 \text{ and}$$

$$\sum_{i=1}^{k} c_i\eta_i^* = C .$$

Thus the sufficient condition for a global maximum is satisfied. □

Using Propositions 1, 2 and 3, the optimum solution (global maximizer) $\eta^* = (\eta_1^*, \eta_2^*, \ldots, \eta_k^*)$ of $(\eta_1, \eta_2, \ldots, \eta_k)$, can be obtained by solving k equations, as given by (5), subject to $\sum_{i=1}^{k} c_i\eta_i = C$, so that $R(\eta_1, \eta_2 \ldots, \eta_k)$ is maximized. Thus we get the optimal solution η_i^* of η_i as

$$\eta_i^* = \frac{\log_e(\frac{\lambda c_i}{\lambda c_i + \log_e(\frac{1}{1-r_i})}) - \log_e q_F^{(i)}}{\log_e(1-r_i)}, \ i = 1,2,\ldots,k, \quad (7)$$

where $\log_e q_F^{(i)} = n_i \log_e(1-p_i)$.

In particular, if $r_i = p_i$, for all $i = 1, 2, \ldots, k$, (7) reduces to

$$\eta_i = \frac{\log_e(\frac{\lambda c_i}{\lambda c_i + \log_e(\frac{1}{1-p_i})})}{\log_e(1-p_i)} - n_i, \ i = 1, 2, \ldots, k. \quad (8)$$

4. A Numerical Example

Let us consider the following system, as shown in Figure 1.

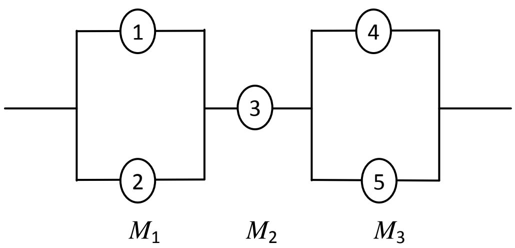

Figure 1. *A hi-fi system*

The above system is decomposed into three subsystems, viz., $M_1 = \{1, 2\}$, $M_2 = \{3\}$, $M_3 = \{4, 5\}$. The reliability of each of the components belonging to subsystem 1, is $p_1 = 0.9$. The component reliabilities for subsystems 2 and 3 are, respectively, $p_2 = 0.85$ and $p_3 = 0.95$. By (1), the system reliability is 0.839396. The cost due to adding a redundant component to the subsystem 1 is $c_1 = 40$. The costs for adding a redundant component to subsystem 2 is $c_2 = 20$ and to subsystem 3 is $c_3 = 30$. Total cost limit is $C = 80$. The reliabilities of the redundant components to be added to the subsystems are, respectively, $r_1 = 0.9$, $r_2 = 0.85$ and $r_3 = 0.95$. The number of components in the subsystems are, respectively, $n_1 = 2$, $n_2 = 1$, $n_3 = 2$. Then using (2) and (8), the solution for λ is found to be $\lambda = 0.000137$, and the number of redundant components that are to be added to different subsystems, subject to the budget constraint, are $\eta_1 = 1$, $\eta_2 = 2$, $\eta_3 = 0$, for which the total cost becomes 80, and system reliability becomes 0.993139, with a gain of 18.316% in reliability.

Table 1 shows the optimal allocation of redundant components for different budget constraints. It reflects how sensitive the optimal solution and system reliability are to the change in budget limit.

Table 1. Sensitivity of optimal redundancy allocation and maximum system reliability to the budget limits

Budget limit	*Optimal allocation* η_1	η_2	η_3	*Total cost*	*Maximum system reliability*	*Gain in reliability (%)*
80	1	2	0	80	0.99314	18.316
100	1	3	0	100	0.99598	18.657
200	2	4	1	190	0.99969	19.097
300	3	6	2	300	0.99998	19.131

4.1. Optimal redundancy allocation under more than one constraint

We can also include the constraints involving weight in the problem and solve it in the similar manner. In such case, the optimal solution will be

$$\eta_i = \frac{\log_e(\frac{\lambda_1 c_i + \lambda_2 w_i}{\lambda_1 c_i + \lambda_2 w_i + \log_e(\frac{1}{1-r_i})}) - \log_e q_F^{(i)}}{\log_e(1-r_i)},$$

$i = 1, 2, \ldots, k$, which are to be obtained subject to

$$\sum_{i=1}^{k} c_i\eta_i = C$$

and

$$\sum_{i=1}^{k} w_i\eta_i = W .$$

In case $r_i = p_i$, the solution becomes

$$\eta_i = \frac{\log_e\left(\dfrac{\lambda_1 c_i + \lambda_2 w_i}{\lambda_1 c_i + \lambda_2 w_i + \log_e(\frac{1}{1-p_i})}\right)}{\log_e(1-p_i)} - n_i, i = 1,2,\ldots,k.$$

4.2. A numerical example

For the system as displayed in Figure 1, if the weights are w_1 = 2, w_2 = 1, w_3 = 3 with a constraint of a total weight of 17 units, and the costs are c_1 = 40, c_2 =20, c_3 = 30 with a constraint of a budget of 300, the optimal solution is $\eta_1 = 3$, $\eta_2 = 5$, $\eta_3 = 2$, for which the total cost becomes 280 with a total weight of 17 units, and the system reliability, becomes 0.999972. The gain in reliability is 19.12999%. This result may be compared with the result obtained before, where only the cost constraint was considered. Because of the additional weight constraint, the number of redundant components to be added to the second subsystem becomes one unit less, and hence the gain in reliability becomes slightly less (0.001%) with a decrease of 20 units in total cost.

Similarly, if there are m constraints, such that

$$\sum_{i=1}^{k} u_{hi}\eta_i \le U_h,\ h = 1, 2, \ldots, m,$$

the optimal solution can be obtained as

$$\eta_i = \frac{\log_e\left(\dfrac{\sum_{h=1}^{m}\lambda_h u_i}{\sum_{h=1}^{m}\lambda_h u_i + \log_e(\frac{1}{1-r_i})}\right) - \log_e q_F^{(i)}}{\log_e(1-r_i)}, i = 1,2,\ldots,k,$$

together with the following m equations:

$$\sum_{i=1}^{k} u_{hi}\eta_i = U_h,\ h = 1, 2, \ldots, m,$$

where $\lambda_1, \lambda_2, \ldots, \lambda_m$ are the Lagrangian multipliers with all $\lambda_i > 0$.

The optimal solution reduces to

$$\eta_i = \frac{\log_e\left(\dfrac{\sum_{h=1}^{m}\lambda_h u_{hi}}{\sum_{h=1}^{m}\lambda_h u_{hi} + \log_e(\frac{1}{1-p_j})}\right)}{\log_e(1-p_i)} - n_i,\ \text{if } r_i = p_i,$$

$i = 1, 2, \ldots, k$.

5. Conclusion and Discussion

Here the problem of reliability maximization by allocating redundancy under cost and other constraints is solved. An explicit expression for determining the optimal solution to the problem has been derived. The method is a generalized one in the sense that it can be applied to any form of component life distributions, and there is no restriction on the number of subsystems that constitute the whole system under consideration. A sensitivity analysis has been done to examine the sensitivity of the optimal allocation of redundant components to the budget allocation. The percentage gain in system reliability becomes more or less stable as budget allocation increases, after a certain point. A solution to a cost minimizing redundancy allocation problem under reliability constraint is in progress, and will be reported in a forthcoming paper.

References

[1] Morrison, D. F., The optimum allocation of spare components in systems, Technometrics, 3(3), 399-406 (1961).

[2] Misra, K.B., Least square approach for system reliability optimization, International Journal of Control, 17(1), 199-207 (1973).

[3] Shaked, M. and Shanthikumar, J.G., Optimal allocation of resources to nodes of parallel and series systems, Advances in Applied. Probability, 24, 894-914 (1992).

[4] Boland, P.J., El-Neweihi, E. and Proschan, F., Stochastic order for redundancy allocation in series and parallel systems, Advances in Applied Probability, 24, 161-171 (1992).

[5] Hsieh, Y-C., A linear approximation for redundant reliability problems with multiple component choices, Computer and Industrial Engineering, 44, 91-103 (2002).

[6] Liu, G-S., A combination method for reliability - redundancy optimization, Engineering Optimization, 38(4), 485-499 (2006).

[7] Prasad, V. R., Kuo, W. and Kim, K. O., Maximization of a percentile life of a series system through component redundancy allocation, IIE Transactions, 33(12), 1071-1079 (2001).

[8] Bhattacharya, D. and Roychowdhury, S., A constrained cost minimizing redundancy allocation problem in coherent systems with non-overlapping subsystems, Advances in

Industrial Engineering and Management, 3(3), 1-6 (2014).

[9] Barlow, R.E. and Proschan, F., Statistical Theory of Reliability and Life Testing, To Begin with, Silver Spring, MD (1981).

Dr. Debasis Bhattacharya is a Professor of Statistics at Visva-Bharati University, Santiniketan, India. He has published a number of research papers in international journals of repute and authored books on Probability and Statistics. His research interest encompasses statistical inference for dependent processes, probability theory, Bayesian inference and reliability engineering.

Dr. Soma Roychowdhury is a Professor of Statistics at Indian Institute of Social Welfare and Business Management, Calcutta, India. She has a number of research papers published in international journals of repute and authored several books on Probability and Statistics. Her research interest includes reliability theory and inventory control.