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A global optimality result using geraghty type contraction

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Abstract. In this paper we prove two proximity point results for finding the distance between two sets. Unlike the best approximation theorems they provide with globally optimal values. Here our approach is to reduce the problem to that of finding optimal approximate solutions of some fixed point equations. We use Geraghty type contractive inequalities in our theorem. Two illustrative examples are given.

Keywords: Contraction; proximity point; metric space; global minima; fixed point.
AMS Classification: 47H10, 54H25.

1. Introduction and Mathematical Preliminaries

A proximity point problem is a problem of achieving the minimum distance between two sets through a function defined on one of the sets to the other. We have addressed the problem in the most general setting of a metric space. A metric is a function which entails the most general notion of distance. The problem of finding minimum distance between two objects is a classical problem. For example, in geometry we have the concept of geodesics, a curve along which the optimal distance between two given points of the space is realized [20]. Examples abound in physical theories, especially in the general theory of relativity, where finding the physically possible shortest path is sometimes the main task [11].

In proximity point problems our objects are sets. Here our aim is to find the distance between two sets A and B with the help of a function f defined from A to B. Mathematically, we want to find a solution to the problem of minimizing $d(x, fx)$ where x is varied over the set A. Equivalently we might want to find the optimal solution of the equation $x = fx$ although the exact solution does not in general exist as in the case where A and B are disjoint. At this point it is worthwhile to draw a comparison with best approximation theorems. A best approximation theorem provides us with best approximate solutions which need not be globally optimal. For instance, let us consider the well known Ky Fan's best Approximation theorem.

Theorem 1. [14] *Let A be a non-empty compact convex subset of a normed linear space X and $T : A \to X$ be a continuous function. Then there exists $x \in A$ such that $\| x - Tx \| = d(Tx, A) = inf\,\{\| Tx - a \| : a \in A\}$.*

The element x in the above theorem need not give the optimum value of $\| x - Tx \|$. On the

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other hand the best proximity point theorems assert that the approximate solution is also optimal, that is, a best proximity point theorem explores the possibility of finding the global minima of the real valued function $x \to d(x, Tx)$ by constraining the approximate solution of $x = Tx$ to satisfy $d(x, Tx) = dist(A, B)$.

Let (X, d) be a metric space. Let A and B be two subsets of X. A pair $(a, b) \in A \times B$ is called a best proximity pair if $d(a, b) = d(A, B) = \inf\{d(x, y) : x \in A \text{ and } y \in B\}$. If A and B are two nonempty subsets of a metric space (X, d), and T is a mapping from A to B, then $d(x, Tx) \geq d(A, B)$ for all $x \in A$. A point $p \in A$ is called a best proximity point (with respect to T) if at the point p the function $d(x, Tx)$ attains its global minimum and the value is $d(A, B)$, that is, $d(p, Tp) = d(A, B)$. Thus the problem is a problem of global minimization. There is another standpoint from which the problem can be viewed. For the mapping $T : A \to B$, the idea of a fixed point, that is, a point for which $x = Tx$ is not relevant in the cases where A and B are disjoint. Even in the cases where $A \cap B \neq \phi$, a fixed point of the function T may not exist. But it is possible to find a sort of approximate fixed point in A by minimizing the function $d(x, Tx)$. The proximity point problem is then to seek an optimal approximate solution of the fixed point equation $x = Tx$ which satisfies $d(p, Tp) = d(A, B)$ although there may not be any exact solution. In this work we adopt this view point of the proximity point problem.

There are several works on this topic in contemporary literature. Some of these works have assumed that certain types of contractive inequalities are satisfied by the concerned mappings. Some examples of these works are noted in [1, 2, 4, 5, 6, 9, 13, 15, 17, 18, 19]. Some of these works are on Banach spaces and utilize the concepts of Banach space geometry. In this paper we have proved two best proximity theorems in general metric spaces settings. The inequalities we have assumed are motivated by an extension of the Banach's contraction mapping principle proved by Geraghty [10] and recently generalized in [3, 8, 12, 16]. We have given two illustrative examples.

The following class of functions given in [7] is a slight modification of the class of functions used by Geraghty.

Let S denote the class of the functions $\alpha : [0, \infty) \longrightarrow [0, 1)$ which satisfies the condition

$$\alpha(t_n) \to 1 \text{ implies } t_n \to 0.$$

The mappings we use are cyclic mappings as described in the following which are non-self maps between two sets A and B.

A mapping $T : A \cup B \to A \cup B$ is called cyclic if $T(A) \subseteq B$ and $T(B) \subseteq A$. Recall that if T is cyclic, then a point $x \in A \cup B$ is called a best proximity point for T if $d(x, Tx) = dist(A, B)$.

2. Main Results

Theorem 2. *Let A and B be two non-empty closed subsets of a complete metric space X. Let $T : A \cup B \to A \cup B$ be a self mapping satisfying the following conditions:*

(i) *$T(A) \subseteq B$ and $T(B) \subseteq A$,*

(ii) *$d(Tx, Ty) \leq \alpha(M(x, y))M(x, y) + (1 - \alpha(M(x, y)))d(A, B)$,*

where $x \in A$, $y \in B$, $\alpha \in S$ and $M(x, y) = max\{d(x, y), d(x, Tx), d(y, Ty)\}$.

Let $x_0 \in A$ be any element and the sequence $\{x_n\}$ be defined as $x_{n+1} = Tx_n$ for all $n \geq 0$. Then $d(x_n, Tx_n) \to d(A, B)$. If $\{x_{2n}\}$ has a convergent subsequence in A, then the subsequence converges to a proximity point.

Proof. By the above construction, for all $n \geq 1$, $x_{n-1} \in A$, $x_n \in B$ or $x_n \in A$, $x_{n-1} \in B$ according as n is odd or even. Then, by an application of condition (ii), for all $n \geq 1$, we have

$$\begin{aligned}
&d(x_n, x_{n+1}) \\
&= d(Tx_{n-1}, Tx_n) \\
&\leq \alpha(M(x_{n-1}, x_n))M(x_{n-1}, x_n) \\
&\quad +(1 - \alpha(M(x_{n-1}, x_n)))d(A, B),
\end{aligned}$$

that is,

$$\begin{aligned}
&d(x_n, x_{n+1}) - d(A, B) \\
&\leq \alpha(M(x_{n-1}, x_n))(M(x_{n-1}, x_n) \\
&\quad -d(A, B)).
\end{aligned} \tag{1}$$

If possible, for some n, let $d(x_n, x_{n+1}) > d(x_{n-1}, x_n)$. Now

$$\begin{aligned}
&M(x_{n-1}, x_n) \\
&= max\{d(x_{n-1}, x_n), d(x_{n-1}, x_n), d(x_n, x_{n+1})\} \\
&= d(x_n, x_{n+1}).
\end{aligned}$$

Then, from (1), we have

$$\begin{aligned}
&d(x_n, x_{n+1}) - d(A, B) \\
&\leq \alpha(M(x_{n-1}, x_n))(M(x_{n-1}, x_n) - d(A, B)) \\
&= \alpha(d(x_n, x_{n+1}))(d(x_n, x_{n+1}) - d(A, B))
\end{aligned}$$

or $\alpha(d(x_n,\ x_{n+1})) \geq 1$, which is a contradiction since $\alpha \in S$. Therefore, for all $n \geq 1$,

$$d(x_n,\ x_{n+1}) \leq d(x_{n-1},\ x_n). \quad (2)$$

Hence the sequence $\{d(x_n,\ x_{n+1})\}$ is a decreasing sequence. Also it is bounded below by $d(A,\ B)$.

Again, by (2), for all $n \geq 1$,

$$M(x_{n-1},\ x_n) = d(x_{n-1},\ x_n). \quad (3)$$

Therefore, there exists $r \geq d(A,\ B) > 0$ such that

$$\lim_{n\to\infty} M(x_{n-1},\ x_n) = \lim_{n\to\infty} d(x_{n-1},\ x_n) = r. \quad (4)$$

Let, if possible, $r > d(A,\ B)$. From (1) and (3), since $\alpha \in S$ and for all $n \geq 1$, we have

$$\frac{d(x_n,\ x_{n+1}) - d(A,\ B)}{d(x_{n-1},\ x_n) - d(A,\ B)} \leq \alpha(d(x_{n-1},\ x_n)) < 1.$$

Taking $n \to \infty$, and using (4), we get

$$\lim_{n\to\infty} \alpha(d(x_{n-1},\ x_n)) = 1.$$

Since $\alpha \in S$, this implies that

$$\lim_{n\to\infty} d(x_n,\ x_{n-1}) = 0,$$

which is a contradiction with $r > d(A,\ B)$. So, $r = d(A,\ B)$, that is,

$$d(x_n,\ Tx_n) \to d(A,\ B) \text{ as } n \to \infty. \quad (5)$$

Now let $\{x_{2n_k}\}$ be a subsequence of $\{x_{2n}\}$ converging to some $z \in A$, that is,

$$\lim_{k\to\infty} x_{2n_k} = z. \quad (6)$$

In this case $\{x_{2n_k-1}\}$ is a sequence in B. Then, for all $k > 0$,

$$\begin{aligned} &d(A,\ B) \\ &\leq d(z,\ x_{2n_k-1}) \\ &\leq d(z,\ x_{2n_k}) + d(x_{2n_k},\ x_{2n_k-1}) \\ &= d(z,\ x_{2n_k}) + d(x_{2n_k-1},\ Tx_{2n_k-1}). \end{aligned}$$

Taking $k \to \infty$, and using (5), we conclude that

$$\lim_{k\to\infty} d(z,\ x_{2n_k-1}) = d(A,\ B). \quad (7)$$

Now,

$$\begin{aligned} &d(z,\ Tz) \\ &\leq d(z,\ x_{2n_k}) + d(x_{2n_k},\ Tz) \\ &= d(z,\ x_{2n_k}) + d(Tx_{2n_k-1},\ Tz) \\ &\leq \alpha(M(x_{2n_k-1},\ z))M(x_{2n_k-1},\ z) \\ &+(1 - \alpha(M(x_{2n_k-1},\ z)))d(A,\ B) + d(z,\ x_{2n_k}) \\ &= \alpha(M(x_{2n_k-1},\ z))(M(x_{2n_k-1},\ z) - d(A,\ B)) \\ &+d(A,\ B) + d(z,\ x_{2n_k}), \end{aligned}$$

or

$$\begin{aligned} &d(z,\ Tz) - d(A,\ B) \\ &\leq \alpha(M(x_{2n_k-1},\ z))(M(x_{2n_k-1},\ z) - d(A,\ B)) \\ &+d(z,\ x_{2n_k}) \\ &\leq M(x_{2n_k-1},\ z) - d(A,\ B) + d(z,\ x_{2n_k}). \quad (8) \end{aligned}$$

Again, by (5) and (7), we have

$$\begin{aligned} &\lim_{k\to\infty} M(x_{2n_k-1},\ z) \\ &= \lim_{k\to\infty} max\{d(x_{2n_k-1},\ z), d(x_{2n_k-1},\ x_{2n_k}), \\ &d(z,\ Tz)\} \\ &= d(z,\ Tz), \text{ (since } d(z,\ Tz)) \\ &\geq d(A,\ B)). \quad (9) \end{aligned}$$

If possible, let $d(z,\ Tz) > d(A,\ B)$. Taking $k \to \infty$ in (8), and using (9), we have

$$\begin{aligned} &d(z,\ Tz) - d(A,\ B) \\ &\leq \lim_{k\to\infty} \alpha(M(x_{2n_k-1},\ z))(d(z,\ Tz) - d(A,\ B)) \\ &\leq d(z,\ Tz) - d(A,\ B). \end{aligned}$$

Hence we have

$$\lim_{k\to\infty} \alpha(M(x_{2n_k-1},\ z)) = 1.$$

Since $\alpha \in S$, this implies that

$$\lim_{k\to\infty} M(x_{2n_k-1},\ z) = 0,$$

that is, by (9), $d(z,\ Tz) = 0$, which is a contradiction with our assumption. Hence $d(z,\ Tz) = d(A,\ B)$, that is, $\{x_{2n_k}\}$ converges to a proximity point.

This completes the proof of the theorem. □

In our next theorem we use a cycle mapping which is not a cyclic contraction.

Theorem 3. *Let A and B be two non-empty closed subsets of a complete metric space X. Let $T : A \cup B \to A \cup B$ be a self mapping which satisfies the following conditions:*

(i) *$T(A) \subseteq B$ and $T(B) \subseteq A$,*
(ii) *$d(Tx,\ Ty) \leq \alpha(d(x,\ y))d(x,\ y)$ where $x, y \in A$ and $\alpha \in S$,*

(iii) *$d(Tx,\ Ty) \leq d(x,\ y)$ where $x, y \in B$,*
(iv) *$d(A,\ B) < d(x,\ y)$ implies that $d(Tx,\ Ty) < d(x,\ y)$ for all $x \in A$ and $y \in B$.*

Then a best proximity point is attained, that is, there exists $x \in A$ such that $d(x,\ Tx) = d(A,\ B)$. If T has two distinct best proximity points, then $d(A,\ B) > 0$ and hence the sets A and B are necessarily disjoint.

Proof. Let $x_0 \in A$ be any element and $x_n = Tx_{n-1}$ for all $n \geq 0$. Then $\{x_{2n}\} \subset A$ and $\{x_{2n+1}\} \subset B$.

Now, by (ii) and (iii), we have

$$\begin{aligned} & d(x_{2n},\ x_{2n+2}) \\ & = d(Tx_{2n-1},\ Tx_{2n+1}) \\ & \leq d(x_{2n-1},\ x_{2n+1}) \\ & = d(Tx_{2n-2},\ Tx_{2n}) \\ & \leq \alpha(d(x_{2n-2},\ x_{2n}))d(x_{2n-2},\ x_{2n}). \quad (10) \end{aligned}$$

Since $\alpha \in S$, we have $d(x_{2n},\ x_{2n+2}) \leq d(x_{2n-2},\ x_{2n})$, for all $n \geq 1$. It follows that the sequence $\{d(x_{2n},\ x_{2n+2})\}$ is a decreasing sequence. Hence, there exists $r \geq 0$ such that

$$\lim_{n\to\infty} d(x_{2n},\ x_{2n+2}) = r \geq 0.$$

Let, if possible, $r > 0$. Then, from (10), we have

$$\frac{d(x_{2n},\ x_{2n+2})}{d(x_{2n-2},\ x_{2n})} \leq \alpha(d(x_{2n-2},\ x_{2n})) < 1.$$

Taking $n \to \infty$, we get

$$\lim_{n\to\infty} \alpha(d(x_{2n-2},\ x_{2n})) = 1.$$

Since $\alpha \in S$, this implies that $\lim\limits_{n\to\infty} d(x_{2n-2},\ x_{2n}) = 0$, which contradicts our assumption that $r > 0$. Hence

$$\lim_{n\to\infty} d(x_{2n},\ x_{2n+2}) = 0. \quad (11)$$

Now we shall show that $\{x_{2n}\}$ is a Cauchy sequence.

Suppose that $\{x_{2n}\}$ is not a Cauchy sequence. Then there exists $\epsilon > 0$ for which we can find subsequences $\{x_{2n(k)}\}$ and $\{x_{2m(k)}\}$ of $\{x_{2n}\}$ with $2m(k) > 2n(k) > k$ such that

$$d(x_{2m(k)},\ x_{2n(k)}) \geq \epsilon, \text{ for all } k \geq 1. \quad (12)$$

Corresponding to $2n(k)$, we can choose $2m(k)$ in such a way that it is the smallest even integer with $2m(k) > 2n(k)$ and satisfying (12). Then, for all $k \geq 1$, we have

$$d(x_{2m(k)},\ x_{2n(k)-2}) < \epsilon. \quad (13)$$

From (12) and (13), for all $k \geq 1$, we have

$$\begin{aligned} \epsilon & \leq d(x_{2m(k)},\ x_{2n(k)}) \\ & \leq d(x_{2m(k)},\ x_{2n(k)-2}) + d(x_{2n(k)-2},\ x_{2n(k)}) \\ & < \epsilon + d(x_{2n(k)-2},\ x_{2n(k)}). \end{aligned}$$

Taking $k \to \infty$, and using (11), we get

$$\lim_{k\to\infty} d(x_{2m(k)},\ x_{2n(k)}) = \epsilon. \quad (14)$$

Again, for all $k \geq 1$,

$$\begin{aligned} & d(x_{2m(k)},\ x_{2n(k)}) \\ & \leq d(x_{2m(k)},\ x_{2m(k)-2}) + d(x_{2m(k)-2},\ x_{2n(k)-2}) \\ & + d(x_{2n(k)-2},\ x_{2n(k)}) \end{aligned}$$

and

$$\begin{aligned} & d(x_{2m(k)-2},\ x_{2n(k)-2}) \\ & \leq d(x_{2m(k)-2},\ x_{2m(k)}) + d(x_{2m(k)},\ x_{2n(k)}) \\ & + d(x_{2n(k)},\ x_{2n(k)-2}). \end{aligned}$$

Taking $k \to \infty$ in the above two inequalities, and using (11), we get

$$\lim_{k\to\infty} d(x_{2m(k)-2},\ x_{2n(k)-2}) = \epsilon. \quad (15)$$

Again, by (ii) and (iii), for all $k \geq 1$,

$$\begin{aligned} & d(x_{2m(k)},\ x_{2n(k)}) \\ & = d(Tx_{2m(k)-1},\ Tx_{2n(k)-1}) \\ & \leq d(x_{2m(k)-1},\ x_{2n(k)-1}) \\ & = d(Tx_{2m(k)-2},\ Tx_{2n(k)-2}) \\ & \leq \alpha(d(x_{2m(k)-2},\ x_{2n(k)-2})) \\ & \quad d(x_{2m(k)-2},\ x_{2n(k)-2}) \\ & \leq d(x_{2m(k)-2},\ x_{2n(k)-2}). \end{aligned}$$

Taking $k \to \infty$ in the above inequality, and using (14) and (15), we get

$$\epsilon \leq \lim_{k\to\infty} \alpha(d(x_{2m(k)-2},\ x_{2n(k)-2}))\epsilon \leq \epsilon.$$

Hence,

$$\lim_{k\to\infty} \alpha(d(x_{2m(k)-2},\ x_{2n(k)-2})) = 1.$$

Since $\alpha \in S$, this implies that

$$\lim_{k\to\infty} d(x_{2m(k)-2},\ x_{2n(k)-2}) = 0,$$

which contradicts (15). Hence, $\{x_{2n}\}$ is a Cauchy sequence. Since the space is complete, and A is closed, there exists $x \in A$ such that

$$x_{2n} \to x \text{ as } n \to \infty. \quad (16)$$

Now, by (ii), for all $m, n \geq 1$,

$$\begin{aligned}&d(x_{2m+1},\ x_{2n+1})\\&= d(Tx_{2m},\ Tx_{2n})\\&\leq \alpha(d(x_{2m},\ x_{2n}))d(x_{2m},\ x_{2n})\\&\leq d(x_{2m}, x_{2n}).\end{aligned}$$

Since $\{x_{2n}\}$ is a Cauchy sequence, we have

$$\lim_{m,n\to\infty} d(x_{2m+1},\ x_{2n+1}) \leq \lim_{m,n\to\infty} d(x_{2m},\ x_{2n}) = 0.$$

This shows that $\{x_{2n+1}\}$ is a Cauchy sequence. Hence

$$x_{2n+1} \to y \text{ as } n \to \infty. \tag{17}$$

Then, by (ii), we have

$$\begin{aligned}&d(Tx,\ y)\\&\leq d(Tx,\ x_{2n+1}) + d(x_{2n+1},\ y)\\&= d(Tx,\ Tx_{2n}) + d(x_{2n+1},\ y)\\&\leq \alpha(d(x,\ x_{2n}))d(x,\ x_{2n}) + d(x_{2n+1},\ y)\\&\leq d(x,\ x_{2n}) + d(x_{2n+1},\ y) \to 0 \text{ as } n \to \infty.\\&\text{(by (16) and (17))}\end{aligned}$$

Hence,

$$Tx = y. \tag{18}$$

Again,

$$\begin{aligned}&d(Ty,\ x)\\&\leq d(Ty,\ x_{2n+2}) + d(x_{2n+2},\ x)\\&= d(Ty,\ Tx_{2n+1}) + d(x_{2n+2},\ x)\\&\leq d(y,\ x_{2n+1}) + d(x_{2n+2},\ x) \to 0 \text{ as } n \to \infty.\\&\text{(by } (iii), (16) \text{ and } (17))\end{aligned}$$

Therefore,

$$Ty = x. \tag{19}$$

If possible, suppose that, $d(A,\ B) < d(x,\ y)$. Then, by (18), (19) and by condition (iv) of the theorem, we get

$$d(x,\ y) = d(Ty,\ Tx) < d(x,\ y),$$

which is a contradiction. Therefore, $d(A,\ B) = d(x,\ y)$. Hence, by (18), we have

$$d(x,\ Tx) = d(A,\ B).$$

If x^* and x^{**} are two best proximity points of T, that is, $x^*, x^{**} \in A$ are such that $d(x^*,\ Tx^*) = d(x^{**},\ Tx^{**}) = d(A,\ B)$, then by (ii)

$$\begin{aligned}&d(x^*,\ x^{**})\\&\leq d(x^*,\ Tx^*) + d(Tx^*,\ Tx^{**}) + d(Tx^{**},\ x^{**})\\&\leq \alpha(d(x^*,\ x^{**}))d(x^*,\ x^{**}) + 2d(A,\ B)\\&< d(x^*,\ x^{**}) + 2d(A,\ B).\end{aligned}$$

Therefore $d(A,\ B) > 0$ and hence the sets A and B are necessarily disjoint. □

3. Example

Let $X = [0,\ 1] \times R$. A metric d is defined by

$$d((x_1,\ y_1), (x_2,\ y_2)) = |x_1 - x_2| + |y_1 - y_2|.$$

Let $A = \{(0,\ x)\colon\ x \in R \text{ and } x \geq 0\}$ and $B = \{(1,\ y)\colon\ y \in R \text{ and } y \geq 0\}$. Then A and B are closed subset of X and $d(A,\ B) = 1$.
Let $\alpha : [0, \infty) \to [0, 1)$ be defined as

$$\alpha(t) = \begin{cases} \frac{1}{1+t}, & \text{for } \ t > 0,\\ c(< 1), & \text{for } \ t = 0.\end{cases}$$

(A)

Let $T : A \cup B \to A \cup B$ be defined as

$$T((0, x)) = (1, \frac{x}{1+x}),\ \text{for all } x \geq 0,$$
$$T((1, y)) = (0, \frac{y}{1+y}),\ \text{for all } y \geq 0.$$

Then all the conditions of Theorem 2 are satisfied and $(0,\ 0)$ is a best proximity of the mapping T.

(B)

Let $T : A \cup B \to A \cup B$ be defined as

$$T((0, x)) = (1, \frac{1}{1+x}),\ \text{for all } x \geq 0,$$
$$T((1, y)) = (0, \frac{1}{1+y}),\ \text{for all } y \geq 0.$$

Then all the conditions of Theorem 3 are satisfied and $(0,\ -\frac{1}{2} + \frac{\sqrt{5}}{2})$ is a best proximity point of the mapping T.

4. Conclusion

The objects we have considered are two closed sets, rather than two points. It is through mappings which satisfy certain contractive conditions that iterations are constructed by which the minimum distances are realized. We have used cyclic mapping in our theorems. The contraction used in one of the theorems is cyclic while in the other is not cyclic. It is well-known that the contractive mappings have several applications in diverse branches of mathematics. This work is another instance of such applications where mappings obeying contractive conditions are applied to problems of global optimality.

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