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Economic production quantity (EPQ) model for three type imperfect items with rework and learning in setup

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Abstract. Imperfect quality items are unavoidable in an inventory system due to imperfect production process, natural disasters, damages, or many other reasons. The setup cost and production cycle time can be related in terms of process deterioration and learning and forgetting effects. Learning reduces production run length and setup cost, whereas deterioration and forgetting increases both. Keeping these facts in mind, this paper investigates an Economic Production Quantity (EPQ) model with imperfect quality items with varying set-up costs. Mathematical model and solution procedures are developed with major insight to its characteristics. Numerical example and sensitivity analysis are provided to illustrate and analyze the model performance. It can be observed that our model has a significant impacts on the optimal lot size and optimal profit of the model.

Keywords: Inventory; EPQ; imperfect quality; learning; rework.
AMS Classification: 90B05; 90B25; 90B30

1. Introduction

One of the most important strategic decisions for a manufacturing system is regarding inventory. Inventory plays the role of lifeblood in every manufacturing organization. Economic Production Quantity (EPQ) model is widely used in industries and academics to study the optimal production lot size that minimizes overall production as well as inventory storage costs. Although, the Economic Order Quantity (EOQ) and the Economic Production Quantity (EPQ) models have been successfully applied in the area of inventory management from a early decades of previous century; yet they bear few unrealistic assumptions. One of the unrealistic assumption in traditional EPQ model is that in order to obtain the optimal policy it is assumed that all the items considered consists of perfect quality. In any production-inventory model, it is observed that defective products are also produced alongside the useful products. The production process is not always perfectly in control due to occurrence of discontinuous power supply, aging, tool wear, overheating or vibration etc. In such situations production process may shift from 'in-control' state to 'out-of-control' state. The production process usually starts in an 'in-control' state and it produces items of 100% perfect quality. After some time, the process may deteriorate and it simultaneously shift to an 'out-of-control' state. From that point the process produces some percentage of non-conforming item. Since final products may contain some imperfect items, so when the production process is completed the inspection is performed before the products are dispatched to the customers. Inspection is nowadays an important aspect of production planning. It does not create quality but helps to control it

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by preventing defects by finding and eliminating defective products. By applying inspection and preventing poor quality products to reach to the customers; the reputation and goodwill of manufacturer can be maintained by providing standard and best quality products consistently. Since performing inspection only cannot improve the quality of the product, the defective products are reworked. Since the inspection separates the perfect quality product by two types of poor quality products: defective (reworkable) and imperfect (non-reworkable). Defective items are reworked to recreate perfect items but the imperfect items are sold at discounted price. During the rework process, a fraction of defective become scrap and discarded from inventory.

Economic order quantity or Economic production quantity type models are used to determine the optimal inventory or production policy when the demand is deterministic and the optimal ordering or production quantity are influenced by two opposite types of costs. The classical EPQ model is developed under the condition of fixed setup cost. However, setup cost is found to be a function of the production run length (Darwish [12]). It is observed that production processes with small production runs require less setup cost than that of long runs. The reason behind this is that, the effort required to perform a setup activity depends on the type and quality of the production process. That is, for long production runs, the production process is more likely to be subjected to higher level of deterioration resulting in a higher setup cost. Deteriorating production processes has been observed in many applications; such as, steel manufacturing, wood stuffs, plastic industry, food processing and machine industry. The dependency between setup cost and production run length is also effected by the learning and forgetting effects. Learning in setup enhances smaller lot-size and shorter runs to be produced more frequently. The effect of forgetting in setup have an opposite impact on the production run length because forgetting increase production runs to long duration, which eventually results in higher setup cost. Thus, the setup cost and run length are interdependent and this fact should not be neglected in developing the mathematical model for a production inventory system. In order to get the idea of present research trends in the vicinity of our problem, literature review is provided in the next section.

2. Literature Review

The classical economic order quantity (EOQ) and economic production quantity (EPQ) models were two traditional inventory model which were popular among researchers and management professionals for their simplicity. But it is a fact that they seem to be based on few unrealistic assumptions. One of them is assuming that all the quantity considered is of perfect quality. In reality, the production process is not always free of defects.A fraction of the items produced always comes with defects. The main key of a successful business is to provide the customer his demand within shortest possible time, with the best quality, and all at a competitive price. That will be easier if the price of production can be reduced by taking advantage of some discount and the production process involves 100 % inspection, so that there is no chance of presence of defective items in the product. During last decades there have been a lot-of work done in the area of EOQ and EPQ of Imperfect quality items. Rosenblatt and Lee [1] discussed an EPQ model where they assumed that the defective items could be reworked instantaneously at a cost and found that the presence of defective products motivates smaller lot sizes. Shwaller [2]presented a procedure and assumed that imperfect quality items are present in a known proportions and considered fixed and variable inspection costs are applied for finding and removing the item. Zhang and Gherchak [3] considered a joint lot sizing and inspection policy here a random proportion of lot size were defective. They assumed that defective items are not reworkable and thus assumed the concept of replacement of them by the good quality items Salameh and Jaber [4] assumed that the defective items could be sold at a discounted price in a single batch by the end of the 100 % testing process and found that the economic lot size quantity tends to increase as the average percentage of imperfect quality items increases. Goyal et al. [5] made some modifications in the model of Salameh and Jaber [4] to calculate actual cost and actual order quantity. Ouyang et al. [6] studied the collective effect of quality improvement, setup cost reduction and lead time reduction in an imperfect production process which is observed to effect the lot size significantly. Papachristos et al. [7] pointed out that the sufficient conditions for prevent shortages given in Salameh and Jaber [4] may not really prevent their occurrence and considering the timing of withdrawing the imperfect quality items from stock, they clarified a point not clearly stated in Salameh and Jaber [4]. Jaber [8] incorporated the concept of learning in setup and quality improvement in the extension of the inventory model with imperfect quality and showed Leaning in setup decreases the cost up to large extent. Wee et al. [9] develops a optimal inventory model for items with imperfects quantity and shortage backorder. They allow 100% testing of items which is greater than the demand rate. Maddaha and Jaber [10] made some suitable corrections in Salameh and Jaber [4] model and showed that error resulting from previous model [4] is minor. Chung [11] et al considered an inventory model with imperfect quality items under the condition of two warehouses for storing items. A detailed survey of the recent inventory models with imperfect items are provided by Khan et al. [13]. Mukhopadhyay and Goswami [18] developed an inventory model for imperfect items with exponentially decreasing time varying demand and constant deterioration where shortages were partially backlogged. Al-Salamah and Alsawafy [14] considered an EOQ model with two types of imperfect items and obtained the optimal policy that maximizes the total profit. They found that lot size increases with the increase in the fraction of scrap items and re-workable item but expected total profit decreases. Mukhopadhyay and Goswami [18] developed an inventory model for imperfect items when demand followed time varying linear pattern. The cost minimization optimal policy was considered by Tsou [15] when the produced item of imperfect production system obeys general distribution pattern, with its quality being either perfect, imperfect or defective. The fractions of such items were restricted to constants and they also established that their model becomes classical EPQ model in case imperfect quality percentage is zero or even close to zero. Recently, Wee [17] used renewal reward theorem to construct economic production quantity model for imperfect items with shortage and screening constraint using time interval as decision variable and shown the robustness of the model.

The purpose of this paper is to bring to point that how the recent models over estimates cost in such an environment where one of the major cost of the model largely reduced because of experience of previous production cycle. Thus, present paper concentrates on detailed aspects of imperfect production inventory model, which focuses on practical facet of production in which the manager get advantages of experience and learning to reduce his total production inventory cost. This model generalists the previous models developed by Al-Salmah et al. [14] and various other models in the literature. This work emphasizes the newer research paradigms and account for accurate cost calculation which will motivate the manager to maximize his profit.

The paper is organized as follows. Section 1 contains introduction and Section 2 contains literature review. In Section 3, our mathematical model is developed. Section 4 contains discussion on some facts about our model. Section 5 contains numerical examples and section 6 contains conclusion and scope of future work based on our model. Reference is provided at the end.

3. Development of Mathematical Model

In our model we have considered the situation of a manufacturer which produces a single type of item at constant production rate P in order to satisfy a constant demand rate of D such that $P > D$. Both production and demand cycle start at $t = 0$ at which inventory is zero; the inventory reaches its maximum value at the production completion. But since the manufactured lot contain a small although detectable fraction of imperfect items, they must be screened out before delivery to the customer through a 100% screening process, the item produced are subjected to screening process slower than the production process. The screening process finishes after production end and it separates the final products in four categories perfectly good, reworkable, non-reworkable and scraps. The perfectly good items are kept for satisfying demand, reworkable items are subjected to rework at some cost to make them perfectly good and stored in main inventory. Those items classified as non-reworkable are sold at a discounted price and the scrap type items are discarded at some disposal cost.

Now, If we include the effect of learning in setup by considering variable setup cost as a function of production run length T_P:

$$\begin{aligned} c_o(T_P) &= c_0(T_P^{\epsilon}) \quad for \quad T_P < T_M \\ &= c_{max} \quad\ \ for \quad T_P \geq T_M \end{aligned} \qquad (1)$$

Here, the term ϵ is the shape factor of setup cost, the constant $c_0 > 0$ can be interpreted as setup cost associated with the basic EPQ model for imperfect items and T_M is the minimum run length beyond which the setup activity requires a cost c_{max}; also, the parameter c_{max} serves as an upper limit on the setup cost. The parameter ϵ estimated by the curve-fitting method using the historical data on setup cost of the previous production processes. Figure 1 shows the setup cost function for particular values of ϵ.

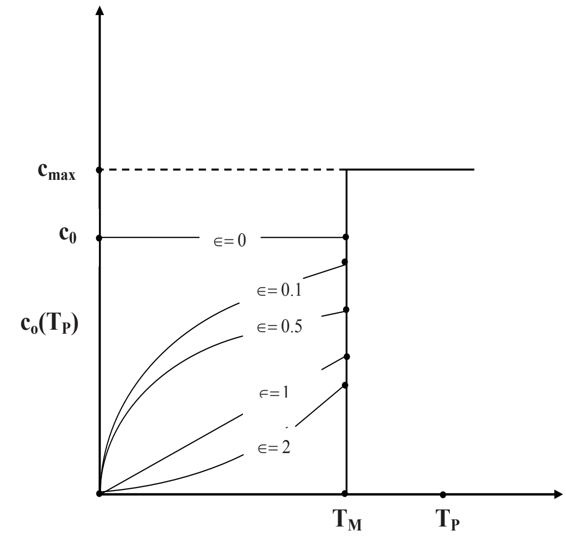

Figure 1. Behavior of set up cost

Notations:

Following notations are used in the model-

D : demand rate;

P : production rate;

x : screening rate;

y : production lot size;

c_h : holding cost per unit, per unit time;

c_o : set-up(organization) cost ;

c_p : production cost per unit;

c_s : screening cost per unit;

c_r : rework cost per unit for defective items;

p_1, p_2 and p_3 : random variables denoting the non-reworkable imperfect items, reworkable defective items and scrap items respectively;

$E[p_i]$: probabilistic expectation of p_i for $i = 1, 2, 3$;

$'$ and $''$: first and second order derivatives w.r.t. decision variable y;

$U(a, b)$: uniform probability distribution with parameters a and b where $a < b$.

Further notations may be introduced when required.

Assumptions:

Following assumptions are considered in the model,

(1) Production process is not perfect and it produce only single product.
(2) Demand rate is constant.
(3) Screening process outputs three types of items-perfect, imperfect and defectives; perfect items are to be sold at full price. Imperfect items are sold at reduced price. Defective items are reworked and which further produce perfect imperfect and scraps. Scraps may be either to be sold at discounted price or just disposed.
(4) The screening process and demand proceeds simultaneously, screening rate is higher than the demand rate. i.e. x : screening rate; $x > D$
(5) The fraction of non-reworkable imperfect items (p_1), reworkable defective items(p_2) and scrap items(p_3) are uniformly distributed random variables for a lot size, i.e. $p_i \sim U(a_i, b_i)$ where $0 \leq a_i \leq b_i < 1$. For validity of the model we assume that: $E[p_1]+E[p_2]+E[p_3] < 1$. Above assumption is realistic, because the produced lot must contain some good quality items.

To avoid shortages during screening period, demand during screening should be at least equal to the number of perfect

items
$y(1 - E[p_1] - E[p_3]) \leq \frac{Dy}{x} \Rightarrow E[p_1] + E[p_3] \leq 1 - \frac{D}{x}$.

(6) Production rate is finite, constant and satisfy the inequality $P(1 - E[p_1] - E[p_3]) > D$.

(7) With course of time manufacturer learn from experience, which results in less setup time and cost.

(8) The time horizon is infinite.

(9) The lot size y is a decision variable in the model.

(10) To avoid lost-sales and loss of goodwill, shortages is ignored.

The behavior of inventory position with course of time is demonstrated in Figure 2.

Now we have:

expected amount of perfect items $= y(1-E[p_1]-E[p_3])$

and expected cycle time $T = \frac{y(1-E[p_1]-E[p_3])}{D}$

Production run- length $T_P = \frac{y}{P}$

Set-up(organization) cost $= c_o(T_P)$.

This implies,

$$\begin{aligned} c_o(T_P) &= c_0(\frac{y^\epsilon}{P^\epsilon})\text{for} T_P < T_M \\ &= c_{max} \text{ for } T_P \geq T_M \end{aligned} \quad (2)$$

Production cost $= c_p y$,

Screening cost $= c_s y$,

Rework cost $= c_r E[p_2] y$,

Holding cost (see Appendix-1)= $c_h\bigg[(E[p_1] + E[p_3])\frac{y^2}{x} + \frac{y^2 E[(1-p_1-p_3)^2]}{2D} - \frac{y^2}{2P}\bigg]$.

The total expected cost is given by the following expression:

$$\begin{aligned} TC(y) &= c_o + c_p y + c_s y + c_r(E[p_2]y + \\ &\quad c_h\bigg[(E[p_1] + E[p_3])\frac{y^2}{x} \\ &\quad + \frac{y^2 E[(1-p_1-p_3)^2]}{2D} - \frac{y^2}{2P}\bigg] \end{aligned} \quad (3)$$

The expected total average cost per cycle is given by:

$$TAC(y) = \frac{TC(y)}{T} = \frac{D \cdot TC(y)}{y(1 - E[p_1] - E[p_3])}$$

Case 1: When $T_P < T_M$ In this case, substituting the appropriate formulation of c_o, we obtain expected total average cost as follows:

$$\begin{aligned} TAC(y) = \Bigg[&\frac{c_0 y^{(\epsilon-1)}}{P^\epsilon} + (c_p + c_s + c_r E[p_2]) + \\ &c_h y\bigg(\frac{(E[p_1] + E[p_3])}{x} - \frac{1}{2P} + \\ &\frac{E[(1-p_1-p_3)^2]}{2D}\bigg)\Bigg]\frac{D}{1 - E[p_1] - E[p_3]}. \end{aligned} \quad (4)$$

From the formulation of the above expected cost function, it is clear that when $\epsilon \geq 1$, the expected cost function is monotonic increasing in y, which shows that $TAC(y)$ will be minimum when $y = 0$; which is quite unrealistic. In practical sense, this may suggest that y should be as minimum as possible so that the total cost may be minimum- which is in accordance with the JIT Philosophy of production. we consider the case only when $\epsilon < 1$. So from this point onwards, whenever we use ϵ it is assumed to be strictly less than 1. Further $c_o(T_P)$ is concave and increasing for $0 < \epsilon \leq 1$ but decreasing for $\epsilon < 0$. The case $\epsilon < 0$ represents the situation when effect of learning overcomes effect of forgetting and deterioration resulting in the decrement of set up cost with time.

So when $\epsilon < 1$, using calculus we obtain that: Thus we have

$$\begin{aligned} TAC'(y) = &\Bigg[\frac{(\epsilon-1)c_0 y^{(\epsilon-2)}}{P^\epsilon} + (c_p + c_s \\ &+ c_r E[p_2])\frac{D}{1 - E[p_1] - E[p_3]} \\ &+ c_h\bigg(\frac{(E[p_1] + E[p_3])}{x} - \frac{1}{2P} \\ &+ \frac{E[(1-p_1-p_3)^2]}{2D}\bigg)\Bigg] \\ &\frac{D}{1 - E[p_1] - E[p_3]} \end{aligned} \quad (5)$$

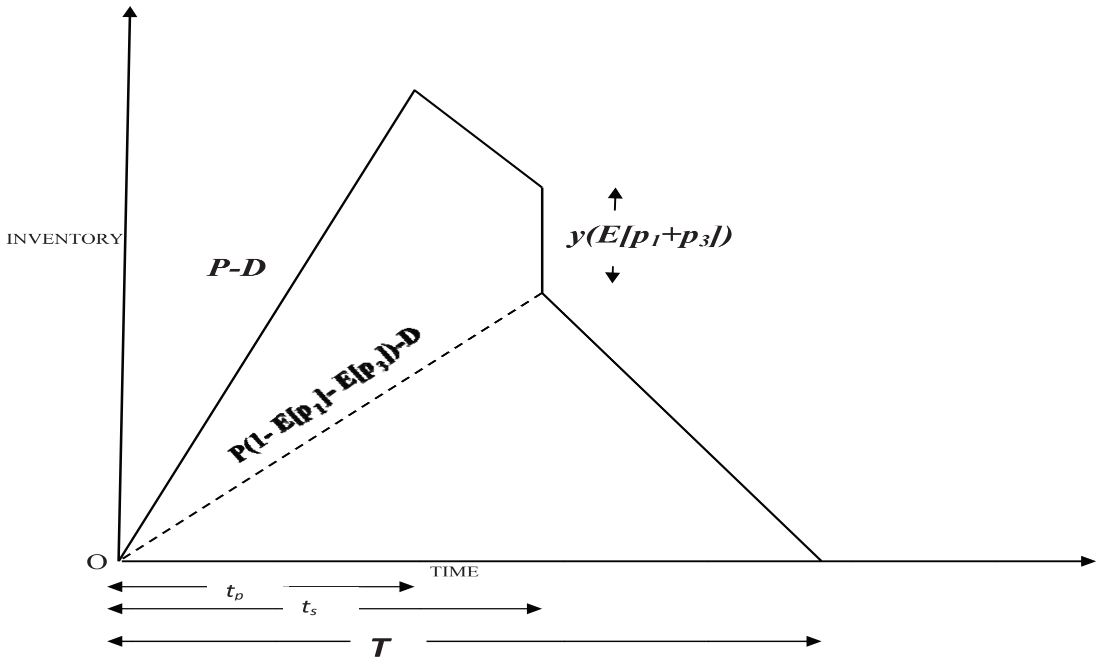

Figure 2. Inventory versus Time in our model

and,

$$TAC''(y) = \frac{(1-\epsilon)(2-\epsilon)Dc_0}{(1-E[p_1]-E[p_3])y^{3-\epsilon}} \qquad (6)$$

Clearly $TAC''(y) > 0$, since numerator and denominator of $TAC''(y)$ are both positive. So, $TAC(y)$ is convex in y. Thus, optimum EPQ is given by the equation $TAC'(y) = 0$.

$$y^* = \left[\frac{2(1-\epsilon)c_0 D}{c_h\left(\frac{2(E[p_1]+E[p_3])D}{x}+E[(1-p_1-p_3)^2]-\frac{D}{P}\right)}\right]^{\frac{1}{2-\epsilon}} \qquad (7)$$

Case 2: When $T_P > T_M$

The expression for expected total average cost in this case is obtained as follows:

$$\begin{aligned} TAC(y) &= \Bigg[\frac{c_{max}}{y} + (c_p + c_s + c_r E[p_2]) \\ &\quad + c_h\Bigg(\Big[\frac{yE[(1-p_1-p_3)^2]}{2D} + \\ &\quad \frac{(E[p_1]+E[p_3])y}{x}\Big](1-\frac{D}{P})^2 \\ &\quad + (1-\frac{D}{P})\frac{y}{2P}\Bigg)\Bigg]\frac{D}{1-E[p_1]-E[p_3]} \end{aligned} \qquad (8)$$

Differentiating with respect to y twice successively, we obtain

$$\begin{aligned} TAC'(y) &= \Bigg[\frac{-c_{max}}{y^2} + c_h\Bigg[\frac{(E[p_1]+E[p_3])}{x} + \\ &\quad \frac{E[(1-p_1-p_3)^2]}{2D} - \frac{1}{2P}\Bigg]\Bigg] \\ &\quad \frac{D}{(1-E[p_1]-E[p_3])} \end{aligned} \qquad (9)$$

and,

$$TAC''(y) = \frac{2Dc_{max}}{(1-E[p_1]-E[p_3])y^3}. \qquad (10)$$

(Since $0 \leq E[p_1] + E[p_3] < 1, y > 0, (P-D) > 0, c_o > 0; TAC''(y) > 0$)

So, $TAC(y)$ is convex in y in this case also. Thus, optimum EPQ is given by the equation $TAC'(y) = 0$

And the optimum EPQ is:

$$y^* = \sqrt{\frac{2Dc_{max}}{c_h\left[\frac{2(E[p_1]+E[p_3])D}{x}+E[(1-p_1-p_3)^2]-\frac{D}{P}\right]}} \qquad (11)$$

Remark: From (7) and (11), it is obvious that when $\epsilon = 0$ and $c_{max} = c_0$; we can have identical results.

<u>*Special cases*</u>:

(i) Substituting $p_2 = p_3 = 0$ in (10), we obtain the simple EPQ with one type of imperfect item.

$$y^* = \sqrt{\frac{2c_0D}{c_h\left[\frac{2(E[p_1])D}{x} + E[(1-p_1)^2] - \frac{D}{P}\right]}} \quad (12)$$

(ii) When $P = \infty$ in (12), we obtain the simple EOQ with one type of imperfect item.

$$y^* = \sqrt{\frac{2c_0}{c_h\left[\left[\frac{E[(1-p_1)^2]}{D} + \frac{2(E[p_1])y}{x}\right]\right]}} \quad (13)$$

It should be noted that above equation corresponds to equation (7) of Maddaha and Jaber[10] EOQ model for imperfect items.

(iii) Further when, $p_1 = 0$ in (13), we get:

$$y^* = \sqrt{\frac{2c_0D}{c_h(1-\frac{D}{P})}} \quad (14)$$

which is only the basic EPQ formula with perfect items.

(iv) Finally putting, $P = \infty$ in (14), we get:

$$y^* = \sqrt{\frac{2c_0D}{c_h}} \quad (15)$$

which is nothing but the classical EOQ formula.

4. Numerical Example

Let us take:
$c_p = 0.1/$ unit, $c_h = 15/$ unit/year , $D = 50000$ units, ,$P = 60000$ units, $c_s = 0.02$, Inspection rate, $x = 1$unit/min $= 175200$ unit / year.

Lest us consider the random variables of imperfect fraction are uniformly distributed as follows:

$p_1 \sim U(0, 0.04)$, $p_2 \sim U(0.01, 0.02)$, $p_3 \sim U(0.01, 0.03)$.

Using the formula in appendix 1 we obtain: $E[(1-p_1-p-2)^2] = 0.9618$.

Now, we define setup cost(from Darwish[12]) as

$$c_s(T_P) = \begin{cases} 100T_P^{0.3}, & T_P \leq T_M, \\ 120, & T_P > T_M, \end{cases}$$

i.e. $c_0 = 100$, $c_{max} = 120$, $\epsilon = 0.3$

For $T_P < T_M$,
optimal production lot size $y^* = 6915.47$ units ; and corresponding optimal cost= 1398963.77 $

For $T_P \geq T_M$,
optimal production lot size y^*=2403.75 and corresponding optimal cost= 1398963.86 $

For various values of ϵ, the effect of change in optimal lot size and optimal cost is provided in Table 1.

Table 1. Effect of ϵ on optimal lot size and total cost

ϵ	y^*	TAC
0.1	3112.25	1398963.85
0.2	4557.39	1398963.80
0.3	6915.47	1398963.75
0.4	10913.63	1398963.67
0.5	17963.03	1398963.55
0.6	30834.89	1398963.47

5. Some Observations

From the model it is observed that:

(i) The presence of reworkable imperfect items does not affect the production lot size which is also clear from the expression of the EPQ. This phenomenon is also consequence of the fact that after the rework the reworked items becomes perfect and it is accumulated together with freshly perfect items. Also the availability of fraction the reworkable defective items increases the cost to some extent because of the additional cost which must be incurred for rework process.

(ii) From the Table 1, it is clear that the parameter ϵ determines the learning factor and thus higher learning reduces the cost and increases the production lot size. Thus the average cost of production per unit item is highly reduced.

(iii) The non-learning model can be trivial case of our model when, $\epsilon = 0$.

6. Conclusion

All items produced are not always perfect in a production system. Learning helps cost reduction in setting up the production. Learning in setting up process is found to decrease the run-length where as forgetting in setup process increase it.In view of these, present paper has considered Economic Production Quantity (EPQ) model with three types of imperfect quality items for learning in setup process. The setup cost and run length can be related in terms of process deterioration and learning and forgetting effects. We have developed a model where the setup cost is a function of production run length. We have considered the case of learning to developed our model. Also we have considered three types of imperfect items fraction which are independent random variables. Above two are major contribution of our work in the literature. It should be noted that although the presence of reworkable items have effect on the optimal lot size of the model by Salmah et al. [14] but this does not happen in our model because reworked items joins the main lot of perfect items. But the presence of more reworkable items of course increase some cost in model due to reworking process which diminish the optimal profit. Above work can be extended to consider error in inspection process. In our working paper we are in the process of modifying this model to one with shortages, time varying demand and Entropy cost.

Appendix-1

Derivation of Holding cost

We have screening time $t_s = \frac{y}{x}$, production time $t_p = \frac{y}{P}$

$\frac{dI(t)}{dt} = P - D$ for $0 \le t \le t_p$

subject to initial condition, $I(0) = 0$;

which gives $I(t) = (P - D)t$ for $0 \le t \le t_p$.

Again, $\frac{dI(t)}{dt} = -D$ for $t_p \le t \le t_s$

subject to initial condition, $I(t_p) = (P - D)t_p = (P - D)\frac{y}{P}$.

This gives $I(t) = y - Dt$ for $t_p \le t \le t_s$.

Initial inventory at t_s, $I(t_s) = y - D\frac{y}{x}$.

Final inventory at t_s, $I(t_s) = y - D\frac{y}{x} - y(p_1 + p_3)$.

Also, $\frac{dI(t)}{dt} = -D$ for $t_s \le t \le T$

subject to the initial condition,

$$I(t_s) = y - D\frac{y}{x} - y(p_1 + p_3).$$

This gives $I(t) = y(E[p_1] + E[p_3]) - Dt$ for $t_s \le t \le T$.

Thus Holding Cost $= c_h\big[\int_0^{t_p} I(t)dt + \int_{t_p}^{t_s} I(t)dt + \int_{t_s}^{T} I(t)dt\big]$
$= c_h\big[(p_1 + p_3)\frac{y^2}{x} + \frac{y^2(1-(p_1+p_3))^2}{2D} - \frac{y^2}{2P}\big]$.

Since p_1 and p_3 are random variables, we have:

Holding Cost $= c_h\big[(E[p_1] + E[p_3])\frac{y^2}{x} + \frac{y^2 E[(1-p_1-p_3)^2]}{2D} - \frac{y^2}{2P}\big]$.

Derivation of $E[(1 - p_1 - p_3)^2]$

We have $E[(1 - p_1 - p_3)^2] = E[1 + (p_1 + p_3)^2 - 2(p_1 + p_3)] = 1 + E[(p_1 + p_3)^2] - 2E[p_1] - 2E[p_3]$

Using the relation: $\mathrm{Var}(X) = (E[X])^2 - E[X^2]$,

$E[(p_1+p_3)^2] = \mathrm{Var}(p_1)+\mathrm{Var}(p_3)+(E[p_1]+E[p_3])^2 = \mathrm{Var}(p_1)+\mathrm{Var}(p_3) + (E[p_1])^2 + (E[p_3])^2 + 2E[p_1]E[p_3]$

Thus,

$E[(1 - p_1 - p_3)^2] = 1+\mathrm{Var}(p_1)+\mathrm{Var}(p_3) + (E[p_1])^2+(E[p_3])^2+2E[p_1]E[p_3]-2E[p_1]-2E[p_3]$.

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