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Generalized transformation techniques for multi-choice linear programming problems

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Abstract. The multi-choice programming allows the decision maker to consider multiple number of resources for each constraint or goal. Multi-choice linear programming problem can not be solved directly using the traditional linear programming technique. However, to deal with the multi-choice parameters, multiplicative terms of binary variables may be used in the transformed mathematical model. Recently, Biswal and Acharya [2] have proposed a methodology to transform the multi-choice linear programming problem to an equivalent mathematical programming model, which can accommodate a maximum of eight goals on the right hand side of any constraint. In this paper we present two models as generalized transformation the multi-choice linear programming problem. Using any one of the transformation techniques a decision maker can handle a parameter with finite number of choices. Binary variables are introduced to formulate a non-linear mixed integer programming model. Using a non-linear programming software optimal solution of the proposed model can be obtained. Finally, a numerical example is presented to illustrate the transformation technique and the solution procedure.

Keywords: Linear programming; mixed integer programming; multi-choice programming; non-linear programming; transformation technique

AMS Classification: 90C11, 90C30, 90C05

1. Introduction

Most of the input parameters in real-world decision making problems exhibit some level of uncertainty due to the scarcity of data (Dantzig [8]). Traditional linear programming (LP) addresses only practical problems in which all parameters are deterministic. This includes objective function costs, constraint coefficients and right-hand side (RHS) parameters. Thus, it is important to find ways of using LP methods with uncertainties in probabilistic, possibilistic or interval formats. The methods developed to deal with different LP models could be grouped into stochastic linear programming (SLP), fuzzy linear programming (FLP) and interval linear programming (ILP). However, in some cases it is believed that the parameters or coefficients in the decision making problems are multi-choice in nature.

The situation of multiple choices for a parameter exists in many managerial decision making problems. The multi-choice programming can not only avoid the wastage of resources but also decide on the appropriate resource from multiple resources. Multi-choice programming is a mathematical programming problem, in which decision maker is allowed to set multiple number of choices for a parameter. Hiller and Lieberman [9] and Ravindran et al. [14] have considered a mathematical model in which an appropriate constraint is to be chosen using binary variables. The number of binary variables required for a constraint is the same as the total number of choices for that constraint. March and Shapira

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[12] discussed that decision makers (DMs) can have two aspiration levels for organizational performance that influence risk-taking. In their model two aspiration levels are considered simultaneously by raising organizational performance to reach silent success level (e.g., market leadership) and keeping organizational performance to avoid falling below survival level (e.g., bankruptcy). In Chang [5], the author has used binary variables to present mixed binary goal programming to take decision on management problems where some goals are met and some are not met. However, in some cases the actual result of the operation is higher than the pre-defined target values. It implies that the experts or DMs underestimate the original aspiration level. In fact, higher target values could be reached under current available resources. Using this concept Chang [6] has proposed formulation of multi-choice goal programming (MCGP), which allows DMs to set multi-choice aspiration levels (MCAL) for each goal (i.e., one goal mapping multiple aspiration levels) to avoid underestimation of decision making. He used multiplicative terms of binary variables to handle the multiple aspiration levels. The transformed mathematical model proposed by Chang [6] is clearly understood by industrialist when the number of aspiration levels assigned to a goal is a power of 2. When the number of aspiration levels is not a power of 2, the model is not clearly understood (i.e. it remains silent on some binary codes). In his other paper [7], he replaces multiplicative terms of the binary variables using continuous variable. Liao [11] proposes a formulation method to solve multi-segment goal programming problem, which obtains a solution close to the DMs multi-segment aspiration levels. The author has handled the multi-choice parameters in the same way as the author has done in Chang [6]. In his work a multi-choice parameter can accommodate at best three choices. Recently, we have established equivalent models for multi-choice linear programming problem (MCLPP) [2] that can accommodate at best eight alternatives for a goal. In this paper we present two generalized transformation techniques to transform an MCLPP to an equivalent mathematical programming model. Using any one of these transformation techniques the transformed model can be derived. Using standard non-linear programming techniques optimal solution of the proposed model can be obtained. For recent developments on multi-choice linear programming problem, one may refer to [1, 3].

The organization of the paper is as follows: following the introduction in Section 1, mathematical model of MCLPP is presented in Section 2. The transformation techniques to derive equivalent mathematical models of MCLPP are presented in Section 3. In order to verify the proposed transformation techniques, an example is presented in Section 4. Finally, results and discussion, and conclusions are presented in Section 5 and Section 6 respectively.

2. Mathematical Model

The mathematical model of a MCLPP is presented as follows:

Find $X = (x_1, x_2, x_3, \ldots, x_n)$ so as to

$$\max : Z = \sum_{j=1}^{n} c_j x_j \tag{1}$$

subject to

$$\sum_{j=1}^{n} a_{ij} x_j \leq \{b_i^{(1)}, b_i^{(2)}, b_i^{(3)}, \ldots, b_i^{(k_i)}\}, \tag{2}$$

$$i = 1, 2, 3, \ldots, m$$

$$x_j \geq 0, \ j = 1, 2, 3, \ldots, n \tag{3}$$

RHS of the constraints (2) has k_i number of parameters where only one parameter is to be selected.

3. Transformation Techniques for MCLPP

We present two transformation techniques of MCLPP to formulate an equivalent mathematical model.

3.1. Transformation technique 1

Restrictions are given on the upper bound of binary variables.

Step-1: Select i-th (i=1,2,3,...,m) constraint from the MCLPP. Find the total number of choices for i-th constraint.

$$\sum_{j=1}^{n} a_{ij} x_j \leq \{b_i^{(1)}, b_i^{(2)}, b_i^{(3)}, \ldots, b_i^{(k_i)}\} \tag{4}$$

The total number of choices for first constraint is k_i. We suppose that $k_i \geq 2$.

Step-2: Find the number of binary variables, which is required to handle the multi-choice parameters in RHS of the constraint in following manner.

Find l_i, for which $2^{(l_i-1)} < k_i \leq 2^{l_i}$. Here l_i number of binary variables are needed. Let the binary variables are $z_i^{(1)}$, $z_i^{(2)}$, $z_i^{(3)}$, $\ldots$, $z_i^{(l_i)}$.

Step-3: Expand 2^{l_i} as $\binom{l_i}{0}+ \binom{l_i}{1}+ \binom{l_i}{2}+ \ldots+\binom{l_i}{r_{i1}} +\ldots+\binom{l_i}{r_{i2}}+ \ldots+ \binom{l_i}{l_i}$ and select the smallest number of consecutive terms whose sum is 'equal to' or 'just greater than' k_i from the expansion. Let the terms be $\binom{l_i}{r_{i1}}$, $\binom{l_i}{r_{i1}+1}$, $\binom{l_i}{r_{i1}+2}$, $\ldots$, $\binom{l_i}{r_{i2}}$.

Step-4: Assign k_i binary codes to k_i number of choices for first constraint as follows:

$$\begin{aligned}\sum_{j=1}^{n} a_{ij}x_j \quad &\leq \quad \sum_{j=1}^{\binom{l_i}{r_{i1}}} P_j^{(r_{i1})}Q_j^{(r_{i1})}b_i^{(j)}\\
&+ \sum_{j=1}^{\binom{l_i}{r_{i1}+1}} P_j^{(r_{i1}+1)}Q_j^{(r_{i1}+1)}b_i^{(\binom{l_i}{r_{i1}}+j)} + \ldots\\
&+ \sum_{j=1}^{\binom{l_i}{(r_{i2}-1)}} P_j^{(r_{i2}-1)}Q_j^{(r_{i2}-1)}b_i^{(\binom{l_i}{r_{i1}}+\ldots+\binom{l_i}{r_{i2}-2}+j)}\\
&+ \sum_{j=1}^{(k_i-L_i^{(1)})} P_j^{(r_{i2})}Q_j^{(r_{i2})}b_i^{(L_i^{(1)}+j)} \qquad (5)\end{aligned}$$

where $L_i^{(1)} = \binom{l_i}{r_{i1}} + \binom{l_i}{r_{i1}+1} + \ldots + \binom{l_i}{r_{i2}-1}$
$j_1 \in \{1, 2, 3, ..., (l_i-s)+1\}$, $j_2 \in \{2, 3, ..., (l_i-s)+2\}$, $\ldots$, $j_s \in \{s, s+1, ..., l_i\}$
$I_s^{(j)} = \{\{j_1, j_2, \ldots, j_s\} | j_1 < j_2 < \ldots < j_s, s = r_{i1}, r_{i1}+1, ..., r_{i2}\}$
$P_j^{(si)}=\{z_i^{(j_1)}z_i^{(j_2)}z_i^{(j_3)}\ldots z_i^{(j_s)} | \{j_1, j_2, j_3, \ldots, j_s\} \in I_s^{(j)}, s = r_{i1}, r_{i1}+1, ..., r_{i2}\}$
$Q_j^{(si)} = \{\prod_{j=1}^{l_i}(1-z_i^{(j)}) | j \notin \{j_1, j_2, \ldots, j_s\}\}$

Step-5: Restrict $(2^{l_i} - k_i)$ number of binary codes to overcome repetitions as follows:

$$z_i^{(1)} + z_i^{(2)} + z_i^{(3)} + \ldots + z_i^{(l_i)} \geq r_{i1} \qquad (6)$$

$$z_i^{(1)} + z_i^{(2)} + z_i^{(3)} + \ldots + z_i^{(l_i)} \leq r_{i2} \qquad (7)$$

$$z_i^{(j_1)} + z_i^{(j_2)} + z_i^{(j_3)} + \ldots + z_i^{(j_{r_{i2}})} \leq r_{i2} - 1,$$
$$j = (k_i - L_i^{(1)}) + 1, (k_i - L_i^{(1)}) + 2, \ldots, \binom{l_i}{r_{i2}} \qquad (8)$$

Restrictions should be imposed on $z_i^{(j_1)}z_i^{(j_2)}z_i^{(j_3)}\ldots z_i^{(j_{r_{i2}})} \in P_j^{(r_{i2}i)}$, but not included in $T_{r_{i2}}$. $T_{r_{i2}}$ contains the terms $P_j^{(r_{i2}i)}$ in transformed constraint (5).

Step-6: Formulate the mathematical model as:

$$\max : Z = \sum_{j=1}^{n} c_j x_j \qquad (9)$$

$$\begin{aligned}\text{subject to } \sum_{j=1}^{n} a_{ij}x_j &\leq \sum_{j=1}^{\binom{l_i}{r_{i1}}} P_j^{(r_{i1})}Q_j^{(r_{i1})}b_i^{(j)}\\
&+ \sum_{j=1}^{\binom{l_i}{r_{i1}+1}} P_j^{(r_{i1}+1)}Q_j^{(r_{i1}+1)}b_i^{(\binom{l_i}{r_{i1}}+j)} + \ldots\\
&+ \sum_{j=1}^{\binom{l_i}{(r_{i2}-1)}} P_j^{(r_{i2}-1)}Q_j^{(r_{i2}-1)}b_i^{(\binom{l_i}{r_{i1}}+\ldots+\binom{l_i}{r_{i2}-2}+j)}\\
&+ \sum_{j=1}^{(k_i-L_i^{(1)})} P_j^{(r_{i2})}Q_j^{(r_{i2})}b_i^{(L_i^{(1)}+j)}, i = 1, ..., m \qquad (10)\end{aligned}$$

$$z_i^{(1)}+z_i^{(2)}+\ldots+z_i^{(l_i)} \geq r_{i1}, \quad i = 1, 2, 3, .., m \qquad (11)$$

$$z_i^{(1)}+z_i^{(2)}+\ldots+z_i^{(l_i)} \leq r_{i2}, \quad i = 1, 2, 3, .., m \qquad (12)$$

$$z_i^{(j_1)} + z_i^{(j_2)} + \ldots + z_i^{(j_{r_{i2}})} \leq r_{i2} - 1,$$
$$j = (k_i-L_i^{(1)})+1, (k_i-L_i^{(1)})+2, \ldots, \binom{l_i}{r_{i2}} \qquad (13)$$

$$x_j \geq 0, \; j = 1, 2, 3, \ldots, n \qquad (14)$$

$$\begin{aligned}z_i^{(l_i)} = 0/1, \; l_i \quad &= \quad 1, 2, 3, \ldots, \lceil \frac{\ln(k_i)}{\ln 2} \rceil, \qquad (15)\\ i \quad &= \quad 1, 2, 3, \ldots, m.\end{aligned}$$

where $L_i^{(1)} = \binom{l_i}{r_{i1}} + \binom{l_i}{r_{i1}+1} + \ldots + \binom{l_i}{r_{i2}-1}$.

Step-7: Above mathematical model is a mixed integer non-linear programming problem. Solve the model with the help of existing methodology.

3.2. Transformation technique 2

Restriction is given on the lower bound of the binary variables.

Step-1: Select i-th constraint from the MCLPP. Find the total number of choices for i-th constraint.

$$\sum_{j=1}^{n} a_{ij}x_j \leq \{b_i^{(1)}, b_i^{(2)}, b_i^{(3)}, \ldots, b_i^{(k_i)}\} \qquad (16)$$

The total number of choices for first constraint is k_i. We suppose that $k_i \geq 2$.

Step-2: Find the number of binary variables, which is required to handle the multi-choice parameters in RHS of the constraint in following manner.

Find l_i, for which $2^{(l_i-1)} < k_i \le 2^{l_i}$. Here l_i number of binary variables are needed. Let the binary variables are $z_i^{(1)}$, $z_i^{(2)}$, $z_i^{(3)}$, $\ldots$, $z_i^{(l_i)}$.

Step-3: Expand 2^{l_i} as $\binom{l_i}{0}+\binom{l_i}{1}+\binom{l_i}{2}+\ldots+\binom{l_i}{r_{i1}}+\ldots+\binom{l_i}{r_{i2}}+\ldots+\binom{l_i}{l_i}$ and select the smallest number of consecutive terms whose sum is 'equal to' or 'just greater than' k_i from the expansion. Let the terms be $\binom{l_i}{r_{i1}}$, $\binom{l_i}{r_{i1}+1}$, $\binom{l_i}{r_{i1}+2}$, $\ldots$, $\binom{l_i}{r_{i2}}$.

Step-4: Assign k_1 binary codes to k_1 number of choices for first constraint as follows:

$$\sum_{j=1}^{n} a_{ij}x_j \le \sum_{j=1}^{\binom{l_i}{r_{i2}}} P_j^{(r_{i2})}Q_j^{(r_{i2})}b_i^{(j)}$$

$$+\sum_{j=1}^{\binom{l_i}{r_{i2}-1}} P_j^{(r_{i2}-1)}Q_j^{(r_{i2}-1)}b_i^{(\binom{l_i}{r_{i2}}+j)}+\ldots$$

$$+\sum_{j=1}^{\binom{l_i}{(r_{i1}+1)}} P_j^{(r_{i1}+1)}Q_j^{(r_{i1}+1)}b_i^{(\binom{l_i}{r_{i2}}+\ldots+\binom{l_1}{r_{i1}+2}+j)}$$

$$+\sum_{j=1}^{(k_i-L_i^{(2)})} P_j^{(r_{i1})}Q_j^{(r_{i1})}b_i^{(L_i^{(2)}+j)}, \tag{17}$$

$$i=1,2,\ldots,m \tag{18}$$

where
$L_i^{(2)}=\binom{l_i}{r_{i2}}+\binom{l_i}{r_{i2}-1}+\ldots+\binom{l_i}{r_{i1}+1}$
$j_1\in\{1,2,3,...,(l_1-s)+1\}$, $j_2\in\{2,3,...,(l_1-s)+2\}$, $\ldots$, $j_s\in\{s,s+1,...,l_i\}$
$I_s^{(j)}=\{\{j_1,j_2,\ldots,j_s\}|j_1<j_2<\ldots<j_s, s=r_{i1},r_{i1}+1,...,r_{i2}\}$
$P_j^{(si)}=\{z_i^{(j_1)}z_i^{(j_2)}z_i^{(j_3)}\ldots z_i^{(j_s)}|\{j_1,j_2,j_3,\ldots,j_s\}\in I_s^{(j)}, s=r_{i1},r_{i1}+1,...,r_{i2}\}$
$Q_j^{(si)}=\{\prod_{j=1}^{l_i}(1-z_i^{(j)})|j\notin\{j_1,j_2,\ldots,j_s\}\}$

Step-5: Restrict $(2^{l_i}-k_i)$ number of binary codes to overcome repetitions as follows:

$$z_i^{(1)}+z_i^{(2)}+z_i^{(3)}+\ldots+z_i^{(l_i)}\ge r_{i1} \tag{19}$$

$$z_i^{(1)}+z_i^{(2)}+z_i^{(3)}+\ldots+z_i^{(l_i)}\le r_{i2} \tag{20}$$

$$\sum_{t=1}^{l_i} z_i^{(t)}\ge 1, t\notin\{j_1,j_2,j_3,\ldots,j_{r_{i1}}\},$$

$$j=(k_i-L_i^{(2)})+1,(k_i-L_i^{(2)})+2,\ldots,\binom{l_i}{r_{i1}} \tag{21}$$

Restrictions should be imposed on $z_i^{(j_1)}z_i^{(j_2)}\ldots z_i^{(j_{r_{i1}})}\in P_j^{(r_{i1}1)}$, but not included in $T_{r_{i1}}$. $T_{r_{i1}}$ contains the terms $P_j^{(r_{i1}i)}$ in transformed constraint (18).

Step-6: Formulate the mathematical model as:

$$\max: Z=\sum_{j=1}^{n} c_jx_j \tag{22}$$

$$\text{subject to}\sum_{j=1}^{n} a_{ij}x_j \le \sum_{j=1}^{\binom{l_i}{r_{i2}}} P_j^{(r_{i2})}Q_j^{(r_{i2})}b_i^{(j)}$$

$$+\sum_{j=1}^{\binom{l_i}{r_{i2}-1}} P_j^{(r_{i2}-1)}Q_j^{(r_{i2}-1)}b_i^{(\binom{l_i}{r_{i2}}+j)}+\ldots$$

$$+\sum_{j=1}^{\binom{l_i}{(r_{i1}+1)}} P_j^{(r_{i1}+1)}Q_j^{(r_{i1}+1)}b_i^{(\binom{l_i}{r_{i2}}+\ldots+\binom{l_1}{r_{i1}+2}+j)}$$

$$+\sum_{j=1}^{(k_i-L_i^{(2)})} P_j^{(r_{i1})}Q_j^{(r_{i1})}b_i^{(L_i^{(2)}+j)},$$

$$i=1,\ 2,\ldots,m \tag{23}$$

$$z_i^{(1)}+\ z_i^{(2)}+z_i^{(3)}+\ldots+z_i^{(l_i)}\ge r_{i1},\quad i=1,2,3,\ldots,m \tag{24}$$

$$z_i^{(1)}+\ z_i^{(2)}+z_i^{(3)}+\ldots+z_i^{(l_i)}\le r_{i2},\quad i=1,2,3,\ldots,m \tag{25}$$

$$\sum_{t=1,t\notin I_{r_{i1}}^{(j)}}^{l_i} z_i^{(t)}\ge 1, \tag{26}$$

$$j=(k_i-L_i^{(2)})+1,(k_i-L_i^{(2)})+2,\ldots,\binom{l_i}{r_{i1}},$$
$$i=1,2,\ldots,m$$

$$x_j\ge 0,\ j=1,2,3,\ldots,n \tag{27}$$

$$z_i^{(l_i)}=0/1,\ l_i=1,2,3,\ldots,\lceil\frac{\ln(k_i)}{\ln 2}\rceil,\quad i=1,2,3,\ldots,m \tag{28}$$

where $L_i^{(2)}=\binom{l_i}{r_{i2}}+\binom{l_i}{r_{i2}-1}+\ldots+\binom{l_i}{r_{i1}+1}$ $i=1,2,3,\ldots,m$.

Step-7: Above mathematical model is a mixed integer non-linear programming problem. Solve the model with the help of existing methodology.

Remarks: An MCLPP can be transformed to a standard mathematical programming problem by using any one of the transformation techniques. When we transform an MCLPP, we observe the following cases.

Case-1: $k_i = 2^{l_i}$. If k_i is a complete power of 2, then the expansion will be $2^{l_i} = \binom{l_i}{0} + \binom{l_i}{1} + \binom{l_i}{2} + \ldots + \binom{l_i}{l_i}$. When we move to Step-6 (in both the transformation techniques), the restriction will be

$$z_i^{(1)} + z_i^{(2)} + z_i^{(3)} + \ldots + z_i^{(l_i)} \geq 0, \; for\ some\ i \quad (29)$$

$$z_i^{(1)} + z_i^{(2)} + z_i^{(3)} + \ldots + z_i^{(l_i)} \leq l_i, \; for\ some\ i \quad (30)$$

which is obvious. Hence the presence or absence of (29) and (30) restrictions will not affect the solution of the transformed mathematical programming model.

Case-2: $k_i \neq 2^{l_i}$. If k_i is not a complete power of 2, then from the expansion of 2^{l_i} the smallest number of consecutive terms, whose sum equals or just greater than k_i are $\binom{l_i}{r_{i1}}, \binom{l_i}{r_{i1}+1}, \binom{l_i}{r_{i1}+2}, \ldots, \binom{l_i}{r_{i2}}$. Hence $k_i \leq \binom{l_i}{r_{i1}} + \binom{l_i}{r_{i1}+1} + \binom{l_i}{r_{i1}+2} + \ldots + \binom{l_i}{r_{i2}}$, $i = 1, 2, 3, \ldots, m$. In this case four possibilities may arise:

(1) If $k_i = \binom{l_i}{r_{i1}} + \binom{l_i}{r_{i1}+1} + \binom{l_i}{r_{i1}+2} + \ldots + \binom{l_i}{r_{i2}}$ and $r_{i1} = l - r_{i2}$, for some i then using any of these two transformation techniques exactly one equivalent mathematical programming model is possible.

(2) If $k_i = \binom{l_i}{r_{i1}} + \binom{l_i}{r_{i1}+1} + \binom{l_i}{r_{i1}+2} + \ldots + \binom{l_i}{r_{i2}}$ and $r_{i1} \neq l - r_{i2}$, for some i then there exists another set of smallest number of consecutive terms $\binom{l_i}{l_i - r_{i1}}, \binom{l_i}{l_i-(r_{i1}+1)}, \ldots, \binom{l_i}{l_i - r_{i2}}$ such that $k_i = \binom{l_i}{l_i - r_{i1}} + \binom{l_i}{l_i-(r_{i1}+1)} + \ldots + \binom{l_i}{l_i - r_{i2}}$. Hence in this case using any of the transformation techniques two different equivalent mathematical programming models are possible.

(3) Let's consider where $k_i < \binom{l_i}{r_{i1}} + \binom{l_i}{r_{i1}+1} + \binom{l_i}{r_{i1}+2} + \ldots + \binom{l_i}{r_{i2}}$ and $r_{i1} = l_i - r_{i2}$, for some i.

(a) Transformation Technique-1: Let $\binom{l_i}{r_{i2}}$ -$(k_i - L_i^{(1)}) = \alpha_i^{(1)}$ for some i. That is out of $\binom{l_i}{r_{i2}}$(say $= R_i^{(1)}$) number of binary codes $\alpha_i^{(1)}$ number of binary codes are to be restricted by using auxiliary constraints. This can be done in $\binom{R_i^{(1)}}{\alpha_i^{(1)}}$ different ways. Therefore a total of $\binom{R_i^{(1)}}{\alpha_i^{(1)}}$ different equivalent mathematical programming models are possible.

(b) Transformation Technique-2: Let $\binom{l_i}{r_{i1}}$ - $(k_i - L_i^{(2)}) = \alpha_i^{(2)}$. That is out of $\binom{l_i}{r_{i1}}$ (say $= R_i^{(2)}$) number of binary codes $\alpha_i^{(2)}$ number of binary codes are to be restricted by using auxiliary constraints. This can be done in $\binom{R_i^{(2)}}{\alpha_i^{(2)}}$ different ways. Therefore a total of $\binom{R_i^{(2)}}{\alpha_i^{(2)}}$ different equivalent mathematical programming models are possible.

We state that if the constraints of multi-choice linear programming problem will satisfy $k_i < \binom{l_i}{r_{i1}} + \binom{l_i}{r_{i1}+1} + \binom{l_i}{r_{i1}+2} + \ldots + \binom{l_i}{r_{i2}}$ and $r_{i1} = l_i - r_{i2}$, for some i then $\binom{R_i^{(1)}}{\alpha_i^{(1)}} + \binom{R_i^{(2)}}{\alpha_i^{(2)}}$ number of equivalent mathematical programming models are possible.

(4) Let's consider where $k_i < \binom{l_i}{r_{i1}} + \binom{l_i}{r_{i1}+1} + \binom{l_i}{r_{i1}+2} + \ldots + \binom{l_i}{r_{i2}}$ and $r_{i1} \neq l_i - r_{i2}$, for some i. In this case we have two set of smallest number of consecutive terms i.e. $\binom{l_i}{r_{i1}}, \binom{l_i}{r_{i1}+1}, \binom{l_i}{r_{i1}+2}, \ldots, \binom{l_i}{r_{i2}}$ and $\binom{l_i}{l_i - r_{i1}}, \binom{l_i}{l_i-(r_{i1}+1)}, \ldots, \binom{l_i}{l_i - r_{i2}}$ such that $k_i < \binom{l_i}{r_{i1}} + \binom{l_i}{r_{i1}+1} + \binom{l_i}{r_{i1}+2} + \ldots + \binom{l_i}{r_{i2}}$ and $k_i < \binom{l_i}{l_i - r_{i1}} + \binom{l_i}{l_i-(r_{i1}+1)} + \ldots + \binom{l_i}{l_i - r_{i2}}$. Considering $k_i < \binom{l_i}{r_{i1}} + \binom{l_i}{r_{i1}+1} + \binom{l_i}{r_{i1}+2} + \ldots + \binom{l_i}{r_{i2}}$ and $r_{i1} \neq l_i - r_{i2}$, a total of $\binom{R_i^{(1)}}{\alpha_i^{(1)}} + \binom{R_i^{(2)}}{\alpha_i^{(2)}}$ equivalent mathematical models are possible. Similarly, considering $k_i < \binom{l_i}{l_i - r_{i1}} + \binom{l_i}{l_i-(r_{i1}+1)} + \ldots + \binom{l_i}{l_i - r_{i2}}$ and $r_{i1} \neq l_i - r_{i2}$, a total of $\binom{R_i^{(1)}}{\alpha_i^{(1)}} + \binom{R_i^{(2)}}{\alpha_i^{(2)}}$ equivalent mathematical programming models are possible.

Therefore we state that if the constraints of multi-choice linear programming problem will satisfy $k_i < \binom{l_i}{r_{i1}} + \binom{l_i}{r_{i1}+1} + \binom{l_i}{r_{i1}+2} + \ldots + \binom{l_i}{r_{i2}}$ and $r_{i1} \neq l_i - r_{i2}$, for some i then $2(\binom{R_i^{(1)}}{\alpha_i^{(1)}} + \binom{R_i^{(2)}}{\alpha_i^{(2)}})$ number of equivalent mathematical programming models are possible.

4. Numerical Example

A model presented by Kent et al. [10] is used to illustrate the transformation techniques of MCLPP to equivalent mathematical models. The data of the model is taken from Rardin [13].

In the model, U.S. forest service has used an allocation model to address the sensitive task of managing national forest land. The forest service must tradeoff timber, grazing, recreational, national preservation, and other demands on forest land. Models of a forest begin by dividing land into homogeneous analysis areas. Several prescriptions or land management policies are proposed and then evaluated for each. The optimization seeks the best possible allocation of land in the analysis areas to particular prescriptions, subject to forest-wide restrictions on land use. Table-1 provides details of fictional 788 thousand acres Wagonho National Forest in the model. Wagonho is assumed to have 7 analysis areas, each subject to 3 different prescriptions. The first prescription encourages timbering, the second emphasizes grazing and third preserves the land as wilderness.

The symbols of parameters are given below, whose values are presented in 1.

s_i : size of analysis area i (in thousands of acres

p_{ij} : net present value (NPV) per acre of all uses in area i if managed under prescription j,

t_{ij} : projected timber yield (in board feet per acre) of analysis area i if managed under prescription j,

g_{ij} : projected grazing capability (in animal unit months per acre) of analysis area i if managed under prescription j,

w_{ij} : wilderness index rating (0 to 100) of analysis area i if managed under prescription j.

We wish to find an allocation that maximizes net present value. The production of timber is not fixed per acre. Past record of production of timber shows that it produces at least either 38.0 million or 38.4 million or 38.9 million or 40 million or 40.7 million or 40.9 million or 41.2 million or 41.5 million or 42 million board feet of timber. The number of animals grazing is at least 5 thousand animal per month. Depending on previous year record the wilderness index is at least either 68.5 or 69.0 or 69.6 or 70 or 71.2 or 71.5.

Forest Service model seeks an optimal allocation of valuable resources. Corresponding decision variables define the allocation.

x_{ij} : number of thousands of acres in analysis area i managed by prescription j, which is nonnegative. The choices of parameters should be in such a manner that the combination of choices will maximize net present value. The above forest service problem is mathematically formulated as an MCLPP and presented below:

$$\max : Z = \sum_{i=1}^{7}\sum_{j=1}^{3} p_{ij}x_{ij} \tag{31}$$

subject to

$$\sum_{j=1}^{3} x_{ij} = s_i, i = 1, 2, ..., 7 \tag{32}$$

$$\sum_{i=1}^{7}\sum_{j=1}^{3} t_{ij}x_{ij} \geq \{38000, 38400, 38900, 40000, 40700, 40900, 41200, 41500, 42000\} \tag{33}$$

$$\sum_{i=1}^{7}\sum_{j=1}^{3} g_{ij}x_{ij} \geq 5.0 \tag{34}$$

$$\frac{1}{788}\sum_{i=1}^{7}\sum_{j=1}^{3} w_{ij}x_{ij} \geq \{68.5, 69.0, 69.6, 70.0, 71.2, 71.5\} \tag{35}$$

$$x_{ij} \geq 0, i = 1, ..., 7; \; j = 1, 2, 3. \tag{36}$$

Using transformation technique-1, the MCLPP (31)-(36) can be transformed to following equivalent mathematical programming model:

$$\max : Z = \sum_{i=1}^{7}\sum_{j=1}^{3} p_{ij}x_{ij} \tag{37}$$

subject to

$$\sum_{j=1}^{3} x_{ij} = s_i, \; i = 1, 2, ..., 7 \tag{38}$$

$$\begin{aligned}\sum_{i=1}^{7}\sum_{j=1}^{3} t_{ij}x_{ij} \geq\ & 38000z_1^{(1)}(1-z_1^{(2)})(1-z_1^{(3)})(1-z_1^{(4)}) \\ & +38400(1-z_1^{(1)})z_1^{(2)}(1-z_1^{(3)})(1-z_1^{(4)}) \\ & +38900(1-z_1^{(1)})(1-z_1^{(2)})z_1^{(3)}(1-z_1^{(4)}) \\ & +40000(1-z_1^{(1)})(1-z_1^{(2)})(1-z_1^{(3)})z_1^{(4)} \\ & +40700z_1^{(1)}z_1^{(2)}(1-z_1^{(3)})(1-z_1^{(4)}) \\ & +40900z_1^{(1)}(1-z_1^{(2)})z_1^{(3)}(1-z_1^{(4)}) \\ & +41200z_1^{(1)}(1-z_1^{(2)})(1-z_1^{(3)})z_1^{(4)} \\ & +41500(1-z_1^{(1)})z_1^{(2)}z_1^{(3)}(1-z_1^{(4)}) \\ & +42000(1-z_1^{(1)})z_1^{(2)}(1-z_1^{(3)})z_1^{(4)}\end{aligned} \tag{39}$$

$$z_1^{(1)} + z_1^{(2)} + z_1^{(3)} + z_1^{(4)} \geq 1 \tag{40}$$

$$z_1^{(1)} + z_1^{(2)} + z_1^{(3)} + z_1^{(4)} \leq 2 \tag{41}$$

$$z_1^{(3)} + z_1^{(4)} \leq 1 \tag{42}$$

$$\sum_{i=1}^{7}\sum_{j=1}^{3} g_{ij}x_{ij} \geq 5.0 \quad (43)$$

$$\frac{1}{788}\sum_{i=1}^{7}\sum_{j=1}^{3} w_{ij}x_{ij} \geq 68.5z_2^{(1)}(1-z_2^{(2)})(1-z_2^{(3)})$$
$$+69.0(1-z_2^{(1)})z_2^{(2)}(1-z_2^{(3)})$$
$$+69.6(1-z_2^{(1)})(1-z_2^{(2)})z_2^{(3)}$$
$$+70.0z_2^{(1)}z_2^{(2)}(1-z_2^{(3)})$$
$$+71.2z_2^{(1)}(1-z_2^{(2)})z_2^{(3)}$$
$$+71.5(1-z_2^{(1)})z_2^{(2)}z_2^{(3)} \quad (44)$$

$$z_2^{(1)}+z_2^{(2)}+z_2^{(3)} \geq 1 \quad (45)$$

$$z_2^{(1)}+z_2^{(2)}+z_2^{(3)} \leq 2 \quad (46)$$

$$z_1^{(p_1)}, z_2^{(p_2)} = 0/1,\ p_1 = 1,2,3,4,\ p_2 = 1,2,3,$$
$$x_{ij} \geq 0,\ i = 1,2,3,..,7;\ j = 1,2,3. \quad (47)$$

Above mathematical programming model is a mixed integer non-linear programming problem. The above model is solved using Lingo[15] software. An optimal allocation is obtained as:

x_{11}=0,	x_{12}=0,	x_{13}=75,	x_{21}=90,
x_{22}=0,	x_{23}=0,	x_{31}=140,	x_{32}=0,
x_{33}=0,	x_{41}=0,	x_{42}=0,	x_{43}=60,
x_{51}=0,	x_{52}=212,	x_{53}=0,	x_{61}=15,
x_{62}=0,	x_{63}=83,	x_{71}=0,	x_{72} =0,
x_{73} =113,	b_8=40700,	b_9=5,	b_{10}=68.5

with a total net present value Z = \$346,900,500. Using transformation technique-2, the MCLPP (31)-(36) can be transformed to following equivalent mathematical model:

$$\max : Z = \sum_{i=1}^{7}\sum_{j=1}^{3} p_{ij}x_{ij} \quad (48)$$

subject to

$$\sum_{j=1}^{3} x_{ij} = s_i,\ i = 1,2,...,7 \quad (49)$$

$$\sum_{i=1}^{7}\sum_{j=1}^{3} t_{ij}x_{ij} \geq 38000z_1^{(1)}z_1^{(2)}(1-z_1^{(3)})(1-z_1^{(4)})$$
$$+38400z_1^{(1)}(1-z_1^{(2)})z_1^{(3)}(1-z_1^{(4)})$$
$$+38900z_1^{(1)}(1-z_1^{(2)})(1-z_1^{(3)})z_1^{(4)}$$
$$+40000(1-z_1^{(1)})z_1^{(2)}z_1^{(3)}(1-z_1^{(4)})$$
$$+40700(1-z_1^{(1)})z_1^{(2)}(1-z_1^{(3)})z_1^{(4)}$$
$$+40900(1-z_1^{(1)})(1-z_1^{(2)})z_1^{(3)}z_1^{(4)}$$
$$+41200(1-z_1^{(1)})(1-z_1^{(2)})(1-z_1^{(3)})z_1^{(4)}$$
$$+41500(1-z_1^{(1)})(1-z_1^{(2)})z_1^{(3)}(1-z_1^{(4)})$$
$$+42000(1-z_1^{(1)})z_1^{(2)}(1-z_1^{(3)})(1-z_1^{(4)}) \quad (50)$$

$$z_1^{(1)}+z_1^{(2)}+z_1^{(3)}+z_1^{(4)} \geq 1 \quad (51)$$

$$z_1^{(1)}+z_1^{(2)}+z_1^{(3)}+z_1^{(4)} \leq 2 \quad (52)$$

$$z_1^{(2)}+z_1^{(3)}+z_1^{(4)} \geq 1 \quad (53)$$

$$\sum_{i=1}^{7}\sum_{j=1}^{3} g_{ij}x_{ij} \geq 5.0 \quad (54)$$

$$\frac{1}{788}\sum_{i=1}^{7}\sum_{j=1}^{3} w_{ij}x_{ij} \geq 68.5z_2^{(1)}(1-z_2^{(2)})(1-z_2^{(3)})$$
$$+69.0(1-z_2^{(1)})z_2^{(2)}(1-z_2^{(3)})$$
$$+69.6(1-z_2^{(1)})(1-z_2^{(2)})z_2^{(3)}$$
$$+70.0z_2^{(1)}z_2^{(2)}(1-z_2^{(3)})$$
$$+71.2z_2^{(1)}(1-z_2^{(2)})z_2^{(3)}$$
$$+71.5(1-z_2^{(1)})z_2^{(2)}z_2^{(3)} \quad (55)$$

$$z_2^{(1)}+z_2^{(2)}+z_2^{(3)} \geq 1 \quad (56)$$

$$z_2^{(1)}+z_2^{(2)}+z_2^{(3)} \leq 2 \quad (57)$$

$$z_1^{(p_1)}, z_2^{(p_2)} = 0/1,\ p_1 = 1,2,3,4,\ p_2 = 1,2,3$$
$$x_{ij} \geq 0,\ i = 1,2,3,\ldots,7;\ j = 1,2,3. \quad (58)$$

Above mathematical model is a mixed integer non-linear programming problem. The above model is solved using Lingo [15] software. An optimal allocation is obtained as:

x_{11}=0,	x_{12}=0,	x_{13}=75,	x_{21}=90,
x_{22}=0,	x_{23}=0,	x_{31}=140,	x_{32}=0,
x_{33}=0,	x_{41}=0,	x_{42}=0,	x_{43}=60,
x_{51}=0,	x_{52}=212,	x_{53}=0,	x_{61}=15,
x_{62}=0,	x_{63}=83,	x_{71}=0,	x_{72} =0,
x_{73} =113,	b_8=40700,	b_9=5,	b_{10}=68.5

with a total net present value $Z = \$346,900,500$.

5. Results and Discussion

We can derive a total of 20 equivalent mathematical programming models. In numerical example, seven binary variables are used. The auxiliary constraints involving binary variables have been used in some cases in the transformed models to restrict the repetition of goals. The number of auxiliary constraints required is dependent on the number of aspiration levels associated with each constraints. Five auxiliary constraints are used to restrict the repetition of goals in the numerical example. The same number of binary variables and auxiliary constraints are required for transformation of a constraint having multi-choice parameter in both the transformation techniques. If k_i is a complete power of 2, the presence or absence of restrictions will not affect the solution of the transformed mathematical model. Only one of the twenty models is to be solved for an optimal solution. In stead of solving several problems by changing the goals it is necessary to solve only one optimization problem. We compute all twenty equivalent models of the numerical example and observed the same optimal solution.

6. Conclusions

The present study has explored the possibility of applying transformation techniques for the solution of MCLPP problem having two or more goals in the RHS of the constraints. The complexity of the proposed model is due to presence of several binary variables. It is observed that to transform an MCLPP to an equivalent mathematical model $\lceil \frac{\ln(k_i)}{\ln 2} \rceil$ number of binary variables are needed when k_i number of aspiration levels are associated with i-th goal. Hence $\sum_{i=1}^{m} \lceil (\frac{\ln(k_i)}{\ln 2} \rceil$ number of binary variables are needed in an equivalent model. The performance of the proposed model becomes much better when the number of choices for a constraint is increased. It can be used as a powerful decision making tool for a decision maker to take right decision. The transformation techniques, which are proposed here can be used with success not only for MCLPP but also can be extended to various other similar problems with multi-choice parameters.

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Table 1. Data of Forest Service

Analysis area	Acres, s_i	Prescrip-tion,	NPV (per acre)	Timber (per acre)	Grazing (per acre)	Wilderness, index
i	(000)'s	j	p_{ij}	t_{ij}	g_{ij}	w_{ij}
1	75	1	503	310	0.01	40
		2	140	50	0.04	80
		3	203	0	0	95
2	90	1	675	198	0.03	55
		2	100	46	0.06	60
		3	45	0	0	65
3	140	1	630	210	0.04	45
		2	105	57	0.07	55
		3	40	0	0	60
4	60	1	330	112	0.01	30
		2	40	30	0.02	35
		3	295	0	0	70
5	212	1	105	40	0.05	60
		2	460	32	0.08	60
		3	120	0	0	90
6	98	1	490	105	0.02	35
		2	55	25	0.03	50
		3	180	0	0	75
7	113	1	705	213	0.02	40
		2	60	40	0.04	45
		3	400	0	0	95

i represents analysis area number ($i = 1, 2, \ldots, 7$)
j represents prescription number ($j = 1, 2, 3$)