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RESEARCH ARTICLE

Observer design for a class of irreversible port Hamiltonian systems

Saida Zenfari*, Mohamed Laabissi, Mohammed Elarbi Achhab

Department of Mathematics, Faculty of sciences, University Chouaib Doukkali 24000 El Jadida, Morocco
saida.zenfari1991@gmail.com, laabissi.m@ucd.ac.ma, elarbi.achhab@gmail.com

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ABSTRACT

In this paper we address the state estimation problem of a particular class of irreversible port Hamiltonian systems (IPHS), which are assumed to be partially observed. Our main contribution consists to design an observer such that the augmented system (plant + observer) is strictly passive. Under some additional assumptions, a Lyapunov function is constructed to ensure the stability of the coupled system. Finally, the proposed methodology is applied to the gas piston system model. Some simulation results are also presented.

1. Introduction

Port Hamiltonian systems (PHS) encompass a very large class of systems including electrical, mechanical, and in general multi-energy systems [1–4]. This formalism has been suggested as a way for modeling and analysis of free and controlled physical systems, due mainly to its essential feature of underlying the crucial role played by the energy function, the interconnection structure, and the dissipation in the control of the system.

Although the PHS frame expresses the first principle of thermodynamics (the conservation of the energy), it is not suited for systems describing irreversible phenomena, as it is necessary to express the irreversible entropy creation, i.e. the second principle of thermodynamics. To solve this problem, the PHS frame has been revised and many quasi-PHS formulations have been presented in [5–7]. In [7], the PHS frame has been extended to a class of systems called IPHS. These systems are defined with respect to a skew symmetric structure matrix, and have the advantage of representing the first and the second principles of thermodynamics as theoretical properties of the system. (The reader is referred to [7] for more details on the IPHS construction and properties).

In most realistic problems, we do not have full information about the system state. Hence, the need to estimate the unknown part of the vector state is of great interest. For PHS many research papers have been developed to investigate the observer design problem [8–12]. In [10, 11], an observer design method based on passivation of the error dynamics is presented. By combining the interconnection and damping assignment method and the dissipativity theory, two observer design strategies are proposed in [8]. In [12], a full order observer design method based on contraction analysis is suggested for a particular class of PHS. For the class of systems considered in this paper (IPHS) regarding the control, a globally stabilizing controller preserving the IPHS structure in closed loop is proposed in [13] and [14]. In [15], an energy shaping and damping injection IPHS

*Corresponding Author

controller is constructed for an IPHS. Concerning the state estimation problem, to our modest knowledge, there is no observer design method developed for IPHS.

In this paper, our contribution is to present an observer design method for a class of IPHS by extending the approach suggested in [10,11] for PHS to the IPHS setting. Although our methodology is following that of [11], it is not obvious or simple to establish the same results for our class of systems. Some specific hypotheses are introduced in order to take into account the conservation of energy and the positivity of entropy production. It is assumed that the system is partially observed and that the observations are depending on the measured state only. That case is the most popular in practice and does not constitute any restriction as the availability of all state variables measurements is infrequent. Our observer is globally exponentially stable, and it is a copy of the original system in which the vector state components are directly the estimates of the plant ones.

The main advantage of the present study is that it is the first approach devoted to the observer design problem of IPHS. Unlike to [10,11] where the irreversibility is not considered, in this paper some specific hypotheses are introduced in order to take into account the conservation of energy and the positivity of entropy production. In addition, the use of the passivity technique renders the observer more stronger and robust against perturbations. Although the efficiency of our design method has been proven, the proposed strategy is restricted to minimum phase systems.

The rest of the paper is organized as follows. In section 2, a brief overview of the considered IPHS, the used observer, and some motivation will be given. Section 3 will be devoted to the description of our main result. In section 4, an application of the proposed approach on the gas piston system model will be presented. The paper is wrapped up in section 5 with a summary and an outlook.

2. Irreversible port Hamiltonian systems

Irreversible Port Hamiltonian Systems (IPHS) have been introduced in [7] as an extension of port Hamiltonian systems. In particular, the IPHS formulation is used to express simultaneously the energy conservation and the irreversible entropy creation. This article will be limited to the class of IPHS given by the following definition.

Definition 1. *The input affine representation of IPHS is defined by the dynamic equation and the output relation:*

$$\begin{aligned} \dot{x} &= R(x, \frac{\partial U}{\partial x}, \frac{\partial S}{\partial x}) J \frac{\partial U}{\partial x} + g(x, \frac{\partial U}{\partial x}) u(t), \quad (1) \\ y &= g^T(x, \frac{\partial U}{\partial x}) \frac{\partial U}{\partial x}(x) \end{aligned}$$

where:

1. $x(t) \in \mathbb{R}^n$ *is the state vector.*
2. $u(t) \in \mathbb{R}^m$ *is an input time dependent function,* $g(x, \frac{\partial U}{\partial x}) \in \mathbb{R}^{n \times m}$.
3. $U(x) \in \mathbb{R}$, $S(x) \in \mathbb{R}$ *represent respectively the internal energy (the Hamiltonian) and the entropy functions.*
4. $J \in \mathbb{R}^n \times \mathbb{R}^n$ *is a constant skew symmetric matrix, the structure matrix of the Poisson bracket* $\{.,.\}_J$*, where* $\{S, U\}_J = \frac{\partial S^T}{\partial x}(x) J \frac{\partial U}{\partial x}(x)$.
5. $R(x, \frac{\partial U}{\partial x}, \frac{\partial S}{\partial x})$ *is the product of a positive definite function* γ *and the Poisson bracket of* S *and* U.

$$R(x, \frac{\partial U}{\partial x}, \frac{\partial S}{\partial x}) = \gamma(x, \frac{\partial U}{\partial x}) \{S, U\}_J \quad (2)$$

with $\gamma(x, \frac{\partial U}{\partial x}) : \mathbb{R}^n \times \mathbb{R}^n \longrightarrow \mathbb{R}$, $\gamma \geq 0$, *a non linear positive function.*

By construction, it is clear that IPHS satisfy the first principle of thermodynamics (conservation of energy):

$$\frac{dU}{dt} = y^T u, \quad (3)$$

which expresses that system (1) is lossless dissipative with energy supply rate $y^T u$ (See e.g. [13]). Moreover, IPHS obey the second principle of thermodynamics (positivity of the internal entropy production):

$$\begin{aligned} \frac{dS}{dt} &= R(x, \frac{\partial U}{\partial x}, \frac{\partial S}{\partial x}) \frac{\partial S^T}{\partial x} J \frac{\partial U}{\partial x} \quad (4) \\ &+ \frac{\partial S^T}{\partial x} g(x, \frac{\partial U}{\partial x}) u(t), \\ &= \gamma(x, \frac{\partial U}{\partial x}) \{S, U\}_J^2 + (g^T(x, \frac{\partial U}{\partial x}) \frac{\partial S}{\partial x})^T u, \end{aligned}$$

where $\gamma(x, \frac{\partial U}{\partial x}) \{S, U\}_J^2 = \sigma(x, \frac{\partial U}{\partial x}) \geq 0$, and $\{S, U\}_J^2 = \{S, U\}_J^T \{S, U\}_J$, (see [13], [16] for more details).

The energy and entropy functions are usually extensive variables. They satisfy the additivity [17]

$$\begin{aligned} S(X, Y) &= S(X) + S(Y), \\ U(X, Y) &= U(X) + U(Y), \end{aligned}$$

where X and Y are two states. In addition, they satisfy the scaling relation [17]

$$\begin{aligned} S(\lambda X) &= \lambda S(X), \\ U(\lambda X) &= \lambda U(X), \end{aligned}$$

where λ is an arbitrary scaling function.

In most realistic problems, we do not have full information about the system state. Hence, the need of estimating the unknown part of the vector state is of great interest. This motivates our observer design method in which the state of the original system will be decomposed into two parts. One is measured and hence it is selected to be the output. The other is non measured and it will be estimated by the observer.

3. Problem formulation

In this note we address the partial state observer problem of IPHS of the form:

$$\left\{ \begin{array}{ll} \dot{x} = & R(x_1, \frac{\partial U}{\partial x_1}, \frac{\partial S}{\partial x_1})J(x_1)\frac{\partial U}{\partial x} + g(x_1, \frac{\partial U}{\partial x_1})u(t), \\ y = & x_1. \end{array} \right. \tag{5}$$

Where $J = \begin{bmatrix} J_1(x_1) & N(x_1) \\ -N^T(x_1) & J_2(x_1) \end{bmatrix}$, $g(x_1, \frac{\partial U}{\partial x_1}) = \begin{bmatrix} g_1(x_1, \frac{\partial U}{\partial x_1}) \\ \\ g_2(x_1, \frac{\partial U}{\partial x_1}) \end{bmatrix}$,

$x = (x_1, x_2) \in \mathbb{R}^n$, $x_1 \in \mathbb{R}^p$ is the measured state, $x_2 \in \mathbb{R}^{n-p}$ is the unmeasured state, $u \in \mathbb{R}^m$ is the input (where m, n and p are integers such that $1 \leq p < n$ and $m \leq n$). It is assumed that the system (5) is forward complete, that is trajectories are defined for all $t \geq 0$. The matrices $J_1 \in \mathbb{R}^{p\times p}$, $J_2 \in \mathbb{R}^{(n-p)\times(n-p)}$ are skew symmetric, $N \in \mathbb{R}^{p\times(n-p)}$, $g_1 \in \mathbb{R}^{p\times m}$ and $g_2 \in \mathbb{R}^{(n-p)\times m}$. $U : \mathbb{R}^p \times \mathbb{R}^{n-p} \longrightarrow \mathbb{R}$ is the internal energy of the system. $S : \mathbb{R}^p \times \mathbb{R}^{n-p} \longrightarrow \mathbb{R}$ is the entropy function. The energy and entropy functions are assumed to satisfy

$$U(x) = U_1(x_1) + U_2(x_2), \tag{6}$$
$$S(x) = S_1(x_1) + S_2(x_2), \tag{7}$$

where $U_1 : \mathbb{R}^p \longrightarrow \mathbb{R}$, and $U_2 : \mathbb{R}^{n-p} \longrightarrow \mathbb{R}$ are two energy functions. $S_1 : \mathbb{R}^p \longrightarrow \mathbb{R}$, and $S_2 : \mathbb{R}^{n-p} \longrightarrow \mathbb{R}$ are two entropy functions.

Our aim is to design a full order observer for system (5) in the following form:

$$\begin{aligned} \dot{\hat{x}} &= R(\hat{x}_1, \frac{\partial U}{\partial \hat{x}_1}, \frac{\partial S}{\partial \hat{x}_1})J(\hat{x}_1)\frac{\partial U}{\partial \hat{x}}(\hat{x}) \\ &+ g(\hat{x}_1, \frac{\partial U}{\partial \hat{x}_1})u(t) + L(\hat{x}_1)v, \end{aligned} \tag{8}$$

where $L(\hat{x}_1) = \begin{bmatrix} L_1(\hat{x}_1) \\ L_2(\hat{x}_1) \end{bmatrix}$, $\hat{x} = (\hat{x}_1, \hat{x}_2)$, $\hat{x}_1 \in \mathbb{R}^p$, $\hat{x}_2 \in \mathbb{R}^{n-p}$, $v \in \mathbb{R}^p$.

Where $v = -k(y, \hat{x})y_d + v_d$, y_d and v_d are desired output and input respectively, and $k : \mathbb{R}^p \times \mathbb{R}^n \longrightarrow \mathbb{R}^{+*}$ is a continuous scalar function.

Following ([15], page 20), the total energy of the augmented system composed of (5) and (8) is $U(x, \hat{x}) = U(x) + U(\hat{x})$. This result represents an extension of the composition theory of dirac structures from the port Hamiltonian case to the irreversible one. This result states that the energy of any two port controlled Hamiltonian systems or more is the sum of the energy function of each system. See ([4], page 241) for more details.

The time derivative of the energy of the augmented system may be defined as follows

$$\begin{aligned} \dot{U}(x, \hat{x}) = & \frac{\partial U^T}{\partial x} g(x_1, \frac{\partial U}{\partial x_1})u + \frac{\partial U^T}{\partial \hat{x}} g(\hat{x}_1, \frac{\partial U}{\partial \hat{x}_1})u \\ - & \frac{\partial U^T}{\partial \hat{x}} Lky_d + \frac{\partial U^T}{\partial \hat{x}} Lv_d. \end{aligned}$$

Then under the conditions

$$\begin{aligned} [\frac{\partial U^T}{\partial x} g(x_1, \frac{\partial U}{\partial x_1}) + \frac{\partial U^T}{\partial \hat{x}} g(\hat{x}_1, \frac{\partial U}{\partial \hat{x}_1})]u &= 0, \\ \frac{\partial U^T}{\partial \hat{x}} L &= y_d^T, \end{aligned}$$

we get

$$\dot{U}(x, \hat{x}) = y_d^T v_d - k y_d^T y_d \leq y_d^T v_d, \tag{9}$$

and hence the feedback law $v = -k(y, \hat{x})y_d + v_d$ makes the augmented system composed of (5) and (8) strictly passive, with respect to the manifold $M = \{(x, \hat{x}) : x = \hat{x}\}$, from the new input v_d to the new output y_d. In that case, system (8) is called a passivity based observer for system (5).

Recall a fundamental characterization of passive systems. A system of the form $\dot{x} = f(x) + g(x)u$, $y = h(x)$, $x \in \mathbb{R}^n$ satisfies the KYP property if there exists a nonnegative function $U : \mathbb{R}^n \to \mathbb{R}$, with $U(0) = 0$, such that

$$\begin{aligned} (\nabla U(x))^T f(x) &\leq 0, \\ (\nabla U(x))^T g(x) &= h^T(x), \end{aligned}$$

see [18] for more details.

In order to solve the observer design problem, we shall find gains L_1, L_2, and some function k such that the augmented system is strictly passive (for more details on the passivity definition and its applications see [10,14,18,19]) with respect to a certain manifold that will be specified later. In this manifold, the unmeasurable state can be reconstructed, and hence global exponential stability of the system can be obtained by letting $v_d \equiv 0$. Note that a nonlinear observer is sensitive to measurement disturbances. In [10], it is shown that the passivity property can be used to modify the nonlinear injection gain in order to make the observer robust with respect to measurement disturbances.

4. Observer design

In the beginning of this section , we state the conditions which will make the augmented system strictly passive from the input v_d to the output y_d. To this end, we follow the same idea as in [11] by using the equivalence between the next two statements established in [18]:

(1) Any affine control system can be rendered strictly passive by a smooth static state feedback.
(2) The system has a vector relative degree $\{1, \dots, 1\}$ and is globally minimum phase.

We recall the relative degree is equal to the number of times that one has to differentiate the system in order to have the input explicitly appearing. Moreover, a system is said to be globally minimum phase if its zero dynamics are globally asymptotically stable. See [18] for more details.

Note that in the study of passive systems, the concepts of relative degree and zero dynamics arise naturally. In particular, in our setting, we assume that the system has a vector relative degree $\{1, \dots, 1\}$ in order to ensure the existence of the system zero dynamics.

We make in the sequel the two following assumptions.

Assumption 1. *For any $Z = \hat{x}_2 - x_2$, there exist $Q = Q^T > 0,\ Q \in \mathbb{R}^{(n-p)\times(n-p)}$ such that:*

$$\frac{\partial U}{\partial \hat{x}_2} = \frac{\partial U}{\partial x_2} + QZ. \tag{10}$$

Assumption 2. *There exists a smooth globally invertible matrix $L_1(x_1) \in \mathbb{R}^{p\times p}$ and a smooth matrix $L_2(x_1) \in \mathbb{R}^{(n-p)\times p}$ such that:*

$$B^T(x_1) + B(x_1) > \delta I_{(n-p)\times(n-p)}, \qquad \delta > 0, \tag{11}$$

holds for all x_1, where:

$$B(x_1) = L_2(x_1) L_1^{-1}(x_1) R(x_1, \frac{\partial U}{\partial x_1}, \frac{\partial S}{\partial x_1}) N(x_1).$$

We are now ready to state the passivation result:

Lemma 1. *Assume that assumptions* (1) *and* (2) *are satisfied. Then:*

(1) *The augmented system $(x, \hat{x})$ has a vector relative degree $\{1, \dots, 1\}$ with respect to the input v and the output $y_d = \hat{x}_1 - x_1$.*
(2) *The zero dynamics of the augmented system $(x, \hat{x})$ with respect to the output y_d renders the manifold $\mathcal{P} = \{(x_1, x_2, \hat{x}_2) : \hat{x}_2 = x_2\}$ positively invariant and globally exponentially attractive.*

Proof. (1) Now, we compute the derivative of y_d as:

$$\dot{y}_d = RN[\frac{\partial U}{\partial \hat{x}_2} - \frac{\partial U}{\partial x_2}] + L_1 v.$$

As v is the considered input and L_1 is invertible by assumption for all x_1, the result is achieved.

(2) The zero dynamics of the augmented system with respect to the output y_d consist of (5) and the equations:

$$0 = RN[\frac{\partial U}{\partial \hat{x}_2} - \frac{\partial U}{\partial x_2}] + L_1 v, \tag{12}$$

$$\begin{aligned} \dot{\hat{x}}_2 &= -RN^T \frac{\partial U}{\partial x_1} + RJ_2 \frac{\partial U}{\partial \hat{x}_2} + g_2 u \\ &+ L_2(x_1) v, \end{aligned} \tag{13}$$

we note that these zero dynamics are defined uniformly for all $u \in \mathbb{R}^m$.

Now, consider the manifold $\mathcal{P}$ and denote $Z = \hat{x}_2 - x_2$. By using (12), we compute the derivative of Z along (5) and (13). We get

$$\dot{Z} = [RJ_2 - L_2 L_1^{-1} RN] QZ. \tag{14}$$

Then by skew symmetry of RJ_2 and the use of assumption (2), we have the positive invariance of the manifold $\mathcal{P}$.

Now consider the Lyapunov function

$$V = \frac{1}{2}(\frac{\partial U}{\partial \hat{x}_2} - \frac{\partial U}{\partial x_2})^T Q^{-1} (\frac{\partial U}{\partial \hat{x}_2} - \frac{\partial U}{\partial x_2}), \tag{15}$$

then by assumptions (1) and (2) we obtain

$$\begin{aligned} \dot{V} &= \frac{1}{2}\dot{Z}^T QZ + \frac{1}{2} Z^T Q \dot{Z} \\ &= \frac{1}{2}[Z^T Q(-RJ_2 - B^T + RJ_2 - B)QZ] \\ &= -\frac{1}{2} Z^T Q [B^T + B] QZ \\ &\le -\delta \frac{\lambda_m^2(Q)}{\lambda_M(Q)} V. \end{aligned}$$

where $\lambda_m(Q)$ and $\lambda_M(Q)$ denotes respectively the smallest and the largest eigenvalue of Q. Thus the system exponentially decays to zero with convergence rate $\delta\frac{\lambda_m^2(Q)}{\lambda_M(Q)}$.

□

Remark 1. *In the last lemma we mean by zero dynamics, the dynamics of the augmented system composed by the observer and the plant restricted to the set of initial conditions such that the corresponding output $y_d = \hat{x}_1 - x_1$ is zero (which implies that $\hat{x}_1 = x_1$).*
Hence the observer (8), will be defined by

$$\begin{aligned}\dot{\hat{x}} &= R(x_1, \frac{\partial U}{\partial x_1}, \frac{\partial S}{\partial x_1})J(x_1)\frac{\partial U}{\partial \hat{x}}(\hat{x}) \quad (16)\\ &+ g(x_1, \frac{\partial U}{\partial x_1})u(t) + L(x_1)v,\end{aligned}$$

where $L(x_1) = \begin{bmatrix} L_1(x_1) \\ L_2(x_1) \end{bmatrix}$.
We note that this definition differs from the usual understanding of zero dynamics, as the input $u(t)$ still appearing in the equations.

The following assumption will play a crucial role in our analysis.

Assumption 3. *There exists a smooth function $\beta : \mathbb{R}^p \longrightarrow \mathbb{R}^{n-p}$ such that*

$$L_2(x_1)L_1^{-1}(x_1) = \frac{\partial \beta}{\partial x_1}(x_1) \quad (17)$$

holds for all $x_1 \in \mathbb{R}^p$.

Remark 2. (1) *Assumption 1 is important in the development of our approach. It allows us to easily demonstrate the positive invariance and the exponential stability of the manifold $\mathcal{P}$. This assumption is very crucial and will be helpful in the choice of our example given in section 5. Moreover, it expresses a relation between states variables and co-energy variables, and means that any co-energy variable $\frac{\partial U}{\partial x_2}(\frac{\partial U}{\partial \hat{x}_2})$ may be linearized with respect to the state x_2 ($\hat{x}_2$).*
(2) *Assumption 2 is usually satisfied since L_1 and L_2 represent degrees of freedom. The choice of L_1 and L_2 is done such that the augmented system has a vector relative degree and is globally minimum phase.*
(3) *In assumption 3, a matching condition is defined and has to be solved. This condition requires that the selected gains L_1 and L_2 should be integrable. This assumption will be used to achieve the attractivity of the manifold.*

Now, we proceed to the design of the feedback law and consequently to the construction of the full order observer.

Theorem 1. *Assume $g_1 \equiv 0$.*
Then, the augmented system (5), (8) expressed in the coordinates $(x_1, x_2; \xi_1, \xi_2)$ where

$$\begin{aligned}\xi_1 &= \hat{x}_1 - x_1, \quad (18)\\ \xi_2 &= \hat{x}_2 - x_2 - \{\beta(\hat{x}_1) - \beta(x_1)\} \quad (19)\end{aligned}$$

has global normal form with respect to the input v and the output y_d.
Moreover, the feedback law defined by

$$v = -(\alpha + \phi_1 + \phi_2^2)\xi_1 + v_d, \quad (20)$$

where $\alpha > 0$ and $\phi_i(\xi_1, \hat{x}_1, \hat{x}_2)$, $i = 1, 2$ are non negative scalar functions, renders the system strictly passive with respect to the manifold $\mathcal{P}$, uniformly for all $u \in \mathbb{R}^m$, from the input v_d to the output ξ_1 with the storage function being given by

$$W(\xi_1, \xi_2) = \frac{1}{2}\xi_2^T Q \xi_2 + \frac{1}{2}\xi_1^T X \xi_1,$$

where $X \in \mathbb{R}^{p\times p}$, and $Q \in \mathbb{R}^{n-p\times n-p}$.

Proof. We define the functions $F_i(\xi_1, \xi_2, x_1, x_2) = F_i$, $i = 1, 2, 3$, as:

$$\begin{aligned}F_1 &= \hat{f}_1 - f_1 \quad (21)\\ F_2 &= \hat{f}_2 - f_2 \quad (22)\end{aligned}$$

$$F_3 = \frac{\partial \beta}{\partial \hat{x}_1}(\hat{x}_1)\hat{f}_1 - \frac{\partial \beta}{\partial x_1}(x_1)f_1; \quad (23)$$

where $f_i(x_1, x_2) = f_i$, $f_i(\hat{x}_1, \hat{x}_2) = \hat{f}_i$, $i = 1, 2$, $F_1 \in \mathbb{R}^p$, $F_2 \in R^{n-p}$,

$$\begin{bmatrix} f_1(x_1, x_2) \\ f_2(x_1, x_2) \end{bmatrix} = R(x_1, \frac{\partial U}{\partial x_1}, \frac{\partial S}{\partial x_1})J\frac{\partial U}{\partial x},$$

and hence using the assumption $g_1 \equiv 0$, the system dynamic may be expressed in the coordinate transformation (ξ_1, ξ_2) as

$$\begin{aligned}\dot{\xi}_1 &= F_1(\xi_1, \xi_2, x_1, x_2) + L_1(\hat{x}_1),\\ \dot{\xi}_2 &= (F_2 - F_3)(\xi_1, \xi_2, x_1, x_2).\end{aligned}$$

In addition, we have

$$F_i = F_i|_{x_2 = x_2 + \xi_2} + F_i|_{\xi_1 = 0},$$

where $F_i|_{x_2=x_2+\xi_2} = F_i(\xi_1,\xi_2,x_1,x_2+\xi_2)$, $F_i|_{\xi_1=0} = F_i(0,\xi_2,x_1,x_2)$. We note that when $\xi_1 = 0$, then $F_i(\xi_1,\xi_2,x_1,x_2+\xi_2) = 0$. Hence, the augmented system assumes its global normal form as we have the existence of continuous matrix functions $A_1(\xi_1,x_1,x_2) \in R^{p\times p}$, and $A_i(\xi_1,x_1,x_2) \in R^{n-p\times p}$, for $i = 2,3$ achieving

$$F_i(\xi_1,\xi_2,x_1,x_2+\xi_2) = A_i(\xi_1,x_1,x_2)\xi_1. \quad (24)$$

Now, There exist non-negative continuous scalar functions $\psi_i(\xi_1,x_1,x_2+\xi_2)$, $i = 1,2,3$ such that,

$$\|A_i(\xi_1,x_1,x_2+\xi_2)\| \le \psi_i(\xi_1,x_1,x_2+\xi_2), \quad (25)$$

holds for all $\xi_1,x_1,x_2+\xi_2$, where $\|.\|$ is the induced norm of any general matrix.

The next inequalities will be used to demonstrate that the system is strictly passive with respect to the input v_d and the output y_d:

$$\begin{aligned}\|\xi_2^T Q(F_2-F_3)(\xi_1,\xi_2,x_1,x_2+\xi_2)\| &\le \\ \zeta\{\psi_2+\psi_3\}(\xi_1,x_1,x_2+\xi_2)\sqrt{\delta}\|\xi_1\|\|\xi_2\| & \quad (26)\end{aligned}$$

where $\zeta = \frac{\lambda_M(Q)}{\sqrt{\delta}}$, and $\lambda_M(Q)$ is the largest eigenvalue of Q.

$$\begin{aligned}&\|\xi_1^T X F_1(\xi_1,\xi_2,x_1,x_2+\xi_2)\| \le \\ &\lambda_M(X)\psi_1(\xi_1,x_1,x_2+\xi_2)\|\xi_1\|^2, \quad (27)\end{aligned}$$

$$\begin{aligned}&\|\xi_1^T X F_1(0,\xi_2,x_1,x_2)\| \le \\ &\zeta\lambda_M(X)\|R\|\|N\|\sqrt{\delta}\|\xi_1\|\|\xi_2\|, \quad (28)\end{aligned}$$

Now consider the feedback law v (20) with $\phi_1 = \lambda_M(X)\psi_1$ and $\phi_2 = \zeta(\{\psi_2+\psi_3\} + \lambda_M(X)\|N\|\|R\|)$, and the storage function W. Using the inequalities (26), (27) and (28) we obtain:

$$\begin{aligned}\dot{W} \le &-\alpha\|\xi_1\|^2 + \xi_1^T v_d - \frac{3}{4}\delta\|\xi_2\|^2 - \\ &-\{\frac{1}{2}\sqrt{\delta}\|\xi_2\| - \|\xi_1\|\phi_2\}^2\end{aligned}$$

Thus, we get the result. □

5. Application

We consider a pure ideal gas contained in a cylinder closed by a piston and submitted to gravity. The thermodynamic properties of this system may be decomposed into the properties of the piston in the gravitation field and the properties of the perfect gas. See [16] for more details.

The total energy of the system is:

$$U(x) = TS - PV + H_{mec}(z,p), \quad (29)$$

where $x = [S,V,z,p]^T$ is the vector of state variable, S denotes the entropy variable, V is the volume variable, z is the altitude of the piston and p its kinetic momentum. $H_{mec}(z,p) = \frac{1}{2}mp^2 + mgz$ represents the energy of the piston. The co-energy variables are defined by the gradient of the total energy:

$$\begin{aligned}\frac{\partial U}{\partial S} &\triangleq T \\ \frac{\partial U}{\partial V} &\triangleq -P \\ \frac{\partial U}{\partial z} &= mg = F_g \\ \frac{\partial U}{\partial p} &\triangleq v\end{aligned} \quad (30)$$

where T is the temperature, P is the pressure, F_g is the gravity force and v is the velocity of the piston. This system may be written in the state space representation form (5) as follows:

$$\frac{d}{dt}\begin{bmatrix} S \\ V \\ z \\ p \end{bmatrix} = R\underbrace{\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & A\frac{T}{\nu v} \\ 0 & 0 & 0 & \frac{T}{\nu v} \\ -1 & -A\frac{T}{\nu v} & -\frac{T}{\nu v} & 0 \end{bmatrix}}_{J} \underbrace{\begin{bmatrix} T \\ -P \\ F \\ v \end{bmatrix}}_{\nabla_x U}$$

where A denotes the area of the piston, and

$$\begin{aligned}R &= R(x, \frac{\partial U}{\partial x}, \frac{\partial U}{\partial S}), \\ &= \gamma(x, \frac{\partial U}{\partial x})\{S,U\}_J, \\ &= \frac{\nu v}{T}, \\ &= R(x_1, \frac{\partial U}{\partial x_1}, \frac{\partial U}{\partial S}),\end{aligned}$$

and

$$J = J(x_1),$$

such that $x_1 = [S,V,p]^T$ and $x_2 = z$.

In order to stay in the context of partial state observability, we assume x_1 to be measured and x_2 is non measured. If we let $(\hat{x}_1,\hat{x}_2)$ be the state estimates and define their dynamic as in (8), the error of the system may be expressed as $(e_1,e_2,e_3,e_4) = (\hat{S},\hat{V},\hat{p},\hat{z}) - (S,V,p,z)$.

The assumption 1 is satisfied:

$$\frac{\partial U}{\partial \hat{x}_2} - \frac{\partial U}{\partial x_2} = \rho A g(\hat{z} - z),$$

where g is the gravity force, ρ is the density.

Now let $L_1 = I_3$, where I_3 is the identity matrix of order 3, and $L_2 = [0,1,1]$. Then, assumption 2 is clearly satisfied as we have:

$$B^T(x_1) + B(x_1) = 2A + 2 > 0.$$

The function β of assumption 3 will be defined as: $\beta(x_1) = V + p$. To express the system in its global normal form, we use the following change of coordinates:

$$\begin{aligned}
\xi_1 &= \hat{x}_1 - x_1, \\
&= [\hat{S} - S, \hat{V} - V, \hat{p} - p]^T, \\
&= [\xi_{11}, \xi_{12}, \xi_{13}]^T, \\
\xi_2 &= \hat{x}_2 - x_2 - (\beta(\hat{x}_1) - \beta(x_1)), \\
&= \hat{z} - z - (\hat{V} - V) - (\hat{p} - p),
\end{aligned}$$

where $\xi_{11} = \hat{S} - S$, $\xi_{12} = \hat{V} - V$, and $\xi_{13} = \hat{p} - p$. We choose the total energy as

$$W(\xi_1, \xi_2) = \frac{1}{2}\xi_{11}^2 + \frac{1}{2}\xi_{12}^2 + \frac{1}{2}\xi_{13}^2 + \frac{1}{2}\xi_2^T \xi_2.$$

Now, as all tools are available, we shall design the feedback law (20). Firstly, the functions f_1 and f_2 are given by

$$f_1(x_1, x_2) = \begin{bmatrix} \nu \frac{v^2}{T} \\ Av \\ -\nu v + AP - \rho A g z \end{bmatrix}, \quad f_2 = v.$$

Then

$$F_1 = \begin{bmatrix} \nu(\frac{\hat{v}^2}{\hat{T}} - \frac{v^2}{T}) \\ A(\hat{v} - v) \\ -\nu(\hat{v} - v) + A(\hat{P} - P) - \rho A g(\hat{z} - z) \end{bmatrix},$$

$$F_2 = \hat{v} - v,$$

$$F_3 = (A - \nu)(\hat{v} - v) + A(\hat{P} - P) - \rho A g(\hat{z} - z).$$

Using the inequalities (26), (27), and (28) we get

$$\begin{aligned}
\phi_1 =& max(Anr\frac{|T|}{|V\hat{V}|} + \rho A g, Anr\frac{T_0}{c|\hat{V}|} + T_0\nu\frac{|\hat{v}^2|}{c|T\hat{T}|}, \\
& \rho A g + (A + \nu + \frac{|V + \hat{V}|}{|T|})sup_p(\|\nabla v\|))
\end{aligned}$$

and

$$\begin{aligned}
\phi_2 =& max(Anr\frac{T_0}{|V\hat{V}|} + \rho A g, Anr\frac{T_0}{c|\hat{V}|}, sup_p(\|\nabla v\|) + \\
& + \rho A g) + 1 + A + \nu\frac{|v|}{|T|}.
\end{aligned}$$

Therefore, we get the expression of the feedback law (20) v as

$$\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = -(\alpha + \phi_1 + \phi_2^2)\xi_1 + \begin{pmatrix} v_{d_1} \\ v_{d_2} \\ v_{d_3} \end{pmatrix},$$

where $\xi_1 = (\xi_{11}, \xi_{12}, \xi_{13})^T = (\hat{S} - S, \hat{V} - V, \hat{p} - p)^T$.

The simulations below address respectively the entropy, volume, the altitude of piston and the kinetic momentum. The plant curves are in red and the observer ones are in black.

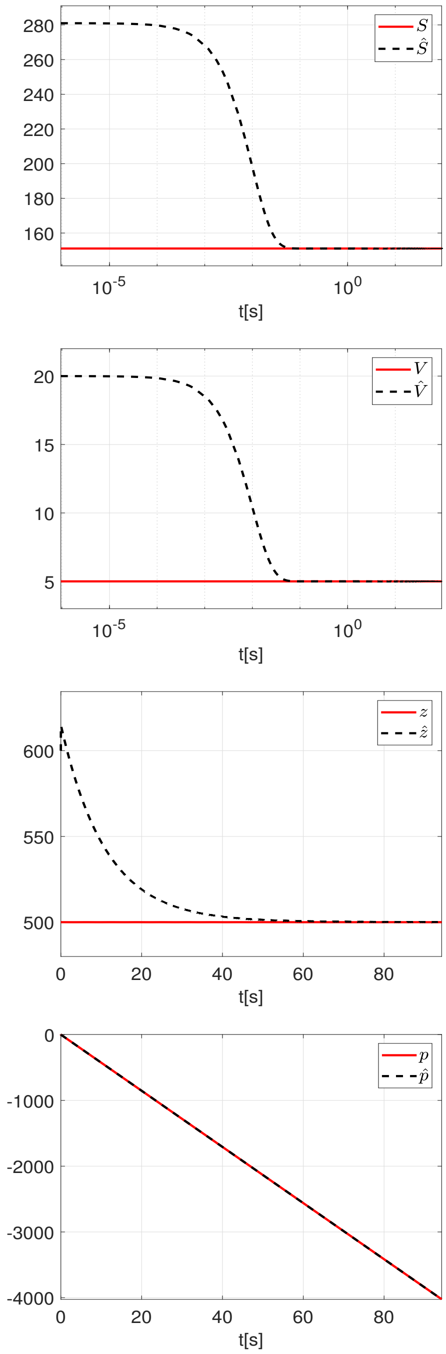

Figure 1. Open-Loop trajectories for the gas piston model and the observer.

The simulation results for the gas piston system model and the observer are given under the initial conditions:

	S_0	$V_0(l)$	p_0	z_0	α
Plant	151.077	5	0	500	0
Observer	281	20	0	600	10

The other parameters are chosen as: $g = 10m/S^2$, $n = 0.1002$ mol, $\nu = 0.05$, $A = 0.01m^2$, $T_0 = 600$ K, $c = 180$ j/Kg/K. $r = 8.31$ $jmol^{-1}K^{-1}$ is the universal gas constant.

6. Conclusion and Outlook

In this note, we have proposed a passivity based observer for a special class of irreversible port Hamiltonian systems. The observer design is done in two steps: The first one is the passivation of the system. In this step we check if assumption 1 is satisfied. Then, the matrices L_1 and L_2 are chosen in such a way to fulfil assumption 2. Finally, we compute the function β by using assumption 3.

The basic idea of the second step is to express the system in its global normal form and compute the feedback law v by using the procedure described in Theorem 1. Finally, the result has been applied to the gas piston system model considered in [7], and some simulation results of the studied example are presented. Since our study involves time derivatives, future works will tackle the investigation of the proposed observer design to the study of fractional differential operators (see [20–28]).

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Saida Zenfari *obtained her bachelor degree in applied mathematics from Chouaib Doukkali University, Morocco, in 2015. She received her Ph.D in Applied mathematics from Chouaib Doukkali university on 2021. Her current research interests include linear and nonlinear control, port Hamiltonian systems and passivity based control.*
https://orcid.org/0000-0003-2715-0159

Mohamed Laabissi *received the "Doctorat de troisième cycle" degree in Mathematical Analysis from the Cadi Ayyad University of Marrakech, Morocco in 1995. He also received The "Doctorat d' Etat" degree in applied mathematics from the University Chouaib Doukkali of El Jadida, Morocco in 2001. He is currently a professor in the Department of mathematics at the University Chouaib Doukkali. His research interests are in infinite dimensional systems theory, positive systems, stabilization of semilinear systems, Analysis and control of tubular chemical reactors nonlinear models.*
https://orcid.org/0000-0001-8287-1034

Mohammed Elarbi Achhab *received the Doctorat de troisieme cycle degree in Applied Mathematics from the University Mohamed V of Rabat, Morocco in 1985 and the Ph.D.degree in engineering sciences (applied mathematics) from the University of Louvain,Belgium in 1993. From 1985 to 1989, he was Assistant Professor at the Department of Mathematics of the University of EL Jadida, Morocco. From 1990 to 1993, he was with the Department of Mechanical Engineering at the University of Louvain, Belgium. He has been a professor and a full professor of applied mathematics at the University Chouaib Doukkali of EL Jadida, Morocco, since 1993 and 1999, respectively. His current research interests include control theory, nonlinear systems, infinite dimensional systems, and applications to mechanical and chemical engineering problems.*
https://orcid.org/0000-0003-4361-4347